PHYSICS
Optics & Modern Physics

B.M. Sharma

CENGAGE Learning

Andover • Melbourne • Mexico City • Stamford, CT • Toronto • Hong Kong • New Delhi • Seoul • Singapore • Tokyo

CENGAGE
Learning

Physics for JEE:
Optics & Modern Physics

B.M. Sharma

ISBN-13: 978-81-315-1492-4
ISBN-10: 81-315-1492-7

Cengage Learning India Pvt. Ltd.
418, F.I.E., Patparganj
Delhi 110092

Cengage Learning is a leading provider of customized learning solutions with office locations around the globe, including Andover, Melbourne, Mexico City, Stamford (CT), Toronto, Hong Kong, New Delhi, Seoul, Singapore, and Tokyo. Locate your local office at: **www.cengage.com/global**

Cengage Learning products are represented in Canada by Nelson Education, Ltd.

For product information, visit **www.cengage.co.in**

Printed in India
First Impression 2012

Brief Contents

Contents

Preface

While the paper-setting pattern and assessment methodology have been revised many times over and newer criteria devised to help develop more aspirant-friendly engineering entrance tests, the need to standardize the selection processes and their outcomes at the national level has always been felt. A combined national-level engineering entrance examination has finally been proposed by the Ministry of Human Resource Development, Government of India. The Joint Entrance Examination (JEE) to India's prestigious engineering institutions (IITs, IIITs, NITs, ISM, IISERs, and other engineering colleges) aims to serve as a common national-level engineering entrance test, thereby eliminating the need for aspiring engineers to sit through multiple entrance tests.

While the methodology and scope of an engineering entrance test are prone to change, there are two basic objectives that any test needs to serve:

1. The objective to test an aspirant's caliber, aptitude, and attitude for the engineering field and profession.
2. The need to test an aspirant's grasp and understanding of the concepts of the subjects of study and their applicability at the grassroot level.

Students appearing for various engineering entrance examinations cannot bank solely on conventional shortcut measures to crack the entrance examination. Conventional techniques alone are not enough as most of the questions asked in the examination are based on concepts rather than on just formulae. Hence, it is necessary for students appearing for joint entrance examination to not only gain a thorough knowledge and understanding of the concepts but also develop problem-solving skills to be able to relate their understanding of the subject to real-life applications based on these concepts.

This series of books is designed to help students to get an all-round grasp of the subject so as to be able to make its useful application in all its contexts. It uses a right mix of fundamental principles and concepts, illustrations which highlight the application of these concepts, and exercises for practice. The objective of each book in this series is to help students develop their problem-solving skills/ accuracy, the ability to reach the crux of the matter, and the speed to get answers in limited time. These books feature all types of problems asked in the examination—be it MCQs (one or more than one correct), assertion-reason type, matching column type, comprehension type, or integer type questions. These problems have skillfully been set to help students develop a sound problem-solving methodology.

Not discounting the need for skilled and guided practice, the material in the books has been enriched with a number of fully solved concept application exercises so that every step in learning is ensured for the understanding and application of the subject. This whole series of books adopts a multi-faceted approach to mastering concepts by including a variety of exercises asked in the examination. A mix of questions helps stimulate and strengthen multi-dimensional problem-solving skills in an aspirant.

It is imperative to note that this book would be as profound and useful as you want it to be. Therefore, in order to get maximum benefit from this book, we recommend the following study plan for each chapter.

Step 1: Go through the entire opening discussion about the fundamentals and concepts.

Step 2: After learning the theory/concept, follow the illustrative examples to get an understanding of the theory/concept.

Overall the whole content of the book is an amalgamation of the theme of physics with ahead-of-time problems, which equips the students with the knowledge of the field and paves a confident path for them to accomplish success in the JEE.

With best wishes!

B.M. Sharma

CHAPTER 1

Geometrical Optics

INTRODUCTION

Light is a form of radiant energy, that is, energy emitted by excited atoms or molecules which can cause the sensation of vision in a normal human eye.

The branch of Physics which deals with the phenomena concerning light is called Optics. There are two branches of Optics:

a. **Geometrical Optics:** This consists the study of light in which light is considered as moving along a straight line as a ray. A ray of light gives the direction of propagation of light. When light meets a surface which separates two media, reflection and refraction take place. An image or an array of images may be formed due to this.

b. **Physical Optics:** It deals with the theories regarding the nature of light and provides an explanation for the different phenomena in light, such as reflection, refraction, interference, diffraction, polarisation, and rectilinear propagation.

SOME DEFINITIONS

(i) **A Ray:** The 'path' along which light travels is called a ray. They are represented by straight lines with arrows directed towards the direction of travel of light.

(ii) **A Beam:** A bundle of rays is called a beam. A beam is parallel when its rays are parallel; divergent when its rays spread out from a point; and convergent when its rays meet at a point.

(iii) **A Pencil:** A narrow beam is called a pencil of light.

NATURE OF OBJECTS AND IMAGES

Ray Optics primarily deals with determining the position and nature of the image formed when an object is placed in front of an optical element. Before we proceed further, let us clearly define what is an object and what is an image.

Types of Objects

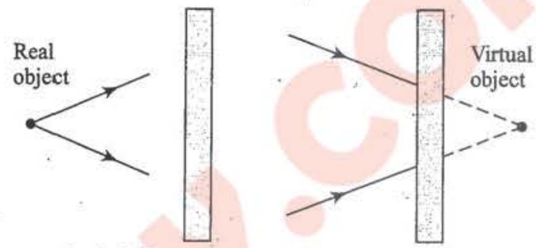
An object is a source of light rays that are incident on an optical element. An object may be a point object or an extended object. Since extended objects can be modeled as a collection of points, we only need to study point objects. Objects are of two kinds: Real objects and Virtual objects.

Real Object

An object is real if two or more incident rays actually emanate or seem to emanate from a point. Fig. 1.1(a) is a typical example of a real object. In this case, two rays emanate from the object and are incident on the optical element and the object is actually present. Hence, it is called a real object.

Virtual Object

Now, consider a converging set of rays as shown in Fig. 1.1(b). If not intercepted the rays will meet at a point. However, if the rays are intercepted by an optical element placed as shown in the figure, then the point of convergence is a virtual point behind the optical element. This point is called the virtual object for the optical element.



A real object placed in front of an optical element
(a)

A virtual object that appears to be behind the optical element
(b)

Fig. 1.1

So, an object is real when two rays emanate or diverge from a point, and an object is virtual when two incident rays seem to converge to that point.

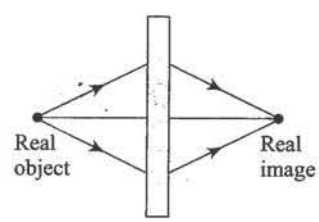
Types of Images

What is an Image?

An image is the point of convergence or apparent point of divergence of rays after they interact with a given optical element. An object provides rays that will be incident on an optical element. The optical element reflects or refracts the incident light rays which then meet at a point to form an image. As in the case of objects, images too can be real or virtual.

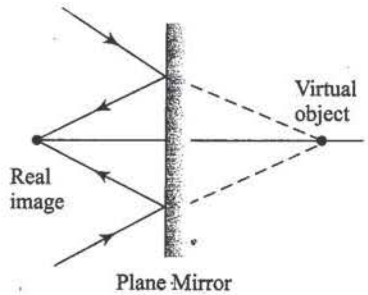
Real Image

Real images are formed when the reflected or refracted rays actually meet or converge to a point. If a screen is placed at that point, a bright spot will be visible on the screen. Thus, a real image can be captured on a screen. Examples of real images are shown in Fig. 1.2(a) and (b). Note that in the former the object is real while in the latter the object is virtual. Thus, both real and virtual objects can form real images.

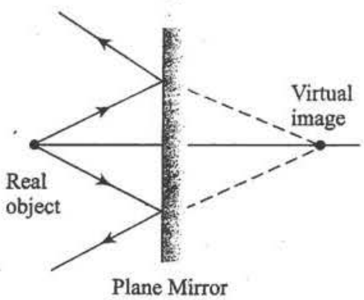


Optical element

A real image formed by a real object placed in front of the convex lens
(a)



A real image formed by a virtual object that appears to be behind the mirror
(b)



A virtual image formed by a real object in front of the mirror
(c)

Fig. 1.2

Virtual Image

When light rays, after interacting with the optical element, actually meet at a point the image formed is a real image. However, if the rays do not meet at a point but appear to emanate from a point, then a virtual image is formed. Consider the case of an object placed in front of a plane mirror. Here, the two reflected rays will never meet at any point. However, if we extend the reflected rays backwards, they appear to emanate from a point. This point is the virtual image of the object. As we will see later in the section on Lenses, it is possible for a virtual image to be formed by a virtual object as well. Thus, we can conclude that both virtual and real images can be formed by either real or virtual objects depending on the optical element.

BASIC LAWS

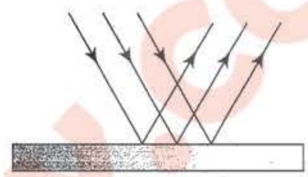
- **Law of Rectilinear Propagation of Light:** It states that light propagates in straight lines in homogeneous media.
- **Law of Independence of Light Rays:** It states that rays do not disturb each other upon intersection.
- **Law of Reversibility of Light Rays:** It states that rays retrace their paths when their direction is reversed.

REFLECTION OF LIGHT

When light rays strike the boundary of two media such as air and glass, a part of light is turned back into the same medium. This is called *Reflection of Light.*

Regular Reflection

When the reflection takes place from a perfect plane surface it is called *Regular Reflection* (see Fig. 1.3(a)). In this case, the reflected light has large intensity in one direction and negligibly small intensity in other directions.



(a) **Regular reflection**

Diffused Reflection

When the surface is rough, we do not get a regular behavior of light. Although at each point light ray gets reflected irrespective of the overall nature of surface, difference is observed because even in a narrow beam of light there are many rays which are reflected from different points of surface. It is quite possible that these rays may move in different directions due to irregularity of the surface. This process enables us to see an object from any position. Such a reflection is called as ***diffused reflection*** (See Fig. 1.3(b)). For example, reflection from a wall, from a newspaper, etc. This is why you cannot see your face in the newspaper and in the wall.

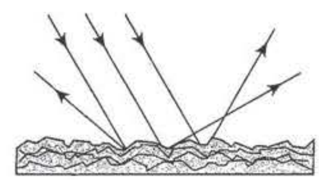


Fig. 1.3(b) Diffused reflection

Laws of Reflection

It has been found experimentally that rays undergoing reflection follow two laws called the Laws of Reflection:

(i) The incident ray, the reflected ray, and the normal at the point of incidence lie in the same plane. This plane is called the *plane of incidence* (or *plane of reflection*).

(ii) The ***angle of incidence*** (the angle between normal and the incident ray) and the ***angle of reflection*** (the angle between the reflected ray and the normal) are equal, i.e.,

$$\angle i = \angle r$$

Special Cases

Normal Incidence: In case light is incident normally [see Fig. 1.4(b)],

$$i = r = 0$$
$$\delta = 180°$$

Grazing Incidence: In case light strikes the reflecting surface tangentially [see Fig. 1.4(c)],

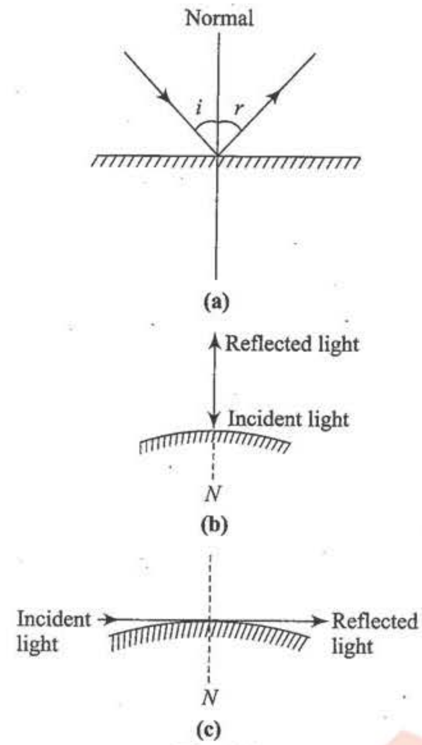


Fig. 1.4

$$i = r = 90^\circ$$
$$\delta = 0^\circ \quad \text{or} \quad 360^\circ$$

Illustration 1.1 **Show that for a light ray incident at an angle 'i' on getting reflected the angle of deviation is $\delta = \pi - 2i$ or $\pi + 2i$.**

Sol. From Fig. 1.5(b), it is clear that light ray bends either by δ_1 anticlockwise or by δ_2 $(= 2\pi - \delta_1)$ clockwise.

From Fig. 1.5(a), $\delta_1 = \pi - 2i$.

$$\therefore \qquad \delta_2 = 2\pi - (\pi - 2i) = \pi + 2i$$

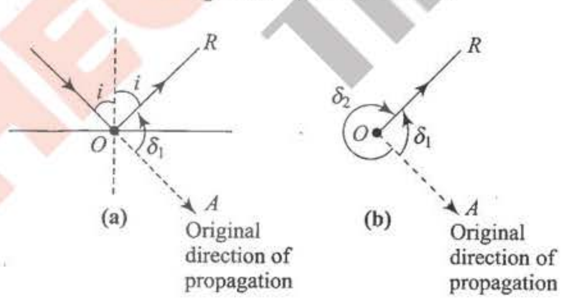


Fig. 1.5

REFLECTION FROM A PLANE SURFACE: PLANE MIRROR

A plane mirror is formed by polishing one surface of a plane thin glass plate (see Fig. 1.6). It is also said to be silvered on one side.

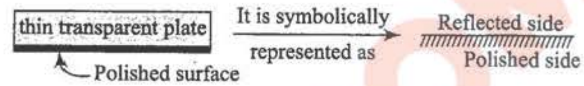


Fig. 1.6 Plane mirror

A beam of parallel rays of light, incident on a plane mirror, will get reflected as a beam of parallel reflected rays.

Illustration 1.2 **For a fixed incident light ray, if the mirror be rotated through an angle θ (about an axis which lies in the plane of mirror and perpendicular to the plane of incidence), show that the reflected ray turns through an angle 2θ in same sense.**

Sol. In Fig. 1.7, M_1, N_1 and R_1 indicate the initial positions of mirror, normal, and direction of reflected light ray, respectively. M_2, N_2 and R_2 indicate the final position, of mirror, normal, and direction of reflected light ray, respectively.

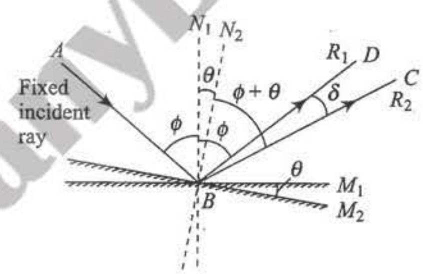


Fig. 1.7

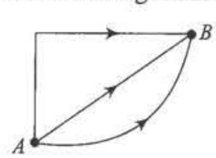
If the mirror is rotated by an angle θ, new angle of incidence $= \angle ABN_2 = \phi + \theta$

Angle of deviation of final ray $= \angle DBC = \delta$

$$\angle DBC = \angle ABC - \angle ABD = 2(\phi + \theta) - 2\phi = 2\theta$$

IMAGE FORMATION FROM PLAIN MIRROR

An explanation for the laws of reflection was provided by Fermat who postulated that a ray of light travels from point A to point B in a path that takes the shortest time. For example, if A and B are two points in the same medium as shown in Fig. 1.8, then the path with the shortest length is the straight line joining A and B. Thus, the light ray will travel in a straight line from A to B.



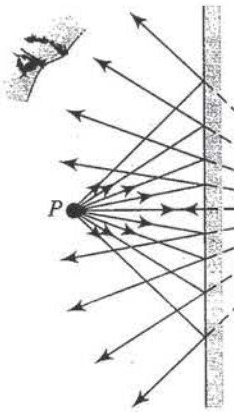
A ray of light can travel from point A to point B in multiple paths. The shortest path is the straight line joining them.

Fig. 1.8

Characteristics of image due to reflection by a plane mirror:

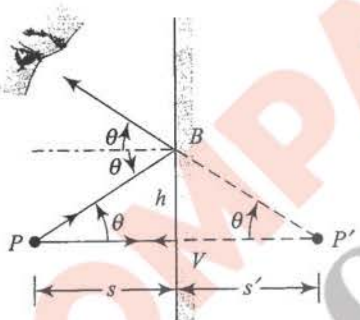
(i) Distance of object from mirror = Distance of image from the mirror.

All the incident rays from a point object after reflection from a plane mirror will meet at a single point which is called image [see Fig. 1.9(a)].



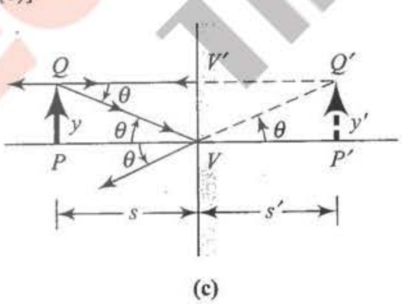
(a)

(ii) The line joining a point object and its image is normal to the reflecting surface [see Fig. 1.9(b)].



(b)

(iii) The size of the image is the same as that of the object [see Fig. 1.9(c)].

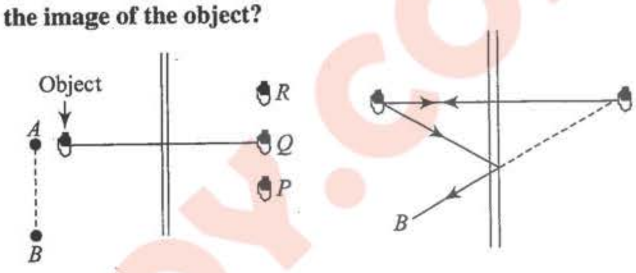


(c)

Fig. 1.9

(iv) For a real object the image is virtual and for a virtual object the image is real.

Illustration 1.3 **Figure 1.10(a) shows an object placed in front of a plane mirror. P, Q and R are the three positions where the image of object may be seen. Observer A is able to see the image at position Q. Where does the observer B see the image of the object?**



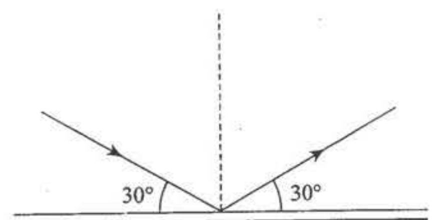
(a) (b)

Fig. 1.10

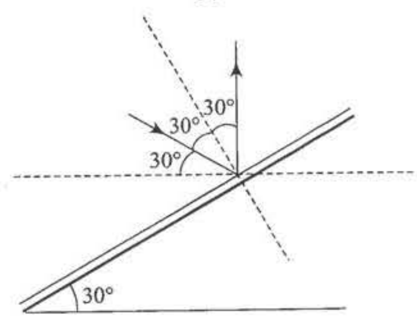
Sol. The observer B also observes the image of the object at the position Q which is explained by the ray diagram in Fig. 1.10(b). ***The position of image will be independent of the position of the observer.***

Illustration 1.4 **A light ray is incident on a plane mirror at an angle of 30° with the horizontal. At what angle with horizontal must a plane mirror be placed in its path so that it becomes vertically upwards after reflection?**

Sol. To make the reflected ray vertical, it should be rotated through an angle of 60°. So, the mirror should be tilted by 60°/2 = 30°, as shown in Fig. 1.11(b).



(a)



(b)

Fig. 1.11

Illustration 1.5 **Figure 1.12 shows a point object A and a plane mirror MN. Find the position of the image of object A,**

in mirror MN, by drawing ray diagram. Indicate the region in which observer's eye must be present in order to view the image. (This region is called field of view.)

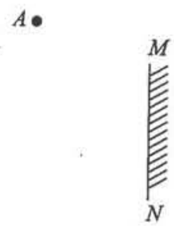


Fig. 1.12

Sol. Consider any two rays emanating from the object (see Fig. 1.13(a)). N_1 and N_2 are normals; $i_1 = r_1$ and $i_2 = r_2$.

The meeting point of reflected rays R_1 and R_2 is image A'. Though only two rays are considered it must be understood that all rays from A reflect from mirror MN such that their meeting point is A'. To obtain the region in which reflected rays are present, join A' with the ends of mirror and extend. Fig. 1.13(b) shows this region as shaded. In the figure, there are no reflected rays beyond the rays 1 and 2, therefore the observers P and Q will not be able to see the image because they do not receive any reflected ray.

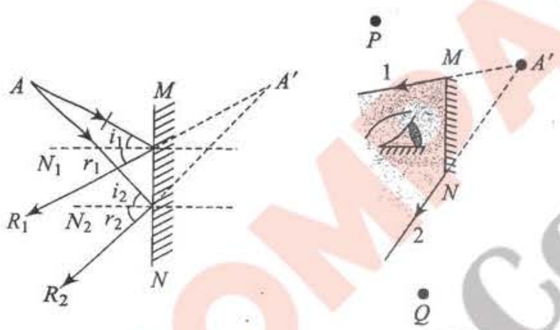
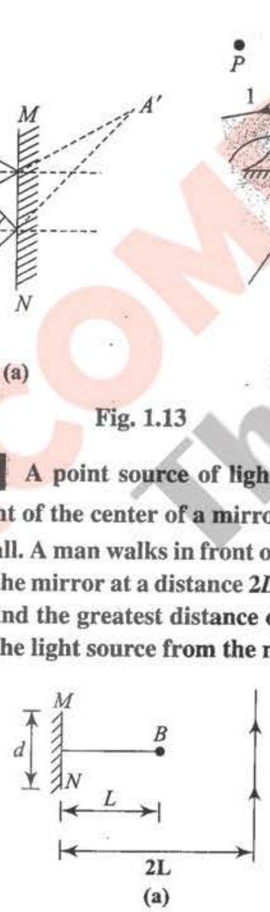


Fig. 1.13

Illustration 1.6 **A point source of light B is placed at a distance L in front of the center of a mirror of width d hung vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror at a distance $2L$ from it, as shown in Fig. 1.14(a). Find the greatest distance over which he can see the image of the light source from the mirror.**



Sol. Draw the rays BM and BN incident on the mirror edges, and draw the reflected rays (see Fig. 1.14(b)). From the figure, it is clear that the man can observe in the angular region $P'B'Q'$. The observer can see through the distance $P'Q'$, where reflected ray from B can meet the line.

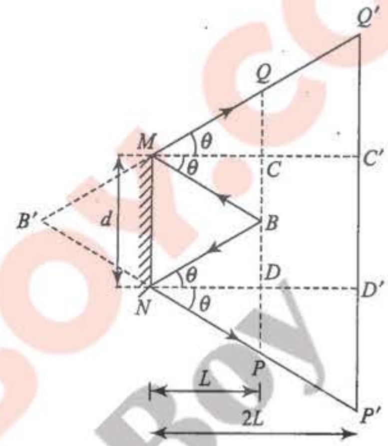


Fig. 1.14

From geometry, $CD = d = C'D'$

$BD = DP = d/2$; $CB = QC = d/2$

$P'Q' = C'D' + P'D' + Q'C'$

$$\Rightarrow \quad \frac{ND}{DP} = \frac{ND'}{D'P'} \Rightarrow \frac{L}{d/2} = \frac{2L}{D'P'} \Rightarrow D'P' = d$$

Similarly, $Q'C' = d$

Hence, distance through which the man can observe the image $= Q'C' + C'D' + D'P' = d + d + d = 3d$

IMAGE OF EXTENDED OBJECT FORMED BY PLANE MIRROR

An extended object like AB shown in Fig. 1.15 is a combination of infinite number of point objects from A to B. Image of every point object will be formed individually and thus infinite images will be formed. A' will be image of A, C' will be image of C, B' will be image of B, etc. All point images together form an extended image. Thus, extended image is formed of an extended object.

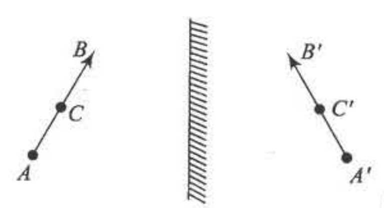
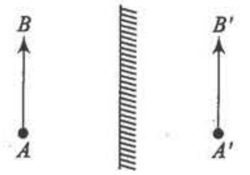


Fig. 1.15

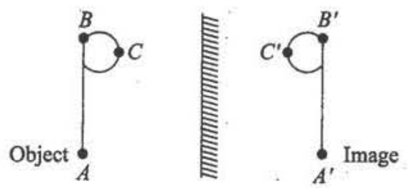
Properties of image of an extended object, formed by a plane mirror:

1. Size of extended object = size of extended image [see Fig. 1.16(a)].



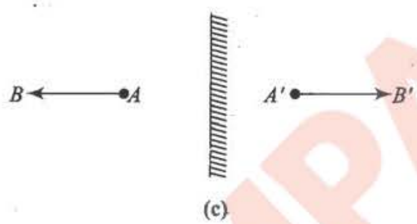
(a)

2. The image is upright, if the extended object is placed parallel to the plane mirror [see Fig. 1.16(b)].



(b)

3. The image is inverted, if the extended object lies perpendicular to the plane mirror [see Fig. 1.16(c)].

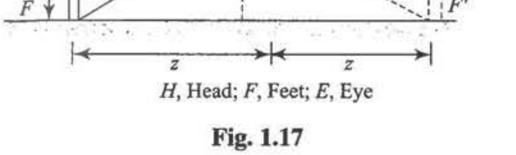


(c)

Fig. 1.16

Illustration 1.7 **Show that the minimum size of a plane mirror required to see the full image of an observer is half the size of that observer.**

Sol. Let HF is the height of the man and 'E' is the eye level. Draw the rays HM_1 and FM_2 incident at the edges of the mirror. Complete the ray diagram as shown in Fig. 1.17. It is self-explanatory if you consider lengths 'x' and 'y' as shown in the figure.



H, Head; F, Feet; E, Eye

Fig. 1.17

Aliter:

ΔEM_1M_2 and $\Delta EH'F'$ are similar

$$\therefore \qquad \frac{M_1M_2}{H'F'} = \frac{z}{2z} \quad \text{or} \quad M_1M_2 = H'F'/2 = HF/2$$

Note: ***The height of the mirror is half the height of eye as shown in the figure.***

RELATION BETWEEN VELOCITY OF OBJECT AND IMAGE

In case of plane mirror, distance of the object from the mirror is equal to distance of image from the mirror (see Fig. 1.18).

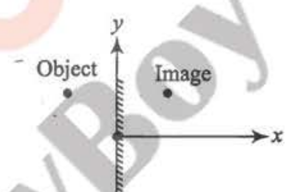


Fig. 1.18

Hence, from the mirror property:

$$x_{\text{im}} = -x_{\text{om}}, \quad y_{\text{im}} = y_{\text{om}} \quad \text{and} \quad z_{\text{im}} = z_{\text{om}}$$

Here, x_{im} means 'x' coordinate of image with respect to mirror. Similarly, others have meaning.

Differentiating w.r.t. time, we get

$$v_{(\text{im})x} = -v_{(\text{om})x}; \quad v_{(\text{im})y} = v_{(\text{om})y}; \quad v_{(\text{im})z} = v_{(\text{om})z},$$

Here, $\quad v_{\text{om}}$ = velocity of the object w.r.t. mirror

$\quad v_{\text{im}}$ = velocity of the image w.r.t. mirror

$$\Rightarrow \qquad v_{\text{i}} - v_{\text{m}} = -(v_{\text{o}} - v_{\text{m}}) \qquad \text{(for } x\text{-axis)}$$

In the direction normal to the mirror = | Relative velocity of image w.r.t. mirror | = | Relative velocity of object w.r.t. mirror |

But $\qquad v_{\text{i}} - v_{\text{m}} = (v_{\text{o}} - v_{\text{m}})$

or $\qquad v_{\text{i}} = v_{\text{o}} \qquad$ for y- and z-axis.

Here, $\quad v_{\text{i}}$ = velocity of image with respect to ground

$\quad v_{\text{o}}$ = velocity of object w.r.t. ground

Velocity of object is equal to velocity of image parallel to mirror surface.

We can write

$$\vec{v}_{\text{om}} = \vec{v}_{\text{o}} - \vec{v}_{\text{m}} \quad \text{and} \quad \vec{v}_{\text{im}} = \vec{v}_{\text{i}} - \vec{v}_{\text{m}}$$

Illustration 1.8 **Figure 1.19 shows a plane mirror and an object that are moving towards each other. Find the velocity of image.**

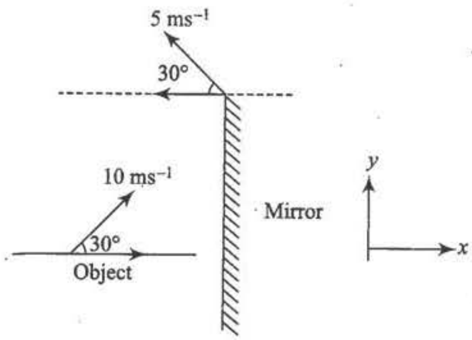


Fig. 1.19

Sol. Along x-direction, applying (relative velocity of image w.r.t. mirror) = – (relative velocity of object w.r.t. mirror).

$\Rightarrow \quad v_i - v_m = -(v_0 - v_m)$

$\Rightarrow \quad v_i - (-5\cos 30°) = -(10\cos 60° - (-5\cos 30°))$

$\therefore \quad v_i = -5(1+\sqrt{3})\ \text{ms}^{-1}$

In the direction parallel to the surface of mirror:

Along y-direction, $v_0 = v_i$

$\therefore \quad v_i = 10\sin 60° = 5\ \text{ms}^{-1}$

$\therefore$ Velocity of the image $= -5(1+\sqrt{3})\,\hat{i} + 5\hat{j}\ \text{ms}^{-1}$.

IMAGES FORMED BY TWO PLANE MIRRORS

If rays after getting reflected from one mirror strike second mirror, the image formed by first mirror will act as an object for second mirror, and this process will continue for every successive reflection. Let us understand this with the illustration discussed below.

Illustration 1.9 **Figure 1.20 shows a point object placed between two parallel mirrors. Its distance from M_1 is 2 cm and that from M_2 is 8 cm. Find the distance of images from the two mirrors considering reflection on mirror M_1 first.**

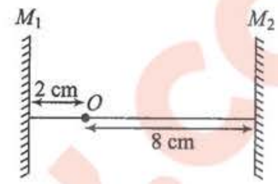


Fig. 1.20

Sol. To understand how images are formed see Fig. 1.21 and the following table. The image formed from the object will act as an object for next reflection. This process will continue again and again upto infinite reflection. You will require to know what symbols like I_{121} stands for. See the following diagram.

Similarly, images will be formed by the rays striking mirror M_2 first. Total number of images = ∞.

Incident rays	Reflected by	Reflected rays	Object	Image	Object distance (cm)	Image distance (cm)
Rays 1	M_1	Rays 2	O	I_1	$AO = 2$	$AI_1 = 2$
Rays 2	M_2	Rays 3	I_1	I_{12}	$BI_1 = 12$	$BI_{12} = 12$
Rays 3	M_1	Rays 4	I_{12}	I_{121}	$AI_{12} = 22$	$AI_{121} = 22$
Rays 4	M_2	Rays 5	I_{121}	I_{1212}	$BI_{121} = 32$	$BI_{1212} = 32$

And so on ...

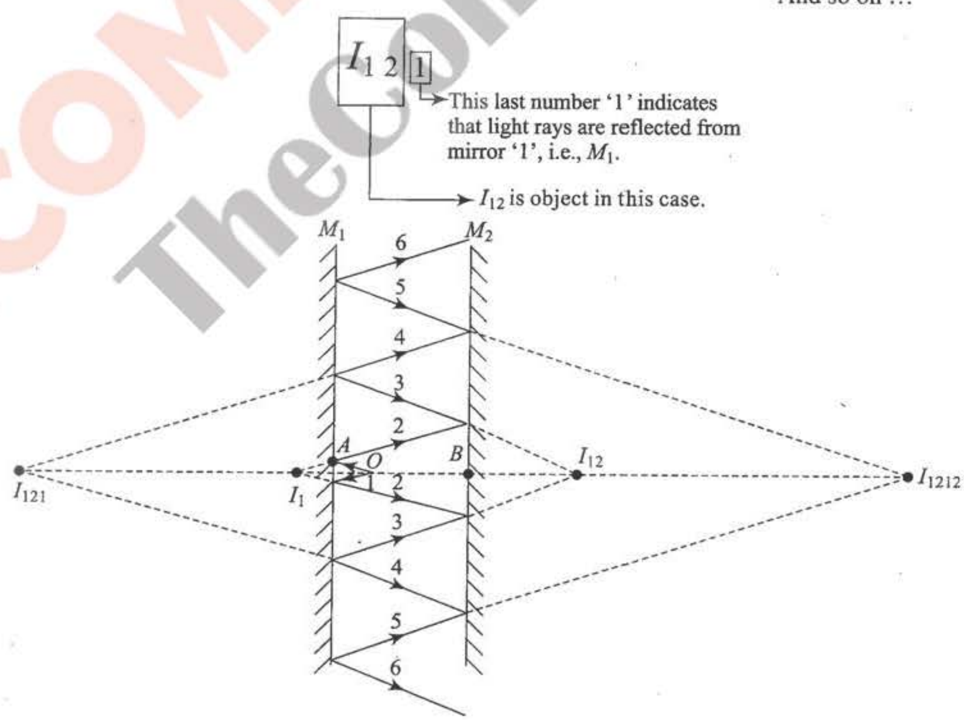


Fig. 1.21

LOCATING ALL THE IMAGES FORMED BY TWO PLANE MIRRORS

Consider two plane mirrors M_1 and M_2 inclined at an angle $\theta = \alpha + \beta$ as shown in Fig. 1.22.

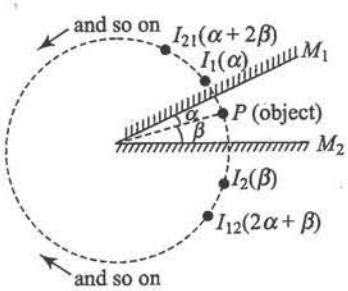


Fig. 1.22

Point P is an object kept such that it makes angle α with mirror M_1 and angle β with mirror M_2. Image of object P formed by M_1, denoted by I_1, will be inclined by angle α on the other side of mirror M_1. This angle is written in bracket in the figure besides I_1. Similarly image of object P formed by M_2, denoted by I_2, will be inclined by angle β on the other side of mirror M_2. This angle is written in bracket in the figure besides I_2.

Now, I_2 will act as an object for M_1 which is at an angle $(\alpha + 2\beta)$ from M_1. Its image will be formed at an angle $(\alpha + 2\beta)$ on the opposite side of M_1. This image will be denoted as I_{21} and so on. Think when will this process stop. [*Hint:* The virtual image formed by a plane mirror must not be in front of the mirror or its extension.]

Number of images formed by two inclined mirrors:

(i) If $\dfrac{360°}{\theta}$ = even number; number of images $= \dfrac{360°}{\theta} - 1$

(ii) If $\dfrac{360°}{\theta}$ = odd number; number of images $= \dfrac{360°}{\theta} - 1$, if the object is placed on the angle bisector.

(iii) If $\dfrac{360°}{\theta}$ = odd number; number of images $= \dfrac{360°}{\theta}$, if the object is not placed on the angle bisector.

(iv) If $\dfrac{360°}{\theta} \neq$ integer, then count the number of images as explained above.

Illustration 1.10 **Consider two perpendicular mirrors M_1 and M_2 and a point object O. Taking origin at the point of intersection of the mirrors and the coordinates of object as (x, y), find the position and number of images.**

Sol. As shown in Fig. 1.23, rays 'a' and 'b' strike mirror M_1 only and these rays will form image I_1 at $(x, -y)$, such that O and I_1 are equidistant from mirror M_1. These rays do not form further image because they do not strike any mirror again. Similarly, rays 'd' and 'e' strike mirror M_2 only and these rays will form image I_2 at $(-x, y)$, such that O and I_2 are equidistant from mirror M_2.

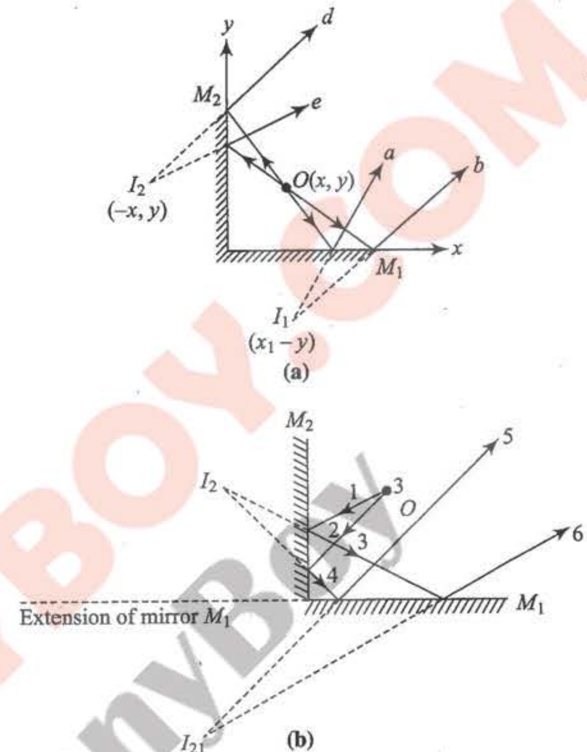


Fig. 1.23

Now, consider those rays which strike mirror M_2 first and then the mirror M_1.

For incident rays 1, 2 object is O, and reflected rays 3, 4 form image I_2.

Now, rays 3, 4 incident on M_1 (object is I_2) reflect as rays 5, 6 and form image I_{21}. Rays 5, 6 do not strike any mirror, so image formation stops.

I_2 and I_{21} are equidistant from M_1. To summarize, see Fig. 1.24.

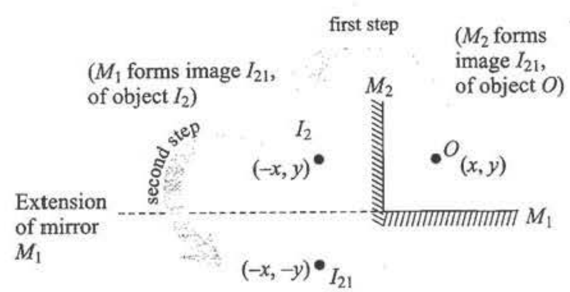


Fig. 1.24

For rays reflecting first from M_1 and then from M_2, first image I_1 (at $(x, -y)$) will be formed. I_1 will act as object for mirror M_2 and then its image I_{12} (at $(-x, -y)$) will be formed. I_{12} and I_{21} coincide.

Hence, ***three images are formed.***

Illustration 1.11 **Two mirrors are inclined at an angle of 30°. An object is placed making 10° with the mirror M_1. Find the positions of first two images formed by each mirror. Find the total number of images using (i) direct formula and (ii) counting the images.**

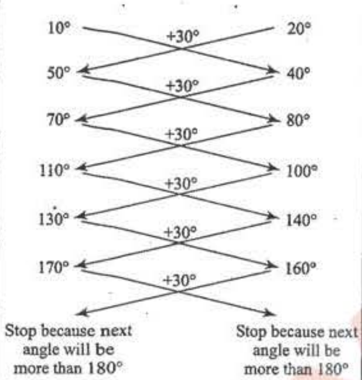
Sol. Number of images:

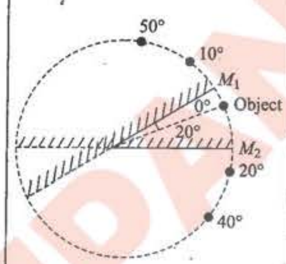
(i) Using direct formula: $360°/30° = 12$ (even number)

Therefore, number of images = $12 - 1 = 11$

(ii) By counting: See the following table

Image formed by mirror M_1 (angles are measured from the mirror M_1)	Image formed by mirror M_2 (angles are measured from the mirror M_2)	





To check whether the final images formed by the two mirrors coincide or not: add the last angles and the angle between the mirrors. If it comes out to be exactly 360°, it implies that the final images formed by the two mirrors coincide. Here, last angles made by the mirrors + the angles between the mirrors = $160° + 170° + 30° = 360°$. Therefore, in this case the final images coincide.

Therefore, the number of images = number of images formed by mirror M_1 + number of images formed by mirror $M_2 - 1$ (as the final images coincide) = $6 + 6 - 1 = 11$.

Concept Application Exercise 1.1

1. Two plane mirrors M_1 and M_2 are inclined at angle as shown in Fig. 1.25. A ray of light 1, which is parallel to M_1, strikes M_2 and after two reflections, the ray 2 becomes parallel to M_2. Find the angle θ.

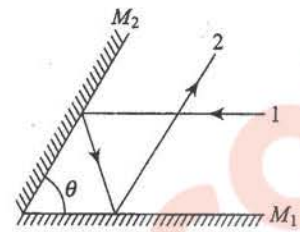


Fig. 1.25

2. Can we project the image formed by a plane mirror on to a screen? Give reasons.
3. Real object means that the object is actually present at the point where the incident ray originates. (True/ False)
4. Two plane mirrors are inclined at an angle of 60° as shown in Fig. 1.26. A ray of light parallel to M_1 strikes M_2. At what angle will the ray finally emerge?

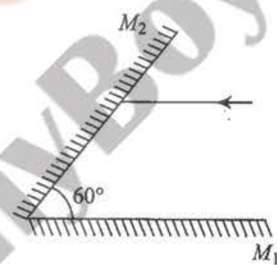


Fig. 1.26

5. Figure 1.27 shows two rays A and B being reflected by a mirror and going as A' and B'. The mirror

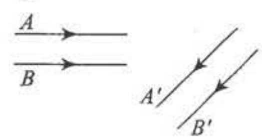


Fig. 1.27

a. is plane **b.** is convex
c. is concave **d.** may be any spherical mirror

6. Two plane mirrors are placed parallel to each other. The distance between the mirrors is 10 cm. An object is placed between the mirrors at a distance of 4 cm from one of them, say M_1. What is the distance between the first image formed at M_1 and the second image formed at M_2?
7. A ray of light travels from a light source S to an observer after reflection from a plane mirror. If the source rotates in the clockwise direction by 10°, by what angle and in what direction must the mirror be rotated so that the light ray still strikes the observer?

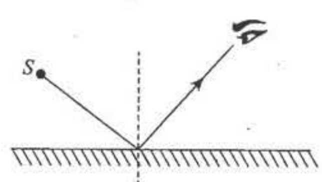


Fig. 1.28

8. Determine image location for the object in Fig. 1.29.

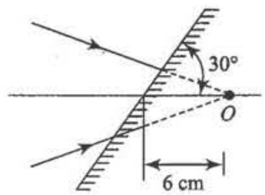


Fig. 1.29

9. Find the region on Y-axis in which reflected rays are present. Object is at A (2, 0) and MN is a plane mirror, as shown in Fig. 1.30.

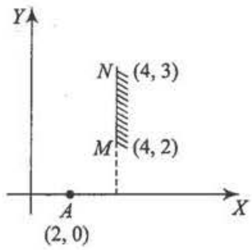


Fig. 1.30

10. An object moves with 5 ms^{-1} toward right while the mirror moves with 1 ms^{-1} toward the left as shown in Fig. 1.33. Find the velocity of image.

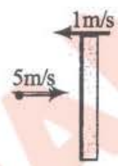


Fig. 1.31

11. There is a point object and a plane mirror. If the mirror is moved by 10 cm away from the object, find the distance which the image will move.
12. A man is standing at distance x from a plane mirror in front of him. He wants to see the entire wall in mirror which is at distance y behind the man. Find the minimum size of the mirror required.
13. Find the velocity of the image when the object and mirror both are moving towards each other with velocities 2 and 3 ms^{-1}. How are they moving?
14. In Fig. 1.32, a plane mirror is moving with a uniform speed of 5 ms^{-1} along negative x-direction and observer O is moving with a velocity of 10 ms^{-1}. What is the velocity of image of a particle P, moving with a velocity as shown in the figure, as observed by observer O? Also find its direction.

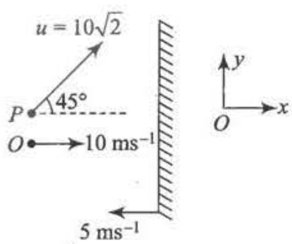


Fig. 1.32

REFLECTION FROM A CURVED SURFACE

Spherical Mirrors

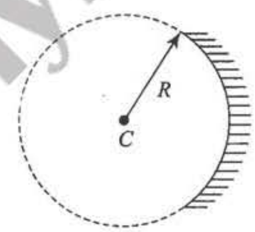
A spherical mirror is formed by polishing one surface of a part of sphere. A spherical mirror is a reflecting surface whose shape is a section of a spherical surface (see Fig. 1.33).

Spherical mirror

Fig. 1.33

Depending upon which part is shining, the spherical mirror is classified as:

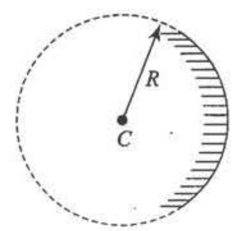
a. Concave Mirror: If the inside surface of the mirror is polished, it is a concave mirror (see Fig. 1.34).



Concave mirror

Fig. 1.34

b. Convex Mirror: If the outside surface of the mirror is polished, it is a convex mirror (see Fig. 1.35).



Convex mirror

Fig. 1.35

IMPORTANT TERMS

Pole (P): It is the geometrical center of the spherical reflecting surface (see Fig. 1.36).

A point on the surface of the mirror from where the position of the object can be specified easily is called pole. The pole is generally taken at the mid point of reflecting surface.

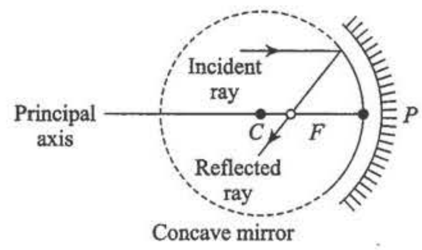


Fig. 1.36

Center of Curvature and Radius of Curvature: The center of the sphere of which the mirror is a part, is called *center of curvature(C)* (see Figs. 1.37, 1.38). The radius of the sphere of which the mirror is a part is called *radius of curvature(R)*. A plane mirror can be treated as a special case of a spherical mirror: one which has an *infinite radius of curvature*.

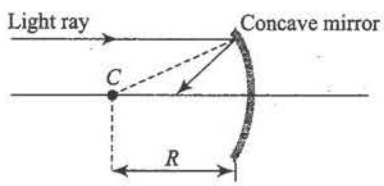


Fig. 1.37

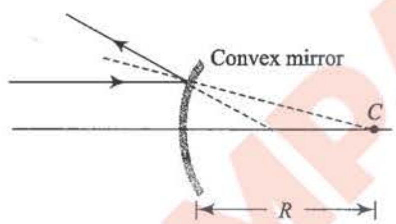


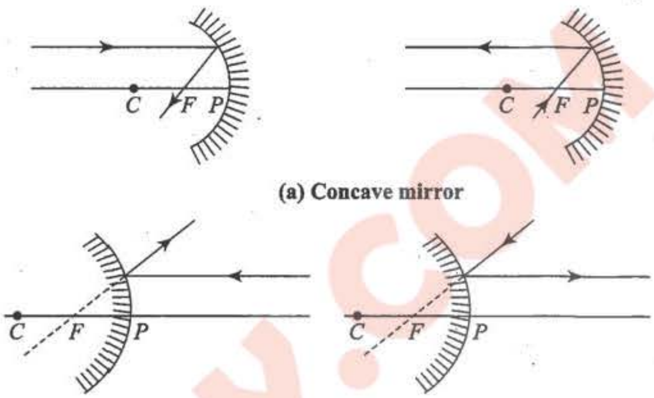
Fig. 1.38

Principal Axis: It is the straight line joining the center of curvature to the pole.

Principal Focus (*F*): It is the point of intersection of all the reflected rays for which the incident rays strike the mirror (with small aperture) parallel to the principal axis. In a concave mirror it is real and in a convex mirror it is virtual. The distance from pole to focus is called *focal length*.

Aperture (related to the size of mirror): It is the diameter of the mirror.

Focus (*F*): When a narrow beam of rays of light, parallel to the principal axis and close to it, is incident on the surface of a mirror, the reflected beam is found to coverage to or appears to diverge from a point on the principal axis. This point is called the focus (see Fig. 1.39).



(a) Concave mirror

(b) Convex mirror

Fig. 1.39

Focal Length (*f*): It is the distance between the pole and the principal focus. For spherical mirrors, $F = R/2$.

Illustration 1.12 **Find the angle of incidence of the ray shown in Fig. 1.40 for which it passes through the pole, given that $MI \parallel CP$.**

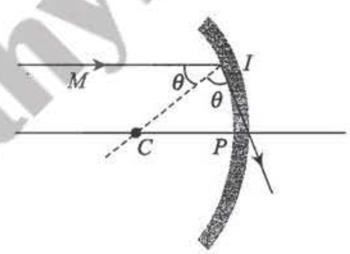


Fig. 1.40

Sol. From the figure, the ray MI is making an angle θ with normal IC. From law of reflection, $\angle MIC = \angle CIP = \theta$

As $\quad MI \parallel CP \Rightarrow \angle MIC = \angle ICP = \theta$

Now, $\quad CI = CP$

$\Rightarrow \quad \angle CIP = \angle CPI = \theta$

$\therefore \quad$ In ΔCIP, all angles are equal, i.e., $3\theta = 180°$

$\Rightarrow \quad \theta = 60°$

Illustration 1.13 **Find the distance CQ if incident light ray parallel to principal axis is incident at an angle i. Also, find the distance CQ if $i \to 0$.**

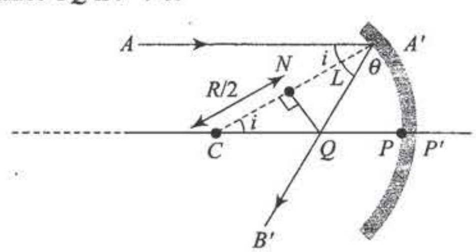


Fig. 1.41

Sol. In triangle CNQ, $\cos i = R/2CQ \Rightarrow CQ = R/2\cos i$

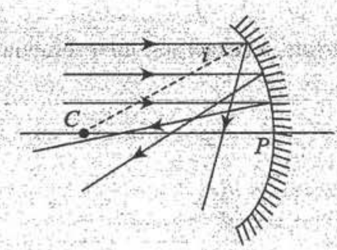
As i increases $\cos i$ decreases. Hence, CQ increases.

If i is a small angle, $\cos i \approx 1$ (see Fig. 1.42(b)).

$\therefore \quad CQ = R/2$

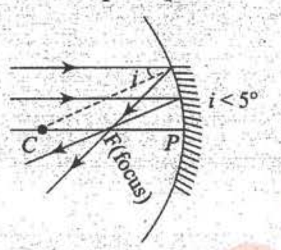
So, paraxial rays meet at a distance equal to $R/2$ from center of curvature, which is called focus.

Note:



(a)

(a) If angle of incident 'i' is more, the rays will focus at different points on the principal axis.



(b)

Fig. 1.42

(b) If angle of incident 'i' is small, the rays will focus at one point on principal axis. This the point is called Focus

SIGN CONVENTION: CARTESIAN CONVENTION

We will use following sign convention for problem solving in case of reflection as well as refraction.

- All distances are measured from the pole.
- Distances measured in the direction of incident rays are taken as positive.
- Distances measured in the direction opposite to that of the incident rays are taken as negative.
- Distances above the principal axis are taken as positive.
- Distances below the principal axis are taken as negative.

Angles measured from the normal in anticlockwise sense are positive, while that in clockwise sense are negative.

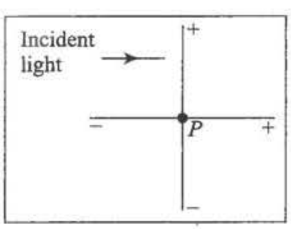
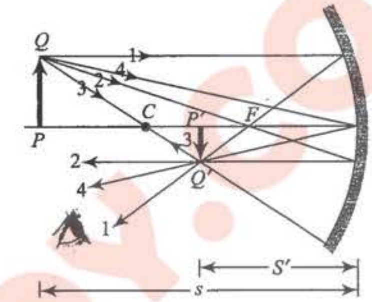


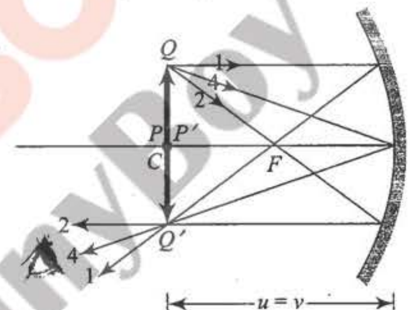
Fig. 1.43

RULES FOR RAY DIAGRAMS

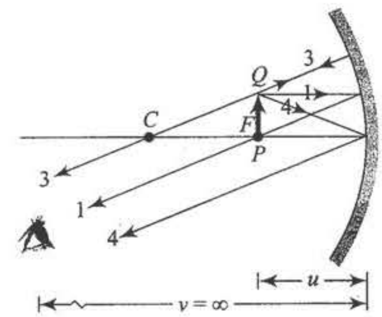
The position, size, and nature of the images formed by mirrors are conventionally expressed by ray diagrams (see Fig. 1.44).



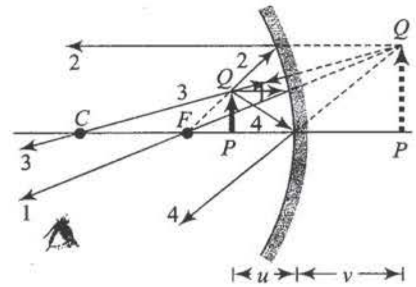
(a)



(b)



(c)



(d)

Fig. 1.44

We can locate the image of any extended object graphically by drawing any two of the following four special rays:

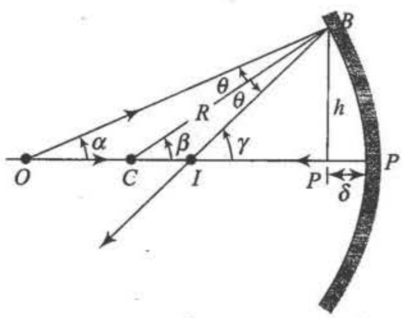
- A ray initially parallel to the principal axis is reflected through the focus of the mirror. (1)

- A ray passing through the center of curvature is reflected back along itself. (3)
- A ray initially passing through the focus is reflected parallel to the principal axis. (2)
- A ray incident at the pole is reflected symmetrically. (4)

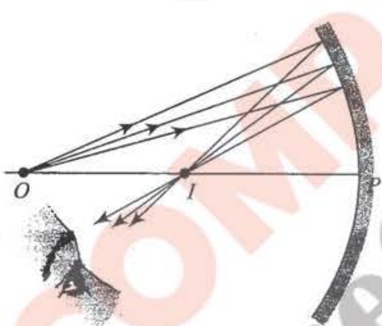
POSITION, SIZE AND NATURE OF IMAGE FORMED BY SPHERICAL MIRRORS

Mirror Formula

Consider Fig. 1.45(a) where O is a point object and I is the corresponding image.



(a)



(b)

Fig. 1.45

CB is normal to the mirror at B. By laws of reflection,

$$\angle OBC = \angle CBI = \theta$$

$$\alpha + \theta = \beta, \quad \beta + \theta = \gamma, \quad \alpha + \gamma = 2\beta$$

For small aperture of the mirror, α, β, γ

$$\Rightarrow \quad \alpha \approx \tan\alpha, \quad \beta \approx \tan\beta, \quad \gamma = \tan\gamma, \quad P' \to P$$

$$\Rightarrow \quad \tan\alpha + \tan\gamma = 2\tan\beta$$

$$\Rightarrow \quad \frac{BP'}{OP'} + \frac{BP'}{IP'} = 2\,\frac{BP'}{CP'}$$

Applying sign convention,

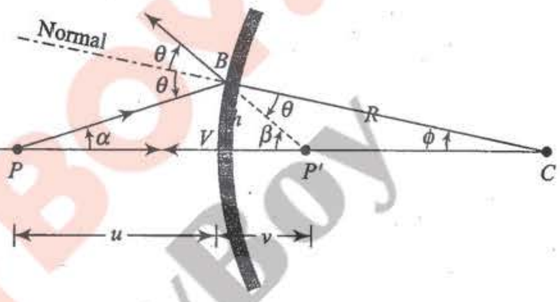
$$u = -OP, v = -IP, R = -CP$$

$$\Rightarrow \quad -\frac{1}{u} + \left(-\frac{1}{v}\right) = -\frac{2}{R}$$

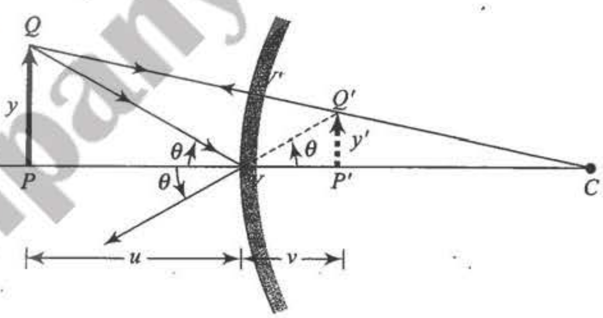
If $\quad u = \infty, \frac{1}{v} = \frac{2}{R}$, but by definition, if $u = \infty, v = f$.

Hence, $f = \frac{R}{2}$ and $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

For convex mirrors, an exactly similar formula emerges (see Fig. 1.46).



(a)



(b)

Fig. 1.46

Illustration 1.14 **When the position of an object reflected in a concave mirror of 0.25 m focal length is varied, the position of the image varies. Plot the image distance as a function of the object distance, taking the object distance from 0 to + ∞.**

Sol. Figure 1.47 shows the required graph.

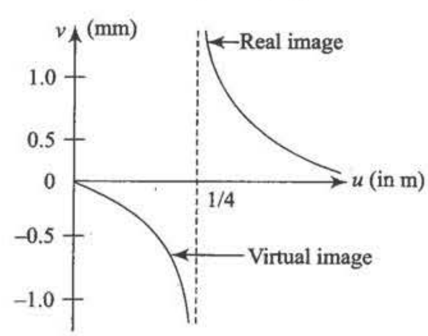


Fig. 1.47

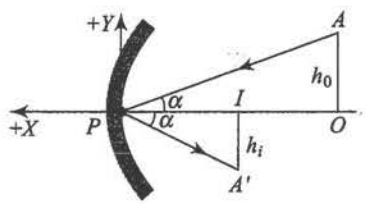
IMAGE FORMATION IN CONVEX MIRROR

Magnification

The lateral magnification is defined as the ratio

$$m_v = \frac{\text{height of image}}{\text{height of object}} = \frac{h_i}{h_0}$$

To compute the vertical magnification, consider the extended object OA shown in Fig. 1.48. The base of the object, O, will map on to a point I on the principal axis which can be determined from the equation $(1/u) + (1/v) = (1/f)$. The image of the top of the object, A, will map on to a point A' that will lie on the perpendicular through I. The exact location can be determined by drawing a ray from A passing through the pole and intercepting the line through I at A'.



A real image of an extended object in front of a concave mirror

Fig. 1.48

Consider the triangles APO and $A'PI$ in the figure. As the two triangles are similar, we get

$$\tan\alpha = \frac{AO}{PO} = \frac{A'I}{PI} \quad \text{or} \quad \frac{A'I}{AO} = \frac{PI}{PO}$$

Applying the sign convention, we get, $u = -PO$

$$v = -PI \;\Rightarrow\; h_0 = +AO \;\Rightarrow\; h_i = -A'I$$

Therefore, $$-\frac{h_i}{h_0} = \frac{v}{u} \quad \text{or} \quad m_v = \frac{h_i}{h_0} = -\frac{v}{u}$$

Magnification from a Concave Mirror

Figure 1.49 shows a concave mirror of focal length f_0 in front of which an object O is placed at a distance x from the pole P. According to Cartesian sign convention, the mirror formula may be modified as $u = -x; f = -f_0$. Thus,

$$\frac{1}{v} + \frac{1}{-x} = \frac{1}{-f_0} \quad \text{or} \quad v = \frac{xf_0}{f_0 - x}$$

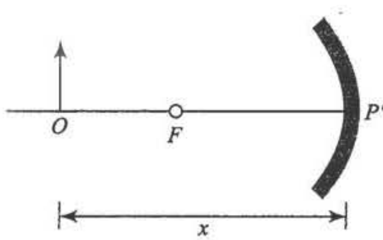


Fig. 1.49

And the magnification formula may be modified as

$$m = \frac{-v}{u} = \frac{y}{x} = \frac{f_0}{f_0 - x}$$

Magnification from a Convex Mirror

Figure 1.50 shows a convex mirror of focal length f_0 in front of which an object O is placed at a distance x from the pole P. According to Cartesian Sign Convention, the formulae may be modified as $u = -x$ and $f = +f_0$.

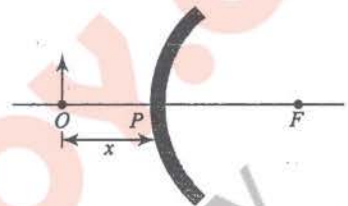


Fig. 1.50

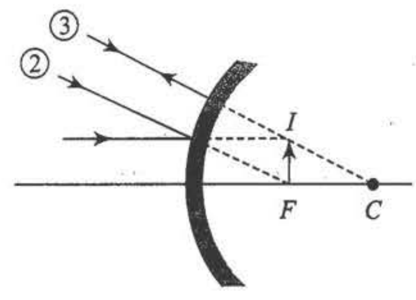
Thus, $$v = \frac{x f_0}{f_0 + x}.$$

The above expression shows that whatever may be the value of $x(|x| > f_0)$, v is always positive and its value is always less than or equal to f_0. The magnification formula may be modified as $m = f_0/(f_0 + x)$. It explains that m always lie between 0 and +1.

Nature of Image Formed by a Convex Mirror

(i) When the object is placed at infinity, a virtual, erect, and very diminished image is formed at the focus.

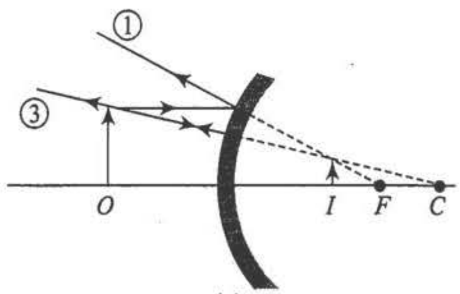
$$x = \infty; \quad v = f_0; \quad m << 1$$



(a)

(ii) When the object is placed in front of the convex mirror, a virtual, erect, and diminished image is formed between F and P.

$$0 < x < \infty; \quad v < f_0; \quad m < 1$$



(a)

Fig. 1.51

Nature of Image Formed by a Concave Mirror

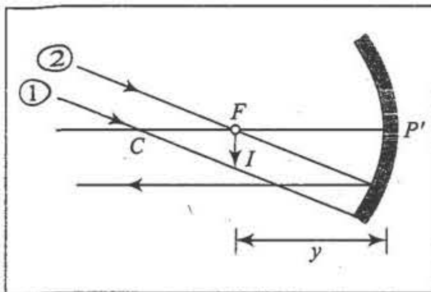
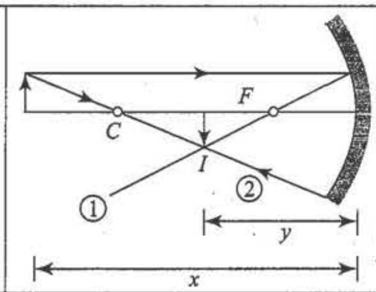
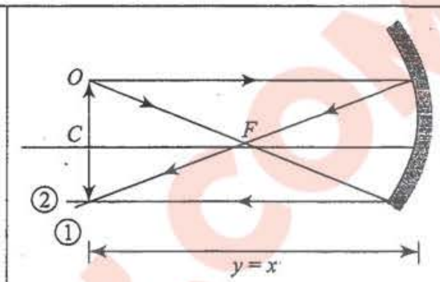
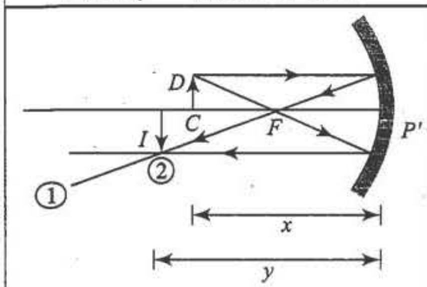
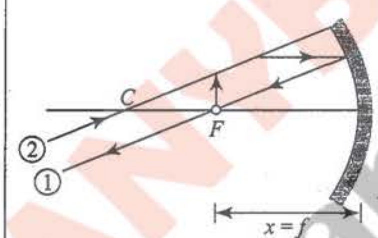
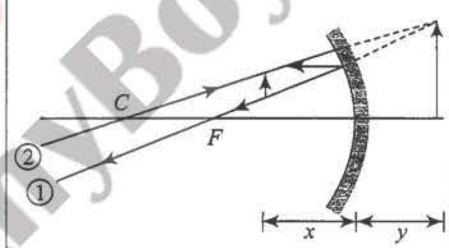
 	 	
(i) When the object is placed at infinity, a real, inverted, and very small image is formed at the focus. $x = \infty$ $v = -y$, where $y = f_0$ $m = -\delta$, where $\delta << 1$	**(ii)** When the object is placed beyond C ($2f_0 < x < \infty$), a real, inverted and diminished image is formed between F and C. $v = -y$, where $f_0 < y < 2f_0$ $m = -\delta$, where $0 < \delta < 1$	**(iii)** When the object is placed at C ($x = 2f_0$), a real, inverted, and equal size image is formed at C. $v = -y$, where $y = 2f_0$ $m = -\delta$ where $\delta = 1$
 	 	
(iv) When the object is placed between F and C ($f_0 < x < 2f_0$), an erect, real, inverted, and large image is formed beyond C. $v = -y$, where $y > 2f_0$ $m = -\delta$, where $\delta > 1$	**(v)** When the object is placed at focus F, a real, inverted, and very large image is formed at infinity. $v = -y$, where $y = \infty$ $m = -\delta$, where $\delta >> 1$	**(vi)** When the object is placed at C ($x = 2f_0$), a real, inverted, and equal size image is formed at C. $v = +y$ $m = +\delta$, where $\delta > 1$

Illustration 1.15 **Can a convex mirror form a real image! Explain.**

Sol. Yes, only when the object is virtual and is placed between F and P. Fig. 1.52 shows a convex mirror exposed to a converging beam which converges to a point that lies between F and P.

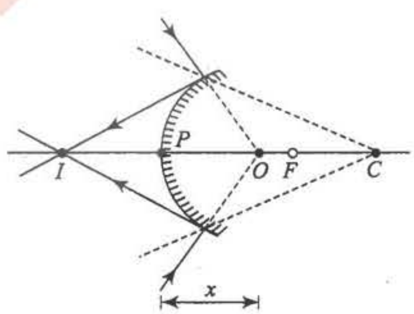


Fig. 1.52

$$v = \frac{-xf}{f_0 - x}$$; v becomes negative (real image) only when $x < f_0$.

Illustration 1.16 **An extended object is placed perpendicular to the principal axis of a concave mirror of radius of curvature 20 cm at a distance of 15 cm from the pole. Find the lateral magnification produced.**

Sol. Given $u = -15$ cm, $f = -10$ cm

Using $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, we get $v = -30$ cm

$$\therefore \quad m = -\frac{u}{v} = -2$$

Aliter: Using direct formula:

$$m = \frac{f}{f-u} = \frac{-10}{-10-(-15)} = -2$$

Illustration 1.17 **A person looks into a spherical mirror. The size of image of his face is twice the actual size of his face. If the face is at a distance of 20 cm, then find the radius of curvature of the mirror.**

Sol. The person will see his face only when the image is virtual. Virtual image of a real object is erect. Therefore,

$$m = 2$$

$$\therefore \quad \frac{-v}{u} = 2 \quad (\text{Here } u = -20 \text{ cm})$$

$$\Rightarrow \quad v = 40 \text{ cm}$$

Applying $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$; $f = -40$ cm or $R = 80$ cm.

Aliter: $$m = \frac{f}{f-u}$$

$$\Rightarrow \quad 2 = \frac{f}{f-(-20)}$$

$$\Rightarrow \quad f = -40 \text{ cm} \quad \text{or} \quad R = 80 \text{ cm}$$

Illustration 1.18 **An image of a candle on a screen is found to be double its size. When the candle is shifted by a distance of 5 cm, then the image becomes triple its size. Find the nature and radius of curvature of the mirror.**

Sol. Since the image is formed on the screen, it is real. Real object and real image implies concave mirror.

Applying $$m = \frac{f}{f-u} \quad \text{or} \quad -2 = \frac{f}{f-u} \qquad \text{(i)}$$

After shifting, $$-3 = \frac{f}{f-(u+5)} \qquad \text{(ii)}$$

Here, care should be taken that distance of the object becomes $u + 5$ not $u - 5$: In a concave mirror the size of real image will increase only when the real object is brought closer to the mirror. In doing so, its x-coordinate will increase.

From (i) and (ii), we get

$$f = -30 \text{ cm} \quad \text{or} \quad R = 60 \text{ cm}$$

Illustration 1.19 **A point object is placed 60 cm from the pole of a concave mirror of focal length 10 cm on the principal axis.**

Find:

a. **the position of image.**

b. **If the object is shifted 1 mm towards the mirror along principal axis, find the shift in image. Explain the result.**

Sol.

a. $u = -60$ cm

$f = -10$ cm

$$v = \frac{fu}{u-f} = \frac{-10(-60)}{-60-(-10)} = \frac{600}{-50} = -12 \text{ cm.}$$

b. $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

Differentiating, we get

$$dv = -\frac{v^2}{u^2}\, du = -\left(\frac{-12}{-60}\right)^2 [1 \text{ mm}] = -\frac{1}{25} \text{ mm}$$

[$\because$ $du = 1$ mm; sign of du is +ve because it is shifted in +ve direction defined by sign convention.]

(i) –ve sign of dv indicates that the image will shift towards negative direction.

(ii) The sign of v is negative. Which implies that the image is formed on the negative side of the pole. (i) and (ii) together imply that the image will shift away from the pole.

Note that differentials dv and du denote small changes only.

Illustration 1.20 **The distance between a real object and its image in a convex mirror of focal length 12 cm is 32 cm. Find the size of image if the object size is 1 cm.**

Sol. Let x and y be the magnitudes of object and image distances.

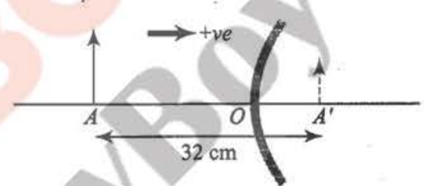


Fig. 1.53

We have $$AA' = 32 \text{ cm}$$

$$\Rightarrow \quad AO + A'O = 32 \text{ cm}$$

$$\Rightarrow \quad (x + y) = 32 \qquad \text{(i)}$$

And also, $$u = -x, \; v = +y$$

$$\frac{1}{-x} + \frac{1}{y} = \frac{1}{+12} \qquad \text{(ii)}$$

Solving (i) and (ii) simultaneously, we can get u and v.

The relevant answers are $u = -24$ cm, $v = +8$ cm

Using $$I = -1\left(\frac{+8}{-24}\right) = +\frac{1}{3}$$

So, the image size is 1/3 cm.

Illustration 1.21 **A plane mirror is placed at a distance of 50 cm from a concave mirror of focal length 16 cm. Where should a short object be placed between the mirrors and facing both the mirrors so that its virtual image in the plane mirror coincides with the real image in concave mirror? What is the ratio of the sizes of the two images?**

Sol. Let the object be at a distance x from the plane mirror.

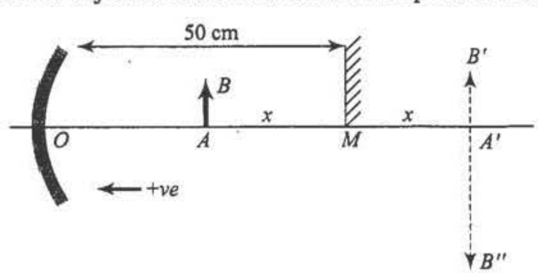


Fig. 1.54

The distance of object from concave mirror = $u = 50 - x$
For the plane mirror, object and image distances are equal.

$$A'M = AM = x$$

$$\Rightarrow \qquad OA' = OM + A'M = 50 + x$$

For the concave mirror, $v = 50 + x$

Using $u = -(50 - x)$, $v = -(50 + x)$, $f = -16$ cm

$$\frac{1}{50-x} + \frac{1}{50+x} = \frac{1}{16}$$

$$\frac{100}{50^2 - x^2} = \frac{1}{16} \quad \Rightarrow \quad x = 30 \text{ cm}$$

The object must be placed at a distance of 30 cm from the plane mirror.

The ratio of image sizes = $\dfrac{A'B'}{A'B''} = \dfrac{AB}{A'B''} = \dfrac{u}{v} = \dfrac{50+x}{50-x} = \dfrac{1}{4}$

The real image formed by the concave mirror is 4 times the size of virtual image formed by the plane mirror.

Illustration 1.22 **Two concave mirrors are placed 40 cm apart and are facing each other. A point object lies between them at a distance of 12 cm from the mirror of focal length 10 cm. The other mirror has a focal length of 15 cm. Find the location of final image formed after two reflections— first at the mirror nearer to the object and second at the other mirror.**

Sol.

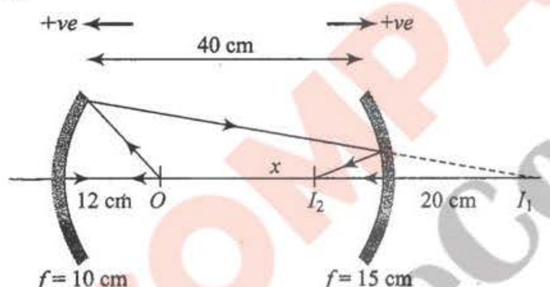


Fig. 1.55

First Reflection:
As incident rays are directed toward left, all quantities to the left of plane are taken positive.

$$u_1 = -12 \text{ cm}, \quad f_1 = -10 \text{ cm}$$

$$v_1 = \frac{u_1 f_1}{u_1 - f_1} = \frac{120}{-12+10} = -60 \text{ cm}$$

The rays after reflection are going to converge at a point $I_1 = 60$ cm from the first mirror, i.e., $60 - 40 = 20$ cm behind the other mirror.

Second Reflection:
As incident rays are directed toward right, all quantities towards right will be taken positive.

The converging rays falling at the second mirror create a virtual object for this mirror at I_1 with object distance

$$u_2 = +20 \text{ cm} \quad \text{and} \quad f_2 = -15 \text{ cm},$$

$$v_2 = \frac{u_2 f_2}{u_2 - f_2} = \frac{-20 \times 15}{+20+15} = \frac{-300}{35} = -8.57 \text{ cm}$$

As right is positive, image will be formed to the left of mirror. Hence, the final image is formed at a distance of 8.57 cm from the second mirror and is real.

Illustration 1.23 **Figure 1.56 shows a spherical concave mirror with its pole at (0, 0) and principal axis along x-axis. There is a point object at (–40 cm, 1 cm), find the position of image.**

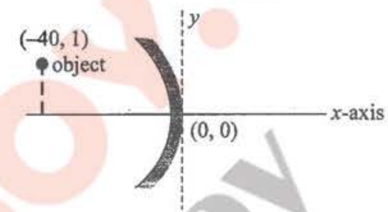


Fig. 1.56

Sol. According to sign convention,

$$u = -40 \text{ cm}$$
$$h_1 = +1 \text{ cm}$$
$$f = -5 \text{ cm}$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} + \frac{1}{-40} = \frac{1}{-5}; \quad \Rightarrow \quad v = \frac{-40}{7} \text{ cm}$$

$$\frac{h_2}{h_1} = \frac{-v}{u}$$

$$\Rightarrow \qquad h_2 = -\frac{-v}{u} \times h_1 = \frac{-\left(\frac{-40}{7}\right) \times 1}{-40} = -\frac{1}{7} \text{ cm.}$$

$\therefore$ The position of image is $\left(\dfrac{-40}{7} \text{ cm}, -\dfrac{1}{7} \text{ cm}\right)$.

Illustration 1.24 **A thin rod of length $f/3$ is placed along the optical axis of a concave mirror of focal length f such that its image which is real and elongated just touches the rod. Calculate the magnification.** **(IIT-JEE, 1991)**

Sol. As in question, image touches the rod, i.e., image and object coincides, hence one end of the rod should be at the center of curvature. It is also written that image is enlarged, it indicates that the orientation of rod should be toward focus then only we can get enlarged image along the principal axis. Let l be the length of the image.

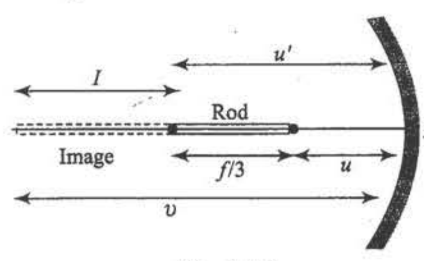


Fig. 1.57

Then, $m = \dfrac{l}{f/3} \Rightarrow l = \dfrac{mf}{3}$

Also, one end of the image coincides with the object, $u' = 2f$.

Now, $u' = u + \dfrac{f}{3} \Rightarrow u = 2f - \dfrac{f}{3} = \dfrac{5f}{3}$

$$v = -\left(u + \frac{f}{3} + \frac{mf}{3}\right).$$

Putting in mirror formula, we get

$$\frac{1}{u + f/3 + mf/3} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{3}{5f + f + mf} + \frac{3}{5f} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{m+6} = \frac{2}{15} \Rightarrow m = \frac{3}{2}$$

Illustration 1.25 **A concave mirror and a convex mirror of focal lengths 10 cm and 15 cm are placed at a distance of 70 cm. An object *AB* of height 2 cm is placed at a distance of 30 cm from the concave mirror. First ray is incident on the concave mirror then on the convex mirror. Find size, position, and nature of the image.**

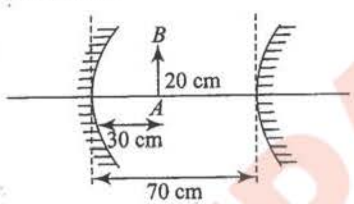


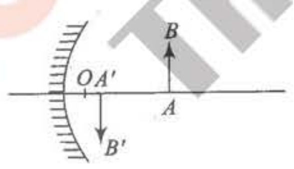
Fig. 1.58

Sol. For concave mirror,

$$u = -30 \text{ cm}, \quad f = -10 \text{ cm}$$

Using $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f} \Rightarrow \dfrac{1}{v} - \dfrac{1}{30} = \dfrac{-1}{10} \Rightarrow v = -15 \text{ cm}$

Now, $\dfrac{A'B'}{AB} = \dfrac{-v}{u} = \dfrac{(-15)}{(-30)} \Rightarrow A'B' = -1 \text{ cm}$



(a)

Image formed by first reflection will be real, inverted, and diminished.

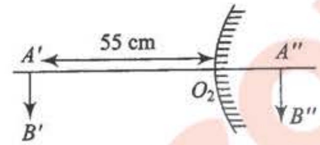
For convex mirror, the image formed by concave mirror will act as object for convex mirror. Now object distance for convex mirror

$$O_2A' = 70 - 15 = 55 \text{ cm}$$

$$u' = -55 \text{ cm}, f' = +15 \text{ cm}$$

Using $\dfrac{1}{v'} + \dfrac{1}{u'} = \dfrac{1}{f'} \Rightarrow \dfrac{1}{v'} - \dfrac{1}{55} = \dfrac{1}{15}$

$$\Rightarrow \quad v' = \frac{165}{14} \text{ cm}$$



(b)

Fig. 1.59

Now, $\dfrac{A''B''}{A'B'} = -\dfrac{v'}{u'} = -\dfrac{\left(+\dfrac{165}{14}\right)}{(-55)}$

$$\Rightarrow \quad A''B'' = \left(-\frac{3}{14}\right)(-1) = -0.2 \text{ cm}$$

Final image will be virtual, inverted, and diminished.

Relation Between Object and Image Velocity

Case (i). Differentiate equation $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{v}$ with respect to time.

$$\Rightarrow \quad -\frac{1}{v^2}\frac{dv}{dt} - \frac{1}{u^2}\frac{du}{dt} = 0$$

$$\Rightarrow \quad -\frac{1}{v^2}V_{im} - \frac{1}{u^2}V_{OM} = 0$$

$\dfrac{dv}{dt} = v_{im}$ =velocity of image w.r.t. mirror

$$\Rightarrow \quad V_{im} = -\frac{v^2}{u^2}V_{OM};$$

$\dfrac{du}{dt} = v_{OM}$ =velocity of object w.r.t. mirror

$$V_{lm} = -m^2 V_{OM}$$

The negative sign shows that if u is decreasing, v will increase, i.e., if real object approaches the mirror, its real image will recede from the mirror.

In this case, $|m| = 1$, hence $\left|\dfrac{dv}{dt}\right| < \left|-\dfrac{du}{dt}\right|$

When the object is at center of curvature, $\left|\dfrac{dv}{dt}\right| = \left|-\dfrac{du}{dt}\right|$

Case (ii). Object moves between center of curvature and focus.

In this case $|m| = 1$, hence $\left|\dfrac{dv}{dt}\right| > \left|-\dfrac{du}{dt}\right|$.

Speed of image is more than speed of object.

Case (iii). Object moves between focus and pole of the mirror.

In this case image is virtual, hence $\frac{1}{(+v)}+\frac{1}{(-u)}=\frac{1}{(-f)}$

$$\Rightarrow \quad \frac{dv}{dt}=\frac{v^2}{u^2}\frac{du}{dt}$$

If u is decreasing, v will also decrease, ie., if real object approaches mirror, image will also do so.

As $|m|>1$, speed of image will be greater than speed of object.

Size of image for a small size object placed along principal axis.

Differentiating $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ w.r.t. t, we get

$$\frac{dv}{dt}=-m^2\frac{du}{dt} \quad \Rightarrow \quad dv=-m^2du$$

Note: ***du/dt and dv/dt and are the velocities with respect to mirror not w.r.t. ground. When the mirror is at rest, then velocity of object or image w.r.t. mirror is same as velocity of object or mirror w.r.t. ground.***

Illustration 1.26 **A mirror of radius of curvature 20 cm and an object which is placed at a distance of 15 cm are both moving with velocities 1 ms^{-1} and 10 ms^{-1} as shown in Fig. 1.60. Find the velocity of image at this situation.**

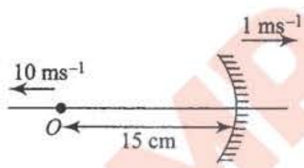


Fig. 1.60

Sol. Using $\frac{1}{v}+\frac{1}{u}=\frac{2}{R}$, we get

$$\frac{1}{v}-\frac{1}{15}=-\frac{1}{10} \quad \Rightarrow \quad v=-30\text{ cm}$$

Now, using $V_{im}=-\frac{v^2}{u^2}V_{OM}$

$$(V_i-v_m)=-\frac{v^2}{u^2}(V_o-V_m)$$

$$\Rightarrow \quad V_i-(1)=-\frac{(-30)^2}{(-15)^2}[(-10)-(1)]$$

$$\Rightarrow \quad V_i=45\text{ cm s}^{-1}$$

So, the image will move with velocity 45 cm s^{-1}.

Illustration 1.27 **An object AB is placed on the axis of a concave mirror of focal length 10 cm. End A of the object is at 30 cm from the mirror. Find the length of the image.**

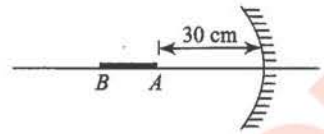


Fig. 1.61

(a) If length of object is 5 cm.

(b) If length of object is 1 mm.

Sol. (a) For point A, $u=-30$ cm, $f=-10$ cm

Using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, $\frac{1}{v'}-\frac{1}{30}=-\frac{1}{10} \Rightarrow v=-15$ cm

Similarly, for point B, $u=-35$ cm, $f=-10$ cm

Using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, we get

$$v'=-14\text{ cm}$$

Now, size of image $|A'B'|=|v-v'|=|(-15)-(-14)|=1$ cm

(b) Here, $u=-30$ cm, $f=-10$ cm

Using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$,

$$\frac{1}{v}+\frac{1}{(-30)}=\frac{1}{(-10)} \quad \Rightarrow \quad v=-15\text{ cm}$$

Now, $\frac{dv}{du}=-\frac{(-15)^2}{(-30)^2} \quad \Rightarrow \quad |dv|=\frac{(15)^2}{(30)^2}|du|$

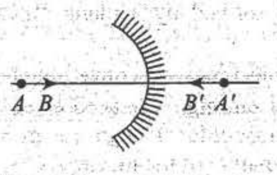
$$\Rightarrow \quad |dv|=\left(\frac{225}{900}\right)(10^{-3})=2.5\times10^{-4}\text{ m}$$

So the length of the image is 2.5×10^{-4} m.

Note: ***Some important points about curved mirror:***

- ***Lateral magnification (or transverse magnification) denoted by m is defined as $m=h_2/h_1$ and is related as $m=v/u$. From the definition of m, positive sign of m indicates erect image and negative sign indicates inverted image.***
  In case of successive reflections from mirrors, the overall lateral magnification is given by $m_1\times m_2\times m_3\times\ldots$, where m_1, m_2, etc. are lateral magnifications produced by individual mirrors.
- ***On differentiating the mirror formula, we get $=dv/du=v^2/u^2$.***
  Mathematically, du′ implies small change in position of object and dv′ implies corresponding small change in position of image. If a small object lies along principal

axis, du may indicate the size of object and dv the size of its image along principal axis. (Note that the focus should not lie in between the initial and final points of object.) In this case, dv/du is called longitudinal magnification. Negative sign indicates inversion of image irrespective of nature of image and nature of mirror.



- ***Velocity of image***
  - ***Object moving perpendicular to the principal axis:***
    We have $h_2/h_1 = v/u$ or $h_2 = (-v/u)h_1$
    If a point object moves perpendicular to the principal axis, x-coordinate of both the object and the image remains constant. On differentiating the above relation w.r.t. time, we get
    $$\frac{dh_2}{dt} = -\frac{v}{v}\frac{dh_1}{dt}$$
    Here, dh_1/dt *denotes velocity of object perpendicular to the principal axis and* dh_2/dt *denotes velocity of image perpendicular to the principal axis.*
  - ***Object moving along the principal axis:*** *On differentiating the mirror formula with respect to time, we get* $dv/dt = -v^2/u^2(du/dt)$, *where* dv/dt *is the velocity of image along the principal axis and* du/dt *is the velocity of object along principal axis. Negative sign implies that the image, in case of mirror, always moves in the direction opposite to that of the object. This discussion is for velocity with respect to mirror and along the x-axis.*
  - ***Object moving at an angle with the principal axis:*** *Resolve the velocity of object along and perpendicular to the principal axis and find the velocities of image in these directions separately and then find the reusltant.*
- *Newton's formula:* $XY = f^2$
  X and Y are the distances (along the principal axis) of the object and image, respectively, from the principal focus. This formula can be used when the distances are mentioned or asked from the focus.
- *Optical power of a mirror (in diopters):* $P = -1/f$
  f *= focal length with sign and is in meters.*
- *If object lying along the principal axis is not of very small size, the longitudinal magnification* $= (v_2 - v_1)/(u_2 - u_1)$ *(it will always be inverted)*

Some Experiments with Curved Mirror

Graphical Method of Determining the Focal Length of a Concave Mirror

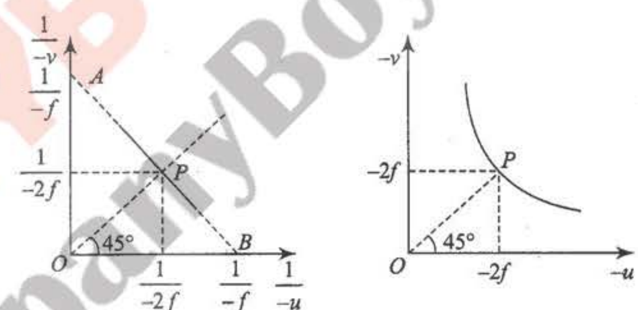
It forms real and inverted image of an object placed beyond its focus. From mirror equation,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

Using Cartesian sign convension, we have $u = -x$, $v = -y$ and $f = -f$

or $$\frac{1}{-y} + \frac{1}{-x} = \frac{1}{-f}$$

A graph between $\frac{1}{-v}$ and $\frac{1}{-u}$ is a straight line, as shown in Fig. 1.62(a).



(a) (b)

Fig. 1.62

Note that the slope of the straight line is –1 and the intercepts on the horizontal and vertical axes are equal. It is equal to $\frac{1}{-f}$.

A straight line OP at an angle of 45° with the horizontal axis is drawn which intersects the line AB at P. The coordinates of the point P are $\left(\frac{1}{-2f}, \frac{1}{-2f}\right)$.

The focal length of the mirror can be calculated by measuring the coordinates of either of the points A, B or P.

Alternatively, a graph between $-v$ and $-u$ can also be plotted, which is a curve as shown in Fig. 1.62 (b). A line drawn at an angle of 45° from the origin intersects it at the point P whose coordinates are $(-2f, -2f)$. By measuring the coordinates of this point, the focal length of the mirror can also be measured.

Measurement of Refractive Index of a Liquid by a Concave Mirror

A concave mirror of large radius of curvature is placed on a table with its principle axis vertical, as shown in Fig. 1.63. A horizontal pin is placed with its tip on the principal axis of the mirror. The pin is moved till there is no parallax between the tip of the pin and its image, when the pin lies at the center of curvature of the mirror.

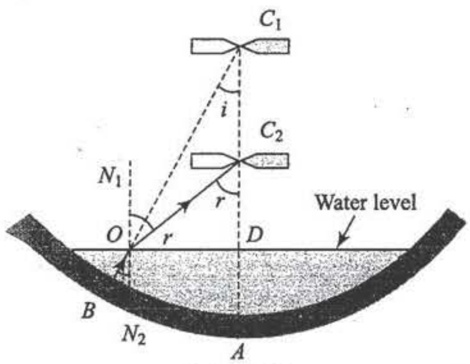


Fig. 1.63

A small quantity of liquid whose refractive index is to be measured is poured into the mirror. The pin is moved down in order to remove the parallax between the tip of the pin and its image.

In the figure, C_1 is the position of the pin when its image coincides with itself without liquid, and C_2 is the position of the pin when its image coincides with itself after pouring the liquid into the concave mirror.

The ray BO that is normal to the mirror passes through C_1 before pouring the liquid. It is refracted away from the normal when the liquid is poured, now it passes through C_2.

In Fig. 1.63, $\sin i = \dfrac{OD}{OC_1}$, $\sin r = \dfrac{OD}{OC_2}$

From Snell's law, $\mu \sin i = 1 \sin r$

$$\mu = \frac{OD}{OC_2} \times \frac{OC_1}{OD} = \frac{OC_1}{OC_2}$$

Taking paraxial ray assumption, $OC_1 \simeq DC_1$; $OC_2 \simeq DC_2$;

Thus, $\mu = \dfrac{DC_1}{DC_2}$

If we neglect the depth of the liquid, we have $\mu = \dfrac{AC_1}{AC_2}$

Concept Application Exercise 1.2

1. State the following statements as TRUE or FALSE.
   a. A convex mirror cannot form a real image for a real object.
   b. The image formed by a convex mirror is always diminished and erect.
   c. Virtual image formed by a concave mirror is always enlarged.
   d. Only in the case of a convex mirror, it may happen that the object and its image move in same direction.
   e. In the case of a concave mirror, the image always move faster than the object.
   f. If an object is placed in front of a diverging mirror at a distance equal to its focal length, then the height of image formed is half of the height of object.
   g. For two positions of an object, a concave mirror can form enlarged image.
   h. Concave mirror is used as a rear view mirror in motor vehicles.
   i. If some portion of the mirror is covered, then complete image will be formed but of reduced brightness.
   j. A plane mirror always forms a real and erect image of same size as that of the object.
   k. The image formed by a plane mirror has left-right reversal.
   l. A virtual object means a converging beam.
2. a. An object 1 cm high is placed at 10 cm in front of a concave mirror of focal length 15 cm. Find the position, height, and nature of the image.
   b. A point source S is placed midway between two converging mirrors having equal focal length f as shown in Fig. 1.64. Find the value of d for which only one image is formed.

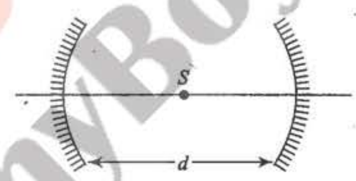


Fig. 1.64

3. Point S' is the image of a point source of light S in a spherical mirror whose optical axis is N_1N_2 (shown in Fig. 1.65). Find by construction the position of the center of the mirror and its focus.

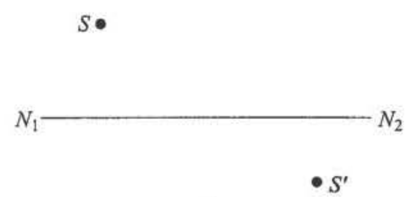


Fig. 1.65

4. The positions of optical axis N_1N_2 of a spherical mirror, the source and the image are known (as shown in Fig. 1.66). Find by construction the positions of the center of the mirror, its focus, and the pole for the cases
   a. A– source, B– image;
   b. B– source, A– image.

Fig. 1.66

5. An object is placed midway between a concave mirror of focal length f and a convex mirror of focal length f. The distance between the two mirrors is $6f$. Trace the ray that is first incident on the concave mirror and then the convex mirror.

6. A particle moves in a circular path of radius 5 cm in a plane perpendicular to the principal axis of a convex mirror with radius of curvature 20 cm. The object is 15 cm in front of the mirror. Calculate the radius of the circular path of the image.
7. An object is placed between a plane mirror and a concave mirror of focal length 15 cm as shown in Fig. 1.67. Find the positions of the images after two reflections.

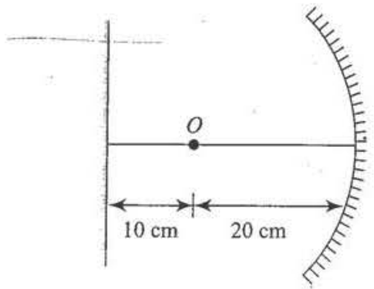


Fig. 1.67

8. A body of length 6 cm is placed 10 cm from a concave mirror of focal length 20 cm. Find the position, size, and nature of the image.
9. An object is placed 15 cm from a mirror and an image is captured on the screen with magnification 2. Calculate the focal length of the mirror and determine if it is concave or convex.
10. An object is placed 15 cm from a mirror and an erect image of size 5 cm is seen. Determine the focal length and the nature of the mirror. Assume the object size is 15 cm.
11. A concave mirror of focal length 10 cm is placed in front of a convex mirror of focal length 20 cm. The distance between the two mirrors is 20 cm. A point object is placed 5 cm from the concave mirror. Discuss the formation of image.

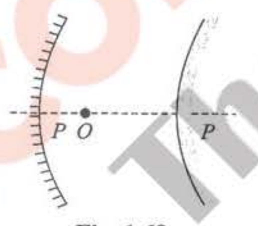


Fig. 1.68

12. A beam of light converges to a point on a screen S. A mirror is placed in front of the screen at a distance of 10 cm from the screen. It is found that the beam now converges at a point 20 cm in front of the mirror. Find the focal length of the mirror.
13. Converging rays are incident on a convex spherical mirror so that their extensions intersect 30 cm behind the mirror on the optical axis. The reflected rays form a diverging beam so that their extensions intersect the optical axis 1.2 m from the mirror. Determine the focal length of the mirror.

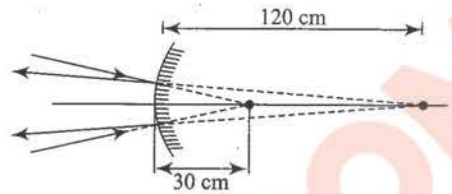


Fig. 1.69

14. Find the position of final image after three successive reflections taking first reflection on m_1.

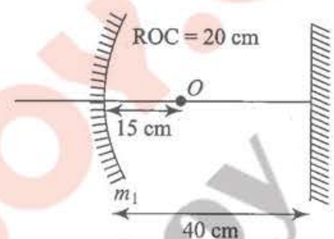


Fig. 1.70

15. A concave mirror gives a real image magnified 4 times. When the object is moved 3 cm the magnification of the real image is 3 times. Find the focal length of mirror.
16. The image of a real object in a convex mirror is 4 cm from the mirror. If the mirror has a radius of curvature of 24 cm, find the position of object and magnification.
17. When an object is placed at a distance of 25 cm from a mirror, the magnification is m_1. The object is moved 15 cm farther away with respect to the earlier position, and the magnification becomes m_2. If $m_1/m_2 = 4$, then calculate the focal length of the mirror.
18. A short linear object is placed at a distance u along the axis of a spherical mirror of focal length f.
    a. Obtain an expression for the longitudinal magnification.
    b. Also, obtain an expression for the ratio of the velocity of image (v) to the velocity of object (u).
19. A convex mirror of focal length 10 cm is shown in Fig. 1.71. A linear object $AB = 5$ cm is placed along the optical axis. Point B is at distance 25 cm from the pole of mirror. Calculate the size of the image of AB.

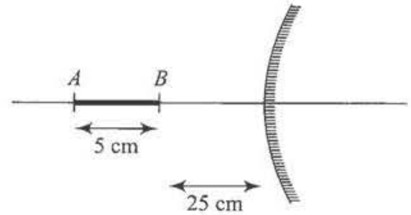


Fig. 1.71

20. A concave mirror forms a real image three times larger than the object on a screen. The object and screen are moved until the image becomes twice the size of the object. If the shift of the object is 6 cm, find the shift of screen.

REFRACTION OF LIGHT

Deviation or bending of light rays from their original path while passing from one medium to another is called *refraction*. It is due to change in speed of light as light passes from one medium to another medium. If the light is incident normally then it goes to the second medium without bending, but still it is called refraction.

When a light ray passes from one medium to another such that it undergoes a change in velocity, refraction takes place. Hence, wavelength of light changes, but frequency remains same.

Refractive index of a medium is defined as the factor by which speed of light reduces as compared to the speed of light in vacuum. $\mu = \frac{c}{v} = \frac{\text{speed of light in vacuum}}{\text{speed of light in medium}}$.

More (less) refractive index implies less (more) speed of light in that medium, which therefore is called denser (rarer) medium.

Illustration 1.28 **Determine the refractive index of glass with respect to water. Given that $\mu_g = 3/2$; $\mu_w = 4/3$.**

Sol. Refractive index of glass with respect to water or relative refractive index of glass w.r.t. water:

$$\frac{\text{R.I. of glass}}{\text{R.I. of water}} = \mu_{gw} = \frac{\mu_g}{\mu_w} = \frac{\frac{3}{2}}{\frac{4}{3}} = \frac{9}{8} = 1.125$$

Laws of Refraction

a. The incident ray, the normal to any refracting surface at the point of incidence, and the refracted ray all lie in the same plane called the plane of incidence or plane of refraction.

b. $\frac{\sin i}{\sin r}$ = constant for any pair of media and for light of a given wavelength (Fig. 1.72). This is known as ***Snell's law***.

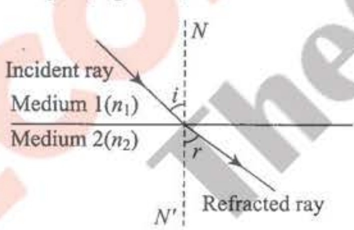


Fig. 1.72

Also, $\frac{\sin i}{\sin r} = \frac{n_2}{n_1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$

For applying in problems remember

$n_1 \sin i = n_2 \sin r$

$\frac{n_2}{n_1} = {}_1n_2$ = refractive index of the second medium with respect to the first medium.

c = speed of light in air (or vacuum) = 3×10^8 ms^{-1}.

Special Cases

Case 1. Normal incidence: $i = 0$ (see Fig. 1.73(a))

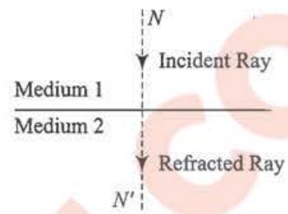


Fig. 1.73(a)

From Snell's law: $r = 0$

Case 2. When light moves from denser to rarer medium, it bends away from the normal (See Fig. 1.73(b))

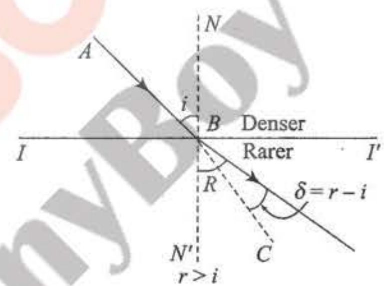


Fig. 1.73(b)

Case 3. When light moves from rarer to denser medium, it bends towards the normal (See Fig. 1.74)

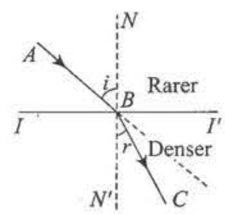


Fig. 1.74

Note:

- ***Higher the value of R.I., denser (optically) is the medium.***
- ***Frequency of light does not change during refraction.***
- ***Refractive index of the medium relative to vacuum*** $= \sqrt{\mu_r \epsilon_r}$

$n_{vacuum} = 1$; $n_{air} \geq 1$; n_{water} ***(average value)*** $= 4/3$; n_{glass} ***(average value)*** $= 3/2$

Deviation of a Ray Due to Refraction

Deviation (δ) of ray incident at $\angle i$ and refracted at $\angle r$ is given by $\delta = |i - r|$ (see Fig. 1.75)

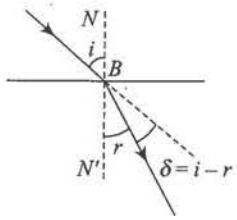


Fig. 1.75

Principle of Reversibility of Light Rays

a. A ray traveling along the path of the refracted ray is refracted along the path of the incident ray.

b. A refracted ray reversed to travel back along its path will get refracted along the path of the incident ray. Thus, the incident and refracted rays are mutually reversible.

c. According to this principle, ${}_1n_2 = \dfrac{1}{{}_2n_1}$.

Illustration 1.29 **Find the angle θ_a made by the light ray when it gets refracted from water to air, as shown in Fig. 1.76.**

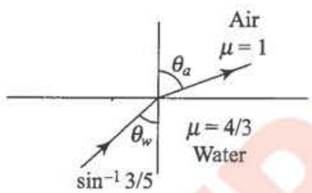


Fig. 1.76

Sol. Snell's law: $\mu_w \sin\theta_w = \mu_a \sin\theta_a$

$$\frac{4}{3}\times\frac{3}{5} = 1\sin\theta_a$$

$$\sin\theta_a = \frac{4}{5} \quad \theta_a = \sin^{-1}\frac{4}{5}$$

Illustration 1.30 **Find the speed of light in medium 'a' if speed of light in medium 'b' is $c/3$, where c = speed of light in vacuum and light refracts from medium 'a' to medium 'b' making 45° and 60°, respectively, with the normal.**

Sol. Snell's law:

$$\mu_a \sin\theta_a = \mu_b \sin\theta_b \Rightarrow \frac{c}{v_a}\sin\theta_a = \frac{c}{v_b}\sin\theta_b.$$

$$\frac{c}{v_a}\sin 45^\circ = \frac{c}{c/3}\sin 60^\circ \Rightarrow v_a = \frac{\sqrt{2}c}{3\sqrt{3}}$$

Illustration 1.31 **A ray of light is incident on a transparent glass slab of refractive index $\sqrt{3}$. If the reflected and refracted rays are mutually perpendicular, what is the angle of incidence?**

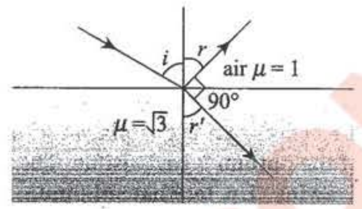


Fig. 1.77

Sol. Let the angle of incidence, angle of reflection, and angle of refraction be i, r and r', respectively.

Now, as per the equation $(90^\circ - r) + (90^\circ - r') = 90^\circ$

$r' = (90^\circ - i)$ (because $i = r$, in case of reflection according to Snell's law, $1\sin i = \mu \sin r'$)

or $\quad \sin i = \mu \sin r'$

or $\quad \sin i = \mu \sin(90^\circ - i) \Rightarrow \tan i = \mu$

or $\quad i = \tan^{-1}\mu = \tan^{-1}\sqrt{3} = 60^\circ$

Illustration 1.32 **A light ray is incident on a glass sphere of reflective index $\mu = \sqrt{3}$ at an angle of incidence 60° as shown in Fig. 1.78. Find the angles r, r', e and the total deviation after two refractions.**

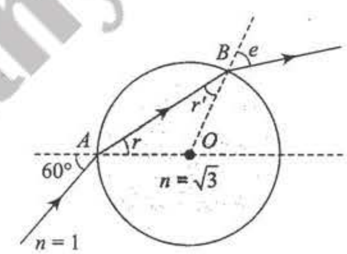


Fig. 1.78

Sol. At point 'A': Applying Snell's law $1\sin 60^\circ = \sqrt{3}\sin r \Rightarrow r = 30^\circ$

From symmetry $r' = r = 30^\circ$.

Again applying Snell's law at second surface (at point 'B') $1\sin e = \sqrt{3}\sin r$.

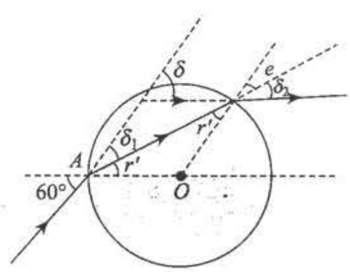


Fig. 1.79

$\Rightarrow \quad e = 60^\circ$

Deviation at first surface, $\delta_1 = i - r = 60^\circ - 30^\circ = 30^\circ$

Deviation at second surface, $\delta_2 = e - r' = 60^\circ - 30^\circ = 30^\circ$

Therefore, total deviation = 60°.

Illustration 1.33 **A cylindrical vessel, whose diameter and height both are equal to 30 cm, is placed on a horizontal surface and a small particle P is placed in it at a distance of 5.0 cm from the center. An eye is placed at a position such that the edge of the bottom is just visible. The particle P is in the plane of drawing. Up to what minimum height should water be poured in the vessel to make the particle P visible ?**

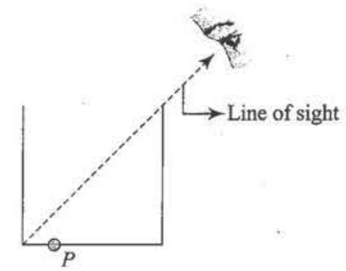


Fig. 1.80

Sol. If we pour water in vessel, refraction will take place at air and water interface.

Applying Snell's law at A, we get $1. \sin 45° = \frac{4}{3} \sin \theta$

Here, θ is the angle which the incidence ray of light makes with the normal.

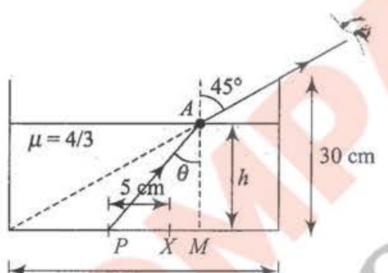


Fig. 1.81

$$\Rightarrow \quad 1\left(\frac{1}{\sqrt{2}}\right) = \frac{4}{3}\sin\theta \quad \Rightarrow \quad \sin\theta = \frac{3}{4\sqrt{2}}$$

$$\tan\theta = \frac{3}{\sqrt{16\times 2-9}} = \frac{3}{\sqrt{23}}$$

$\Rightarrow$ In ΔAPM,

$$\tan\theta = \frac{3}{\sqrt{23}} = \frac{PM}{AM} = \frac{5+x}{h} = \frac{5+h-15}{h}$$

$$\frac{3}{\sqrt{23}} = \frac{h-10}{h} \quad \Rightarrow \quad 3h = h\sqrt{23} - 10\sqrt{23}$$

$$h = \left(\frac{10\sqrt{23}}{\sqrt{23}-3}\right)\text{cm}$$

Hence, water should be poured upto height $\frac{10\sqrt{23}}{\sqrt{23}-3}$ cm to make the particle 'P' visible.

VECTOR REPRESENTATION OF A LIGHT RAY

The angle of incidence of a light ray on an interface is usually determined geometrically by simply drawing the ray diagram. However, in some situations, the vector representation of the line along which the light ray travels is given. The ray may then reflect or refract at an interface and emerge. How do we find the vector representation of the line along which the emergent ray travels? Following are the steps for finding this:

- Determine the unit vector representation of the normal to the interface where the incident ray strikes the surface, say $\hat{e}_n$.
- Find the component of the incident ray along the normal.
- Subtract this component from the original ray to compute the plane in which the incident ray travels. Calculate the unit vector that represents the plane of the incident ray and the normal, say $\hat{e}_p$.
- Using the vector dot product, calculate the angle between the incident ray and the normal.
- From the governing equation calculate the angle of reflection/refraction, say r.
- We know that the emergent ray will be in the same plane as that of the incident ray and the normal. Thus the unit vector representing the direction of the emergent beam is simply $\cos(r)\hat{e}_n + \sin(r)\hat{e}_p$.

Let us learn to apply these steps through following illustrations.

Illustration 1.34 **The XY plane is the boundary between two transparent media. Medium 1 with $z \geq 0$ has a refractive index of $\sqrt{2}$ and medium 2 with $z \leq 0$ has a refractive index of $\sqrt{3}$. A ray of light in medium 1 given by the vector $6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} - 10\hat{k}$ is incident on the plane of separation. Find the unit vector in the direction of the refracted ray in medium 2.** **(IIT-JEE, 1999)**

Sol. Unit vector representing the normal to the plane $\hat{e}_n = \hat{k}$.

Component of the incident ray along the normal is $-10\hat{k}$.

The unit vector that represents the plane of the incident ray and the normal

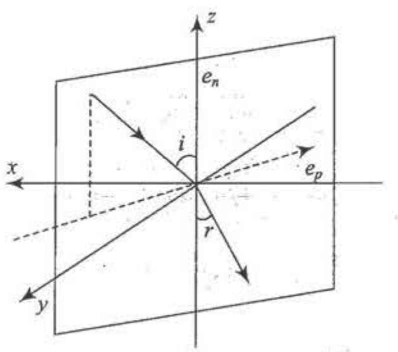
$$\hat{e}_p = \frac{(6\sqrt{3}\,\hat{i} + 8\sqrt{3}\,\hat{j})}{\sqrt{(6\sqrt{3})^2 + (8\sqrt{3})^2}} = 0.6\hat{i} + 0.8\hat{j}$$

Angle between the incident ray and the normal is given by

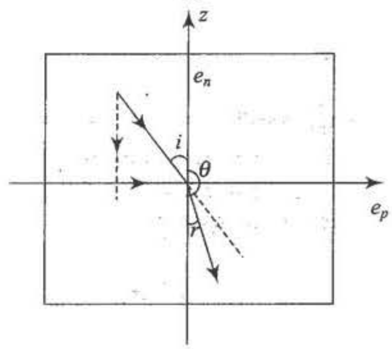
$$\cos\theta = (6\sqrt{3}\,\hat{i} + 8\sqrt{3}\,\hat{j} - 10\hat{k})\cdot\hat{k}\Big/\sqrt{(6\sqrt{3})^2 + (8\sqrt{3})^2 + 10^2}$$

or $\quad \cos\theta = -0.5$

Therefore, the angle $\theta = 120°$



(a)



(b)

Fig. 1.82

The angle of incidence is $i = 180° - 120° = 60°$

The angle of the refracted beam is given by $\sqrt{2}\sin(i) = \sqrt{3}\sin(r)$ or $r = 45°$

The equation of the emergent ray is $\cos(r)\hat{e}_n + \sin(r)\hat{e}_p$

$$= \cos(45°)(-\hat{k}) + \sin(45°)\cdot(0.6\hat{i} + 0.8\hat{j})$$

$$= \frac{1}{\sqrt{2}}(0.6\hat{i} + 0.8\hat{j} - \hat{k})$$

CRITICAL ANGLE AND TOTAL INTERNAL REFLECTION

Consider a ray of light that travels from a denser medium to rarer medium. As the angle of incidence increases in the denser medium the angle of refraction in the rarer medium increases (see Fig. 1.83). The angle of incidence for which the angle of refraction becomes 90° is called *critical angle*.

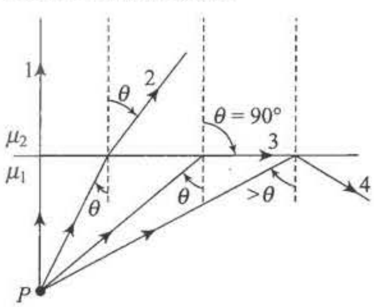


Fig. 1.83

$$\frac{\sin c}{\sin 90°} = {}_2\mu_1 = \frac{\mu_1}{\mu_2} \Rightarrow \sin c = \frac{1}{{}_2\mu_1} = \frac{\mu_2}{\mu_1}$$

$$\text{or} \quad \sin C = \frac{\text{R.I. of rarer medium}}{\text{R.I. of denser medium}}$$

When the angle of incidence of a ray traveling from a denser medium to rarer medium is greater than the critical angle, no refraction occurs. The incident ray is totally reflected back into the same medium. Here, the laws of reflection hold good. Some light is also reflected before the critical angle is achieved, but not totally.

Graph between Angle of Deviation (δ) and Angle of Incidence (i)

(I) For light ray going from denser medium to rarer medium (see Fig. 1.84):

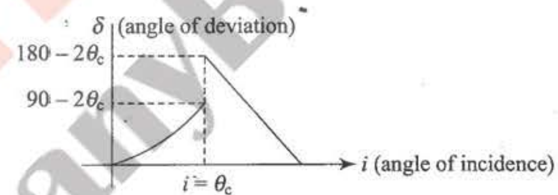


Fig. 1.84

(II) For light ray going from rarer medium to denser medium (see Fig. 1.85):

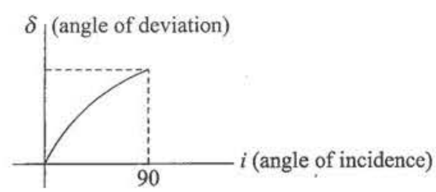


Fig. 1.85

Conditions of Total Internal Reflection

a. Light is incident on the interface from denser medium.

b. Angle of incidence should be greater than the critical angle ($i > c$). Figure 1.86 shows a luminous object placed in denser medium at a distance h from an interface separating two media of refractive indices μ_r and μ_d. Subscript r and d stand for rarer and denser media, respectively.

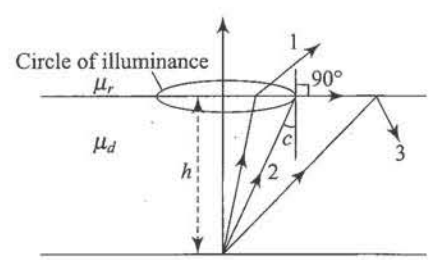


Fig. 1.86

In the figure, ray 1 strikes the surface at an angle less than critical angle c and gets refracted in rarer medium. Ray 2 strikes the surface at critical angle and grazes the interface. Ray 3 strikes the surface making an angle greater than the critical angle and gets internally reflected. The locus of points where ray strikes at critical angle is a circle, called *circle of illuminance* (C.O.I). All light rays striking inside the circle of illuminance get refracted in the rarer medium. If an observer is in the rarer medium, he/she will see light coming out only from within the circle of illuminance. If a circular opaque plate covers the circle of illuminance, no light will get refracted in the rarer medium and then the object cannot be seen from the rarer medium. Radius of C.O.I. can be easily found.

Illustration 1.35 **Find the maximum. angle that can be made in glass medium ($\mu = 1.5$) if a light ray is refracted from glass to vacuum.**

Sol. Maximum angle of refraction from denser medium to rarer medium is the critical angle. Hence,

$1.5 \sin C = 1 \sin 90°$, where C = critical angle.

$$\sin C = 2/3$$

$$C = \sin^{-1} 2/3$$

Illustration 1.36 **Find the angle of refraction in a medium ($\mu = 2$) if light is incident in vacuum, making an angle equal to twice the critical angle.**

Sol. Since the incident light is in rarer medium, total internal reflection cannot take place.

$$C = \sin^{-1}\frac{1}{\mu} = 30°$$

$$\therefore \quad i = 2C = 60°$$

Applying Snell's law, $1 \sin 60° = 2 \sin r$

$$\sin r = \frac{\sqrt{3}}{4} \quad \Rightarrow \quad r = \sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$$

Illustration 1.37 **What should be the value of angle θ so that light entering normally through the surface AC of a prism ($n = 3/2$) does not cross the second refracting surface AB.**

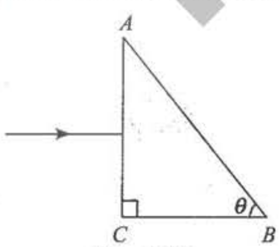


Fig. 1.87

Sol. Light ray will pass the surface AC without bending since it is incident normally. Suppose it strikes the surface AB at an angle of incidence i.

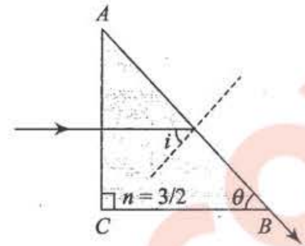


Fig. 1.88

$$i = 90 - \theta$$

For the required condition: $i > C \Rightarrow \quad 90° - \theta > C$

or $\quad \sin(90° - \theta) > \sin C$

or $\quad \cos\theta > \sin C = \dfrac{1}{3/2} = \dfrac{2}{3}$ or $\theta < \cos^{-1}\dfrac{2}{3}$

Illustration 1.38 **A slab of refractive index μ is placed in air and light is incident at maximum angle θ_0 from vertical. Find minimum value of μ for which total internal reflection takes place at the vertical surface.**

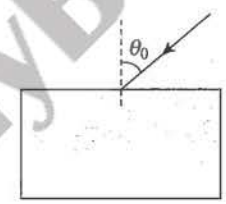


Fig. 1.89

Sol. For vertical surface, $\alpha > C$

$$\Rightarrow \quad \sin\alpha > \sin C \quad \Rightarrow \quad \sin\alpha > \frac{1}{\mu} \qquad \text{(i)}$$

For horizontal surface,

$$\sin\theta_0 = \mu\cos\alpha \quad \Rightarrow \quad \sin\alpha = \sqrt{\frac{\mu^2 - \sin^2\theta}{\mu^2}} \qquad \text{(ii)}$$

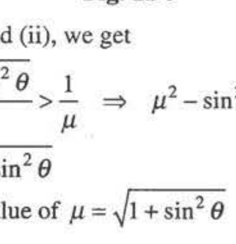


Fig. 1.90

From Eqs. (i) and (ii), we get

$$\Rightarrow \quad \frac{\sqrt{\mu^2 - \sin^2\theta}}{\mu} > \frac{1}{\mu} \quad \Rightarrow \quad \mu^2 - \sin^2\theta > 1$$

$$\Rightarrow \quad \mu > \sqrt{1 + \sin^2\theta}$$

So, minimum value of $\mu = \sqrt{1 + \sin^2\theta}$

Illustration 1.39 **A rectangular slab *ABCD*, of refractive index n_1, is immersed in water of refractive index n_2 ($n_1 < n_2$). A ray of light is incident at the surface *AB* of the slab as shown in Fig. 1.91. Find the maximum value of angle of incidence α_{max}, such that the ray comes out only from the other surface *CD*.**

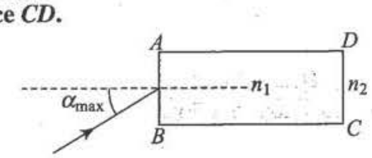


Fig. 1.91

Sol. For a maximum angle of incidence at surface *AB* there will be a maximum angle of incidence at the surface *AD*. A ray to pass through the face *CD* as it should not pass beyond *AD* i.e. it should not refract at *AD*. Hence, the angle θ should be the critical angle.

By Snell's law,

$$\sin\theta = \frac{n_2}{n_1}$$

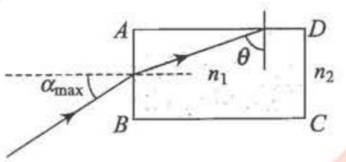


Fig. 1.92

$$n_2 \sin\alpha_{max} = n_1 \sin(90 - \theta)$$

$$\sin\alpha_{max} = \frac{n_1}{n_2}\cos\theta \Rightarrow \alpha_{max} = \sin^{-1}\left[\frac{n_1}{n_2}\cos\left(\sin^{-1}\frac{n_2}{n_1}\right)\right]$$

Illustration 1.40 **A point source of light is placed a distance *h* below the surface of a large and deep lake. What fraction of light will escape through the surface of water?**

Sol. Due to total internal reflection some of the light rays incident at the interface will return back into water. So, only that portion of light will escape for which the angle of incidence at the interface of the medium is less than the critical angle.

If the critical angle is θ_c, then the light rays that reach beyond the base of the cone whose vertical angle is $2\theta_c$ will suffer total internal reflection.

Hence, only the light incident on the base of the cone refracts and escapes.

Method 2:

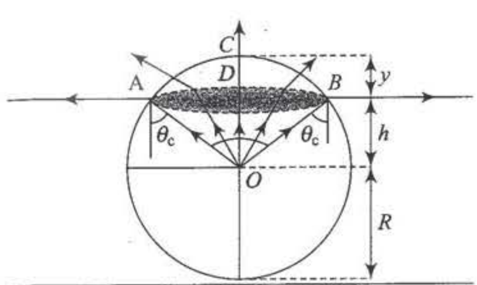


Fig. 1.93

The fraction of light escaping,

$$f = \frac{\text{Area of the cap}}{\text{Area of sphere}} = \frac{2\pi R y}{4\pi R^2}, \text{ i.e., } f = \frac{1}{2}\left[\frac{y}{R}\right] = \frac{1}{2}\left[\frac{R-h}{R}\right]$$

Area of cap *ABCD* can be calculated by using method of integration,

$$\text{i.e., } f = \frac{1}{2}\left[1 - \frac{h}{R}\right] = \frac{1}{2}[1 - \cos\theta_c], \text{ i.e., } f = \frac{1}{2}\left[1 - \sqrt{1 - \sin^2\theta_c}\right]$$

$$\text{i.e., } f = \frac{1}{2}\left[1 - \sqrt{1 - \frac{1}{n^2}}\right].$$

Illustration 1.41 **A spider is on the surface of a glass sphere with a refractive index of 1.5. An insect crawls on the other side of the sphere as shown in Fig. 1.94(a). For what maximum value of θ will the spider be able to still see the insect. Assume the spiders eye is in air.**

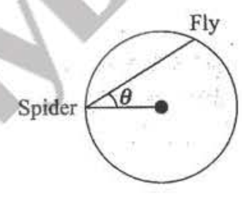


Fig. 1.94(a)

Sol. The spider will be able to see the insect if a ray of light from the insect reaches the spider's eye. If the angle of incidence of the beam is greater than the critical angle for the glass–air interface, the ray will be reflected within the glass sphere and will not emerge from it. Consequently, the spider will not be able to see the insect. Therefore, the angle θ must be greater than the critical angle for the glass–air interface. That is,

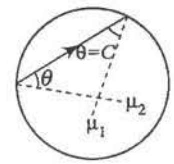


Fig. 1.94(b)

$$\theta > \sin^{-1}\left[\frac{1}{1.5}\right] \Rightarrow \theta > \sin^{-1}\left(\frac{2}{3}\right)$$

Illustration 1.42 **A monochromatic light is incident on the plane interface *AB* between two media of refractive indices μ_1 and μ_2 ($\mu_2 > \mu_1$) at an angle of incidence θ as shown in Fig.1.95.**

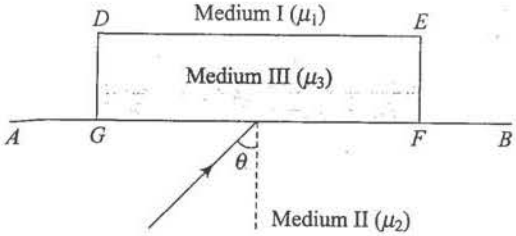


Fig. 1.95

The angle θ is infinitesimally greater than the critical angle for the two media so that total internal reflection takes place. Now, if a transparent slab *DEFG* of uniform thickness and of refractive index μ_3 is introduced on the interface (as shown in the figure), show that for any value of μ_3 all light will ultimately be reflected back into medium II.

Sol. We will use the symbol $\leq$ to mean 'infinitesimally greater than'.

When the slab is not inserted,

$$\theta \leq \theta_c = \sin^{-1}(\mu_1/\mu_2) \text{ or } \sin\theta \geq \mu_1/\mu_2$$

When the slab is inserted, we have two cases

$$\mu_3 \leq \mu_1 \text{ and } \mu_3 > \mu_1.$$

Case I. $\mu_3 < \mu_1$. We have $\sin\theta \geq \mu_1/\mu_2 \geq \mu_3/\mu_2$

Thus, the light is incident on AB at an angle greater than the critical angle $\sin^{-1}(\mu_3/\mu_2)$. It suffers total internal reflection and goes back to medium II.

Case II. $\mu_3 > \mu_1$

$$\sin\theta \geq \mu_1/\mu_2 < \mu_3/\mu_2$$

Thus, the angle of incidence θ may be smaller than the critical angle $\sin^{-1}(\mu_3/\mu_2)$ and hence it may enter medium III. The angle of refraction θ' is given by (figure).

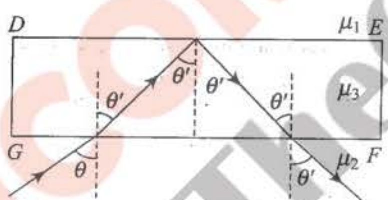


Fig. 1.96

$$\frac{\sin\theta}{\sin\theta'} = \frac{\mu_3}{\mu_2} \quad \text{(i)}$$

$$\Rightarrow \quad \sin\theta' = \frac{\mu_2}{\mu_3}\sin\theta \leq \frac{\mu_2}{\mu_3}\cdot\frac{\mu_1}{\mu_2}$$

Thus, $\quad \sin\theta' \geq \dfrac{\mu_1}{\mu_3} \quad \Rightarrow \quad \theta' \geq \sin^{-1}\left(\dfrac{\mu_1}{\mu_3}\right) \quad$ (ii)

As the slab has parallel faces, the angle of refraction at the face FG is equal to the angle of incidence at the face DE. Equation (ii) shows that this angle is infinitesimally greater than the critical angle here. Hence, the light suffers total internal reflection and falls at the surface FG at an angle of incidence θ'. At this face, it will refract into medium II and the angle of refraction will be θ as shown by Eq. (i). Thus, the total light energy is ultimately reflected back into medium II.

APPARENT SHIFT OF AN OBJECT DUE TO REFRACTION

Due to bending of light at the interface of two different media, the image formed due to refraction appears at a place other than the object position. This image formation due to refraction creates illusion of shifting of the object position.

Locating the position of image formed becomes much simpler if we restrict ourselves to nearly normal incident rays.

Consider an object O in the medium (R.I = μ). After refraction, the ray at the interface bend. When the bent ray falls in our eye our eye perceives it along a straight line and it appears at I. (see Fig. 1.97)

For nearly normal incident rays, θ_1 and θ_2 will be very small.

$$\tan\theta_1 = \sin\theta_1 = \frac{AB}{\text{object distance from the refracting surface}}$$

$$\sin\theta_2 = \frac{AB}{\text{image distance from the refracting surface}}$$

$$\Rightarrow \frac{\text{Image distance from the refracting surface}}{\text{Object distance from the refracting surface}} = \frac{\sin\theta_1}{\sin\theta_2}$$

$$= \mu_2 = \frac{\mu_2}{\mu_1}$$

$$\Rightarrow \frac{\tan\theta_1}{\tan\theta_1} = \frac{\mu_2}{\mu_1}$$

$$\Rightarrow \frac{\dfrac{AB}{OB}}{\dfrac{BA}{BI}} = \frac{\mu_2}{\mu_1} \Rightarrow \frac{BI}{OB} = \frac{\text{Apprant depth}}{\text{Real depth}} = \frac{\mu_2}{\mu_1}$$

So, Shift = Real depth – Apparent depth = Real depth $\left(1 - \dfrac{\mu_2}{\mu_1}\right)$

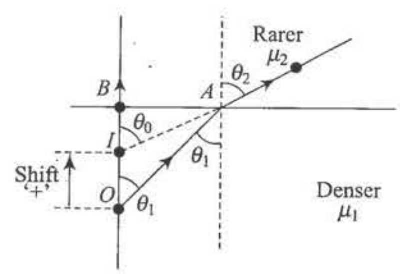


Fig. 1.97

Case I.

If $\mu_1 < \mu_2$, shift becomes negative (in the direction opposite to initial, say travelling). Image distance > object distance, i.e., image is farther from the refracting surface (See Fig. 1.98).

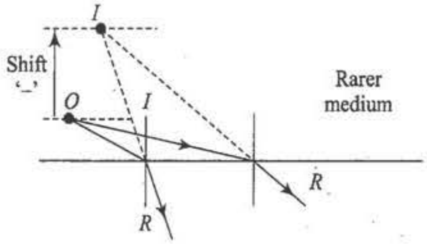


Fig. 1.98

Case II.

If $\mu_1 > \mu_2$, shift becomes positive. Image distance < object distance, i.e., image is closer to the refracting surface (see Fig. 1.99).

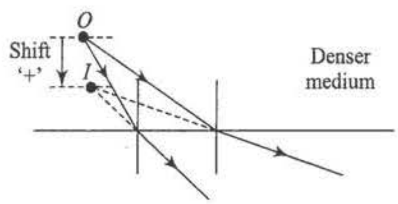


Fig. 1.99

Note: Apparent Depth and Shift of Submerged Object

At near normal incidence (small angle of incidence i), apparent depth (d′) is given by:

$$d' = \frac{d}{n_{relative}}$$

where

$$n_{relative} = \frac{n_i \text{ (R.I. of medium of incidence)}}{n_r \text{ (R.I. of medium of refraction)}}$$

d = distance of object from the interface = real depth

d′ = distance of image from the interface = apparent depth

In general, we can write

$$\frac{n_i}{d} = \frac{n_r}{d'}$$

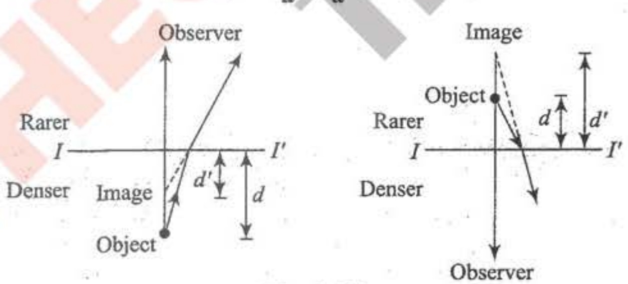


Fig. 1.100

Apparent shift $= d\left(1 - \frac{1}{n_{rel}}\right)$

Illustration 1.43 **An object lies 100 cm inside water (μ = 4/3). It is viewed from air nearly normally. Find the apparent depth of the object.**

Sol: Here, object is placed inside water and the observer is situated in air. Hence, apparent depth

$$d' = \frac{d}{(n_i/n_r)} = \frac{d}{\left(\frac{n_{water}}{n_{air}}\right)} = d' = \frac{d}{n_{relative}} = \frac{100}{\frac{4/3}{1}} = 75 \text{ cm}$$

Illustration 1.44 **See Fig. 1.101 and answer the following questions.**

(i) Find apparent height of the bird.

(ii) Find apparent depth of the fish.

(iii) At what distance will the bird appear to the fish?

(iv) At what distance will the fish appear to the bird?

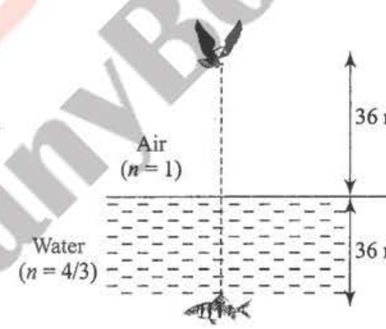


Fig. 1.101

Sol.

(i) Here, bird is an object and fish is an observer. Hence, apparent height observed by the fish

$$d'_B = \frac{d}{n_{rel}} = \frac{d}{\left(\frac{n_{air}}{n_{water}}\right)} \Rightarrow d'_B = \frac{36}{\frac{1}{\left(\frac{4}{3}\right)}} = \frac{36}{3/4} = 48 \text{ m}$$

(ii) Here, the fish is an object and the bird is an observer. Hence, apparent height observed by the bird

$$d'_F = \frac{d}{n_{rel}} = \frac{d}{\left(\frac{n_{water}}{n_{air}}\right)} \Rightarrow d'_F = \frac{36}{4/3} = 27 \text{ m}$$

(iii) For the fish, the bird will be observed at a distance d'_B from the fish: $d_B = 36 + 48 = 84$ m

(iv) For the bird, the fish will be observed at a distance 'd_F' from the bird: $d_F = 27 + 36 = 63$ m

Illustration 1.45 **Consider the situation in Fig. 1.102. The bottom of the pot is a reflecting plane mirror, S is a small fish, and T is a human eye. Refractive index of water is μ.**

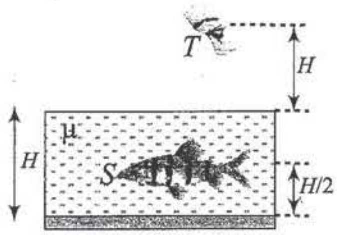


Fig. 1.102

(a) At what distance(s) from itself will the fish see the image(s) of the eye?

(b) At what distance(s) from itself will the eye see the image(s) of the fish?

Sol.

(a) The fish will observe the images of eye one from direct observation and the other reflected image from the plane mirror.

(i) Direct observation of eye from fish

Apparent height, $H' = \dfrac{H}{n_{\text{rel}}} = \dfrac{H}{\left(\dfrac{n_{\text{air}}}{n_{\text{water}}}\right)} = \dfrac{H}{\left(\dfrac{1}{\mu}\right)}$

Hence, $H' = \mu H$

Distance of image of eye from fish

$$d = \frac{H}{2} + \mu H = H\left(\frac{1}{2} + \mu\right)$$

(ii) Observation of reflected image of the eye from the fish

For mirror, the distance of eye from it will be $(H + \mu H)$. Hence, the image of eye from mirror will be $(H + \mu H)$ behind the mirror. Hence, distance of image of eye from the fish

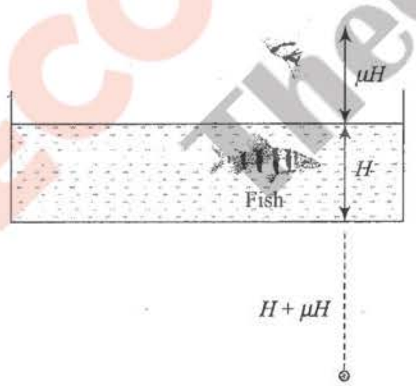


Fig. 1.103

$$d' = \frac{H}{2} + H + \mu H = \frac{3}{2}H + \mu H \quad \Rightarrow \quad d' = H\left(\frac{3}{2} + \mu\right)$$

(b) The eye will also observe two images of the fish, one from direct observation and the other reflected image from the mirror.

(i) Direct observation of fish from eye:

Apparent depth of the fish observed by the eye

$$H' = \frac{H/2}{n_r} = \frac{H/2}{\left(\dfrac{n_{\text{water}}}{n_{\text{air}}}\right)} = \frac{H/2}{\mu} = \frac{H}{2\mu}$$

Distance of image of the fish from the eye,

$$d = H + \frac{H}{2\mu} = H\left(1 + \frac{1}{2\mu}\right)$$

(ii) Eye observing image of fish:

The eye will observe the image of fish reflected from the mirror.

Apparent depth of image of the fish from air and water interface.

$$H' = \frac{\text{Real depth}}{n_{\text{rel}}} = \frac{\frac{3}{2}H}{\left(\dfrac{n_{\text{water}}}{n_{\text{air}}}\right)}$$

Here real depth from top surface of water $= H + \dfrac{H}{2} = \dfrac{3}{2}H$

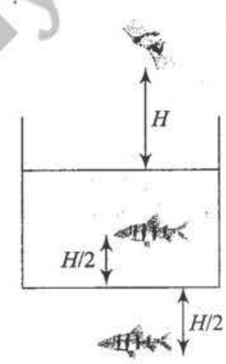


Fig. 1.104

$$H' = \frac{3H}{2\mu}$$

Hence, distance between this image and the eye,

$$d' = H + \frac{3H}{2\mu} = H\left(1 + \frac{3}{2\mu}\right)$$

Illustration 1.46 **A fish in an aquarium approaches the left wall at a rate of 2.5 ms^{-1} observes a fly approaching it at 8 ms^{-1}. If the refractive index of water is (4/3), find the actual velocity of the fly.**

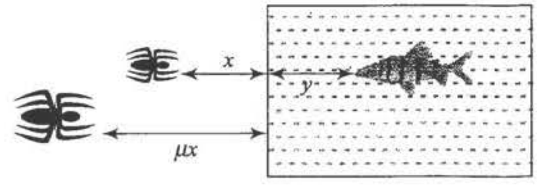


Fig. 1.105

Sol. For the fish, the apparent distance of the fly from the wall of the aquarium is μx, if x is the actual distance.

Then apparent velocity will be $d(\mu x)/dt \Rightarrow (v_{app})_{fly} = \mu v_{fly}$

Now, the fish observes the velocity of the fly to be 8 ms^{-1}

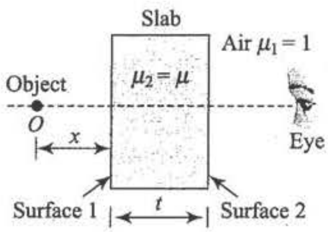
$\Rightarrow$ Apparent relative velocity will be = 8 ms^{-1}

$\Rightarrow \quad v_{fish} + (v_{app})_{fly} = 8$

$\Rightarrow \quad 3 + \mu v_{fly} = 8 \quad \Rightarrow \quad v_{fly} = 5 \times \dfrac{3}{4} = 3.75 \text{ ms}^{-1}$

REFRACTION THROUGH A PARALLEL SLAB

A slab is formed when a medium is isolated from its surroundings by two plane surfaces parallel to each other. In this section, we will determine the position and nature of the image formed when a slab is placed in front of an object.



All objects viewed through a transparent glass slab

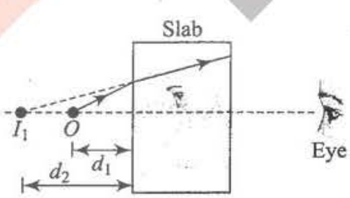
Fig. 1.106

Consider an object O placed a distance x in front of a glass slab of thickness "t" and refractive index μ. The observer is on the other side of the slab. A ray of light from the object first refracts at surface (1) then refracts at surface (2) before reaching the observer (Fig. 1.106). Let us analyse the location of image as sum by observer taking one step at a time.

Refraction at surface 1:

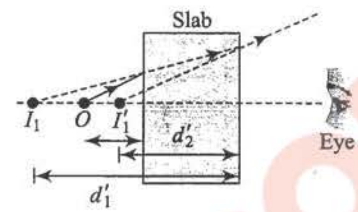
Here, $\mu_1 = 1$, $\mu_2 = \mu$, d_1 = real depth, d_2 = apparent depth

$$\text{Apparent depth, } d_2 = \frac{d_{real}}{n_{relative}} = \frac{d_1}{\left(\dfrac{n_{incident}}{n_{refraction}}\right)} = \frac{d_1}{\left(\dfrac{\mu_1}{\mu_2}\right)}$$



The apparent position of the object after refraction at the first surface.

Fig. 1.107(a)



Final position of the object after refraction at both surfaces

Fig. 1.107(b)

$$d_2 = \frac{x}{(1/\mu)}$$

$$d_2 = \mu x \qquad \text{(i)}$$

Thus, the first image is formed behind the object at the point I_1. I_1 now serves as the object for the second surface [see Fig. 1.107 (a)].

Refraction at surface 2:

Here, $\quad n_{incident} = \mu_2 = \mu$

$\qquad\quad n_{refraction} = \mu_1 = 1$

$$\text{Apparent depth, } d_2' = \frac{d_{real}}{n_{real}}$$

$$d_2' = \frac{d_1'}{\left(\dfrac{\mu}{1}\right)} \text{ but } d_1' = (d_2 + t)$$

$$d_2' = \frac{(\mu x + t)}{\mu} \qquad \text{(ii)}$$

Thus, the final image I_1' is at a distance $x + (t/\mu)$ behind the second interface. The original object was at a distance $x + t$ behind the second interface. Therefore, the image appears shifted by

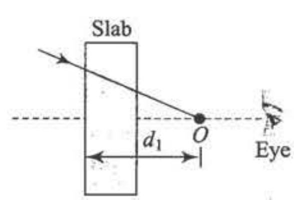
$$s = \left[x + \frac{t}{\mu}\right] - [(x + t)] = t\left[1 - \frac{1}{\mu}\right] \qquad \text{(iii)}$$

We can say a slab appears to shift the object along the perpendicular to the slab by a distance $t[1 - (1/\mu)]$ in the direction of the travelling ray [see Fig. 1.107(b)].

How about the direction of the shift?

In Fig. 1.107, the object is shifted towards the glass slab. Is it always true?

Consider a converging set of rays incident on a glass slab as shown in Fig. 1.108(a). Here, we have a virtual object to the right of the slab. Following the same procedure as that in earlier, we can show that the net shift is same as given by Eq. (iii). However, let us look a little closely at the direction of the shift.



(a)

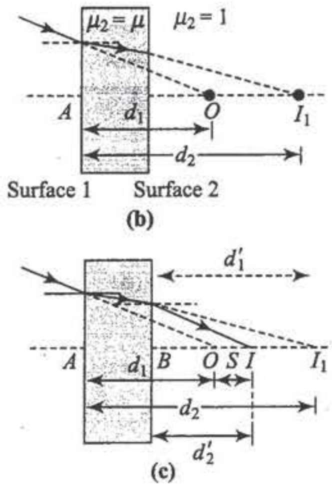


Fig. 1.108

Here, μ is the refractive index of the slab with respect to surrounding medium ($\mu = \mu_2/\mu_1$). If the refractive index of medium is greater than the refractive index of slab, then shifting (s) will be negative. In this case, the shifting will be in the direction opposite to the traveling ray.

Refraction at surface 1:

The converging rays will be focused at O. The point O will act as virtual object for refraction through surface 1.

For surface 1: Real depth for surface 1, $AO = d_1$

Here, $n_{\text{incident}} = \mu_1 = 1$

$n_{\text{refraction}} = \mu_2 = \mu$

Apparent depth, $d_2 = \dfrac{d_{\text{real}}}{n_{\text{relative}}} = \dfrac{d_1}{\left(\dfrac{n_{\text{incident}}}{n_{\text{refraction}}}\right)}$

$$d_2 = \frac{d_1}{\left(\dfrac{1}{\mu}\right)} = \mu d_1$$

Refracion at surface 2:

After refraction from surface 1, the image of 'O' will form at I_1. For surface 2, I_1 will act as an object.

For surface 2: Real depth for surface $BI_1 = d_1' = (d_2 - t)$

$$= (\mu d_1 - t)$$

Here, $n_{\text{incident}} = \mu_2 = \mu$

$n_{\text{refraction}} = \mu_1 = 1$

Apparent depth, $d_2' = \dfrac{d_{\text{real}}}{n_{\text{relative}}} = \dfrac{d_1'}{\left(\dfrac{n_{\text{incident}}}{n_{\text{refraction}}}\right)}$

$$= \frac{d_1'}{\left(\dfrac{\mu}{1}\right)} = \frac{(\mu d_1 - t)}{\mu}$$

$$d_2' = \frac{(\mu d_1 - t)}{\mu} = \left(d_1 - \frac{t}{\mu}\right)$$

Hence, shifting of object position from slab

$$OI = s = BI - BO = d_2' - (d_1 - t)$$

$$s = \left(d_1 - \frac{t}{\mu}\right) - (d_1 - t) = \left(t - \frac{t}{\mu}\right)$$

$$s = t\left(1 - \frac{1}{\mu}\right)$$

Image formation by a slab:

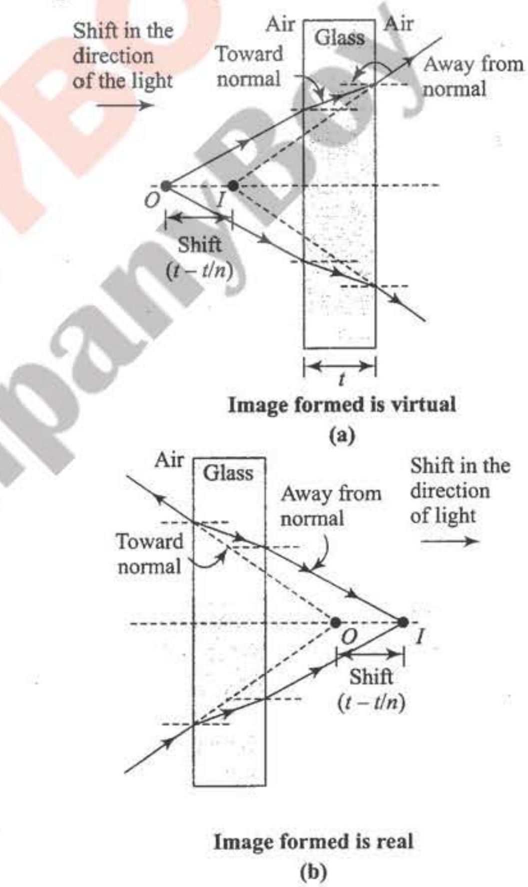


Fig. 1.109

In both cases, shifting is in the direction of the traveling ray.

Lateral Displacement of Emergent Beam Through a Glass Slab

Figure 1.110 represents the refraction of a ray AO incident on the slab at an angle of incidence i through the glass slab $EFGH$. At face EF, the incident ray AO is refracted along OP, the angle of refraction being r.

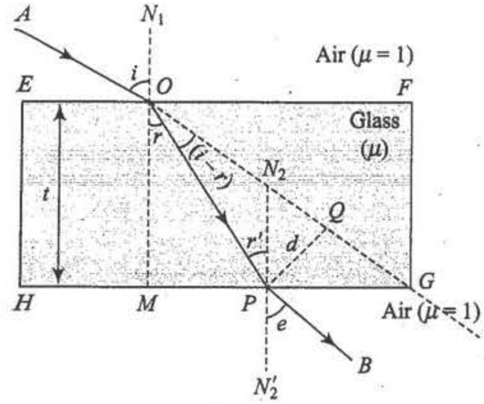


Fig. 1.110

(i) If e is angle of emergence, then from Snell's law at faces EF and HG,

$$\mu_a \sin i = \mu \sin r \quad \text{and} \quad \mu \sin r' = \mu_a \sin e$$

$$r' = r \quad \text{and} \quad \mu_a = 1, \text{ we have}$$

$$\sin i = \sin e \text{ or } e = i.$$

That is the emergent ray is parallel to the incident ray.

(ii) Let PQ be the perpendicular dropped from P on incident ray produced.

Then, the lateral displacement caused by the plate,

$$d = PQ = OP \sin(i - r) = \frac{OM}{\cos r} \sin(i - r)$$

$$= \frac{t \sin(i - r)}{\cos r}$$

Special Case

If i is very small, r is also very small, then

$\sin i \to i$, $\sin r \to r$ and $\cos r \to 1$ so that

$\dfrac{\sin i}{\sin r} = \mu$ takes the form $\dfrac{i}{r} = \mu$.

$\therefore$ The expression for lateral displacement takes the form

$$d = \frac{t(i - r)}{1} = ti\left(1 - \frac{r}{i}\right) = ti\left(1 - \frac{1}{\mu}\right)$$

$$d = \left(1 - \frac{1}{\mu}\right) ti$$

Illustration 1.47 **Find the lateral shift of a light ray while it passes through a parallel glass slab of thickness 10 cm placed in air. The angle of incidence in air is 60° and the angle of refraction in glass is 45°.**

Sol. $$d = \frac{t \sin(i - r)}{\cos r} = \frac{10 \sin(60° - 45°)}{\cos 45°}$$

$$= \frac{10 \sin 15°}{\cos 45°} = 10\sqrt{5} \sin 15°.$$

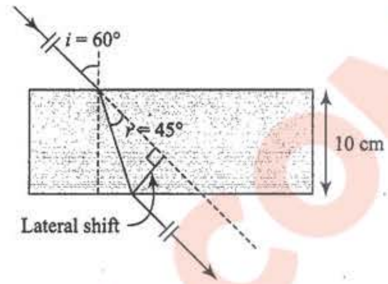
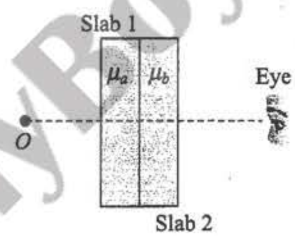


Fig. 1.111

REFRACTION ACROSS MULTIPLE SLABS

In Fig. 1.112, an object is placed in front of two slabs in contact. The thickness and refractive indices of the slabs are t_1, μ_a and t_2, μ_b, respectively. Where will the final image of the object appear to be?



A object placed in front of two glass slabs in contact

Fig. 1.112

A light ray emerging from O now refracts at three surfaces. The first is between air and μ_a the second between μ_a and μ_b while the third is between μ_b and air. Let us solve the problem taking one step at a time.

1st Interface (See Fig. 1.113): Here, $\mu_1 = 1$, $\mu_2 = \mu_a$

$$d_1 = x, d_2 = ?$$

$$d_2 = \frac{d_1}{\mu_{\text{relative}}} = \frac{d_1}{\mu_1/\mu_2}$$

$$\Rightarrow \quad \frac{\mu_1}{d_1} = \frac{\mu_2}{d_2} \quad \text{or} \quad d_2 = \frac{\mu_2}{\mu_1} d_1 = \mu_a d_1$$

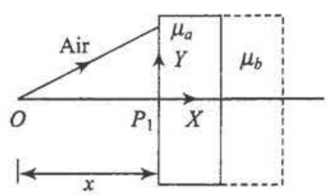


Fig. 1.113

Therefore, the image distance $d_2 = -\mu_a x$ (i)

2nd Interface (See Fig. 1.114): Here, $\mu_1 = \mu_a$, $\mu_2 = \mu_b$

$$d_1 = -(\mu_a x + t_1), d_2 = ?$$

Since $$\frac{\mu_1}{d_1} = \frac{\mu_2}{d_2}$$

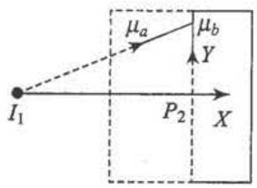


Fig. 1.114

The image distance $d_2 = -\mu_b\left(x + \dfrac{t_1}{\mu_a}\right)$ (ii)

3rd Interface (See Fig. 1.115): Here, $\mu_1 = \mu_b$, $\mu_2 = 1$

$$d_1 = -\mu_b\left(x + \frac{t_1}{\mu_a}\right) + t_2,\ d_2 = ?$$

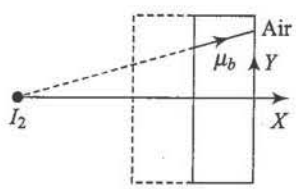


Fig. 1.115

and the final image distance from the 3rd interface is

$$d_2 = -\left[x + \frac{t_1}{\mu_a} + \frac{t_2}{\mu_b}\right] \qquad \text{(iii)}$$

Therefore, the net shift in the position of the image is

$$s = -\left(x + \frac{t_1}{\mu_a} + \frac{t_2}{\mu_b}\right) - [-(x + t_1 + t_2)]$$

or $$s = t_1\left(1 - \frac{t}{\mu_a}\right) + t_2\left(1 - \frac{t}{\mu_b}\right) \qquad \text{(iv)}$$

Looking at the above result, we realize that the net shift in the position of the image is simply the sum of the individual shifts at each of the slabs if they were independently placed in air.

Thus, the simple problem of refraction in a glass slab can be tackled in two ways:

1. By the method of interfaces: Here, the refraction formula, equation ($\mu_1/d_1 = \mu_2/d_2$), is applied at each interface.
2. By the method of elements: Here, the slab itself is an element with a governing equation $t\left[1 - \dfrac{1}{\mu}\right]$.

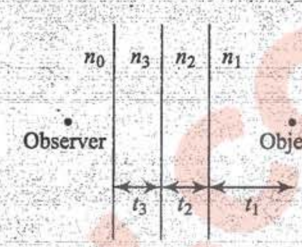
Most problems in ray optics can be solved by either of the two methods. In the following problems, we shall solve problems in both ways and highlight the method that gives a quicker solution.

Note: Refraction Through A Composite Slab

(or Refraction through a number of parallel media, as seen from a medium of R.I. n_0)

- ***Apparent depth (distance of final image from final surface)***

$$= \frac{t_1}{n_{1rel}} + \frac{t_2}{n_{2rel}} + \frac{t_3}{n_{3rel}} + \cdots + \frac{t_n}{n_{nrel}}$$



- ***Apparent shift***

$$= t_1\left[1 - \frac{1}{n_{1rel}}\right] + t_2\left[1 - \frac{1}{n_{2rel}}\right] + \cdots + t_n\left[1 - \frac{n}{n_{nrel}}\right]$$

where 't' represents thickness and 'n' represents the R.I. of the respective media, relative to the medium of observer (i.e. $n_{1rel} = n_1/n_0$, $n_{2rel} = n_2/n_0$ etc.)

Illustration 1.48 **In Fig. 1.116, find the apparent depth of the object seen below surface *AB*.**

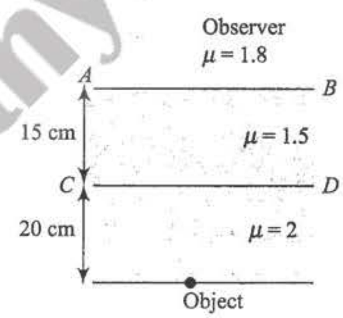


Fig. 1.116

Sol. $D_{app} = \sum \dfrac{d}{\mu} = \dfrac{20}{\left(\dfrac{2}{1.8}\right)} + \dfrac{15}{\left(\dfrac{1.5}{1.8}\right)} = 18 + 18 = 36$ cm

Illustration 1.49 **Light is incident from air on an oil layer at an incident angle of 30°. After moving through the oil 1, oil 2, and glass it enters water. If the refraction index of glass and water are 1.5 and 1.3, respectively. Find the angle which the ray makes with the normal in water.**

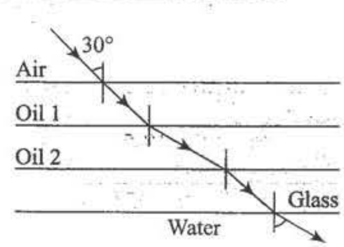


Fig. 1.117

Sol. As we know that $\mu \sin i = \text{constant}$

$$\Rightarrow \quad \mu_{\text{air}} \sin i_{(\text{air})} = \mu_{\text{glass}} \sin r_{(\text{glass})}$$

$$\sin i_{(\text{glass})} = \frac{\mu_{\text{air}}}{\mu_{\text{glass}}} \sin i_{\text{air}} \tag{i}$$

Again $\quad \mu_{\text{glass}} \sin i_{\text{glass}} = \mu_{\text{water}} \sin r_{\text{water}} \quad$ (ii)

From Eqs. (i) and (ii), $\sin 30° = 1.3 \sin r$

$$\Rightarrow \quad \sin r = \frac{1}{2 \times 1.3} = \frac{1}{2.6};\ r = \sin^{-1}\frac{1}{2.6}$$

Note: ***For parallel layers, we can apply Snell's law directly at initial and final parts.***

Illustration 1.50 **A layer of oil 3 cm thick is floating on a layer of coloured water 5 cm thick. Refractive index of coloured water is 5/3 and the apparent depth of the two liquids appears to be 36/7 cm. Find the refractive index of oil.**

Sol. Apparent depth (AI) $= \dfrac{t_1}{\mu_1} + \dfrac{t_2}{\mu_2}$

$$\therefore \quad \frac{36}{7} = \frac{5}{5/3} + \frac{3}{\mu_2} \Rightarrow \frac{3}{\mu_2} = \frac{36}{7} - 3 = \frac{15}{7} \Rightarrow \mu_2 = \frac{7}{5} = 1.4$$

Illustration 1.51 **In Fig. 1.118, determine the apparent shift in the position of the coin. Also, find the effective refractive index of the combination of the glass and water slab.**

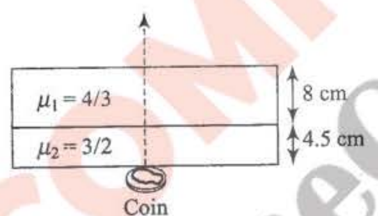


Fig. 1.118

Sol. Total apparent shift is

$$s = t_1\left(1 - \frac{1}{\mu_1}\right) + t_2\left(1 - \frac{1}{\mu_2}\right)$$

or $$s = 8\left(1 - \frac{1}{\frac{4}{3}}\right) + 4.5\left(1 - \frac{1}{\frac{3}{2}}\right)$$

or $$s = 2 + 1.5 = 3.5 \text{ cm}$$

The apparent depth of the coin from the top is $t = (8 + 4.5) - 3.5 = 9$ cm and, the real depth of the coin is

$$t_1 + t_2 = 8 + 4.5 = 12.5$$

Therefore, the effective refractive index is $\mu_{\text{eff}} = \dfrac{\text{Real depth}}{\text{Apparent depth}}$

$$= \frac{t_1 + t_2}{t} = \frac{12.5}{9}$$
$$= 1.39$$

SLAB AND MIRROR COMBINED

Let us observe what happens if one surface of the slab is silvered?

Consider the silvered slab shown in the Fig. 1.119. An object is placed in front of a silvered glass slab.

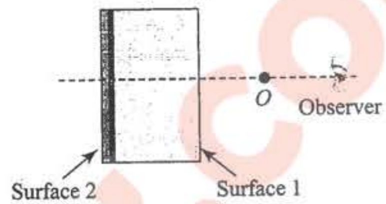


Fig. 1.119

Here, a ray of light from the object first refracts at surface 1. It is then reflected from surface 2 before refracting again at surface 1 and emerging (See Fig. 1.120). So, we can consider a silvered slab as a combination of

1. A refracting surface,
2. A reflecting surface, and
3. A refracting surface again.

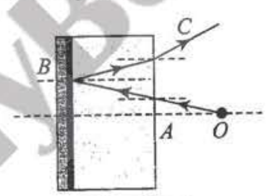


Fig. 1.120

The above situation can be considered as a combination of a slab and a plane mirror placed together. Thus, a silvered slab is a combination of

1. A glass slab,
2. A plane mirror, and
3. A glass slab again.

To learn the concept, we will discuss the situation through an illustration.

Illustration 1.52 **An object is placed in front of a slab (μ= 1.5) of thickness 6 cm at a distance 28 cm from it. Other face of the slab is silvered. Find the position of final image.**

Sol. Method of interface: A ray of light from the object O undergoes refraction, reflection and then refraction.

Refraction at surface 1:

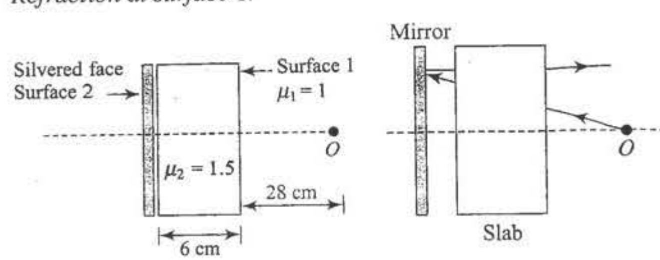


Fig. 1.121

Here, $\quad \mu_1 = 1,\ \mu_2 = 1.5$
$\quad d_1 = 28 \text{ cm},\ d_2 = ?$

Since $\quad d_2 = \dfrac{d_1}{n_{\text{relative}}} = \dfrac{d_1}{(1/\mu)}$; $\left[n_{\text{rel}} = \dfrac{n_{\text{incident}}}{n_{\text{refracted}}} = \dfrac{1}{\mu}\right]$

$\Rightarrow \quad d_2 = \mu d_1 = 1.5 \times 28 = 42$ cm

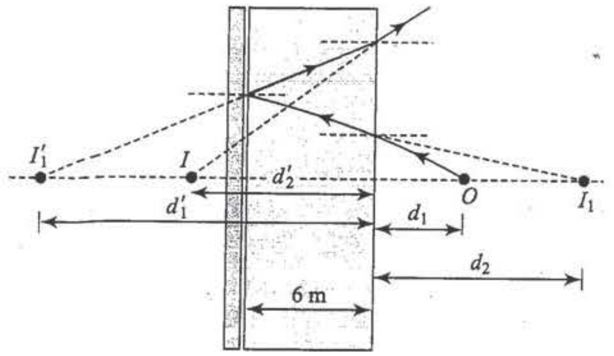


Fig. 1.122

Therefore, $d_2 = 42$ cm from the first interface. The first image I_1 is formed 42 cm in front of the slab.

Reflection at surface 2:

The object for reflection at the second surface is the image from refraction at the first.

Therefore, object distance from the mirror is = 42 + 6 = 48 cm.

As a result of reflection, the image will be formed as far behind the mirror as the object is in front of it. Therefore, the second image I_1' is formed 48 cm behind the mirror.

Second refraction at surface 1:

Object distance from surface 1,

$d_1 = 48 + 6 = 54$ cm

$$d_2' = \frac{d_1'}{n_{\text{relative}}} = \frac{d_1'}{\left(\dfrac{n_{\text{incident}}}{n_{\text{refraction}}}\right)} = \frac{54}{(1.5/1)} = 36 \text{ cm}$$

So, the final image is at a distance $d_2 = 36$ cm behind the first interface.

Hence, final image is formed 36 cm behind surface 1 or 30 cm behind surface 2.

Method 2: Shifting of object

A ray of light from the object first encounters a glass slab, then a mirror, and finally a glass slab again.

Glass slab: A slab simply shifts the object along the axis by a distance $s_1 = t\left(1 - \dfrac{1}{\mu}\right) = 2$ cm

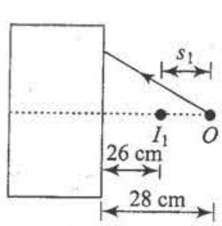


Fig. 1.123

Direction of shift of object is towards left. Therefore, the object appears to be at I_1 which is 28 – 2 = 26 cm from the slab.

For mirror, the object for the mirror is the image I_1 formed after shift due to the slab.

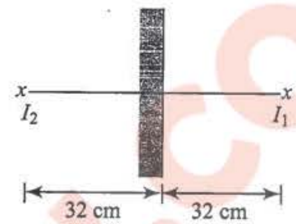


Fig. 1.124

Therefore, object distance from the mirror is 26 + 6 = 32 cm. The image will now be formed 32 cm behind the mirror. Now, reflected rays are travelling from left to right.

The ray now travels through the slab again but this time from right to left. Therefore, it is shifted again by a distance of 2 cm, but towards the right. Thus, final position of the image is 32 – 2 = 30 cm behind the mirror.

Method 3: Shifting of mirror

By the principle of reversibility of light, we can say if light rays are coming from the mirror and passing through the slab, the mirror will shift 2 m towards right for observer in front of the slab.

The position of the object from shifted mirror = 32 cm.

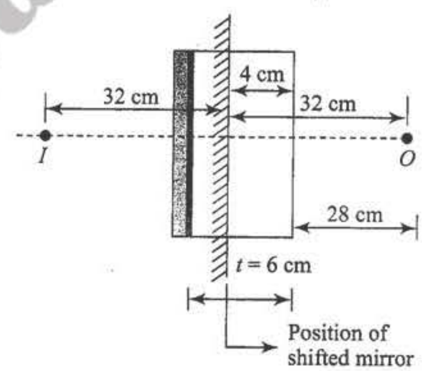


Fig. 1.125

So, the position of the image formed by shifted mirror will be 32 cm behind it. Hence, position of the image from surface 2 is 30 cm left to it and 36 cm left of surface 1.

Let us learn the combination of the slab and mirror through some more illustrations.

Illustration 1.53 **A 20 cm thick glass slab of refractive index 1.5 is kept in front of a plane mirror. Find the position of the image (relative to mirror) as seen by an observer through the glass slab when a point object is kept in air at a distance of 40 cm from the mirror.**

Sol. The rays from O will first pass through slab and produce shifting towards right. The glass slab will form an image of O at I_1 such that

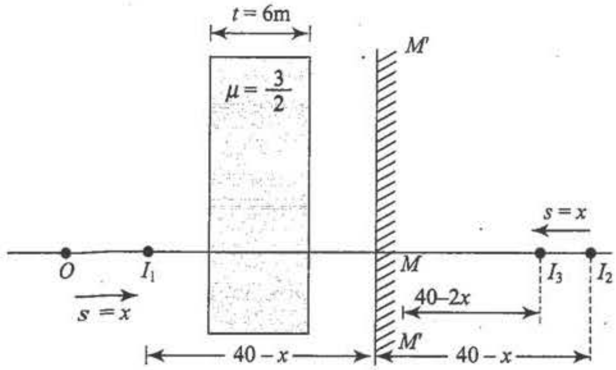


Fig. 1.126

$$OI_1 = x = d\left[1-\frac{1}{\mu}\right] = 20\left[1-\frac{2}{3}\right] = \frac{20}{3}\text{ cm} \qquad \text{(i)}$$

Now, the rays after passing through slab will (towards right)

So, the distance of I_1 from the mirror MM',

$$I_1M = (40 - x).$$

This image I_1 will act as an object for the mirror and the mirror will form an image I_2, such that

$$MI_2 = MI_1 = 40 - x \qquad \text{[with } x \text{ given by Eq. (i)]}$$

Now, image I_2 will act as an object again for the glass slab which by producing a shift of x forms the final image I_3 such that $I_2I_3 = x$. Hence, the distance of final image I_3 from the mirror will be

$$MI_3 = MI_2 - I_3I_2 = (40 - x) - x = 40 - 2x$$

$$MI_3 = 40 - 2\times\left[\frac{20}{3}\right] = \left[\frac{80}{3}\right]\text{ cm}$$

[as from Eq. (i), $x = 20/3$ cm]

Alternative solution: As thickness of glass slab is 20 cm and $\mu = (3/2)$, so shift produced by it will be

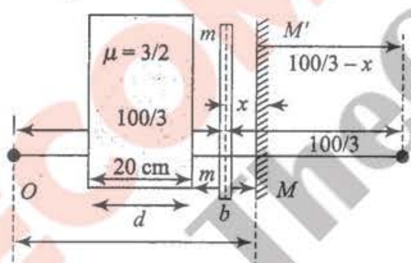


Fig. 1.127

$$x = d\left[1-\frac{d}{\mu}\right] = 20\left[1-\frac{2}{3}\right] = \frac{20}{3}\text{ cm}$$

So, the glass slab will shift the mirror from MM' to mm' as shown in.

The distance of object from this virtual mirror will be

$$40 - x = 40 - (2/3) = (100/3)\text{ cm}$$

This virtual mirror will form the image of object O at a distance (100/3) behind it and so the distance of image from actual mirror MM' will be $\frac{100}{3} - \frac{20}{3} = \frac{80}{3}$ cm [as mm' is 20/3 cm in front of MM']

Illustration 1.54 **A point object O is placed in front of a concave mirror of focal length 10 cm. A glass slab of refractive index $\mu = 3/2$ and thickness 6 cm is inserted between the object and mirror.**

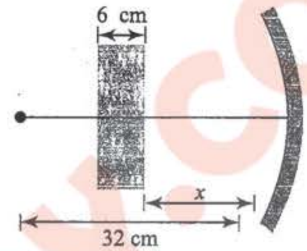


Fig. 1.128

Find the position of final image when the distance x shown in Fig. 1.128 is, (a) 5 cm and (b) 20 cm.

Sol. The normal shift produced by a glass slab is,

$$S = \left(1-\frac{1}{\mu}\right)t = \left(1-\frac{2}{3}\right)(6) = 2\text{ cm}$$

i.e., for the mirror the object is placed at a distanced $(32 - S) = 30$ cm from it.

Applying mirror formula $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ or $\frac{1}{v}+\frac{1}{30}=-\frac{1}{10}$

or $v = -15$ cm

a. When $x = 5$ cm: The light falls on the slab on its return journey as shown. But the slab will again shift it by a distance $S = 2$ cm. Hence, the final real image is formed at a distance $(15 + 2) = 17$ cm from the mirror.

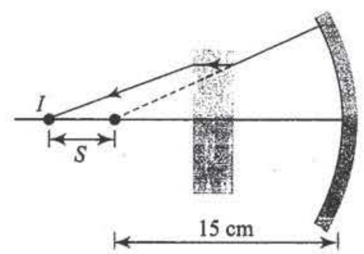


Fig. 1.129

b. When $x = 20$ cm: This time also the final image is at a distance 17 cm from the mirror but it is virtual as shown.

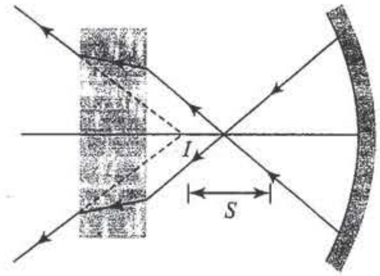


Fig. 1.130

Illustration 1.55 **A vessel having perfectly reflecting plane bottom is filled with water ($\mu = 4/3$) to a depth d. A point source of light is placed at a height h above the surface of water. Find the distance of final image from water surface.**

Sol. As shown in Fig. 1.131, water will form the image of object O at I_1 such that $OI_1 = y = d[1 - (1/\mu)]$, so that the distance of image I_1 from water surface will be $I_1A = h - y = h - d[1 - (1/\mu)]$. Hence, the distance of this image I_1 from mirror MM',

$$I_1M = I_1A + AM = \left[h - d\left(1 - \frac{1}{\mu}\right)\right] + d = h + \left[\frac{d}{\mu}\right]$$

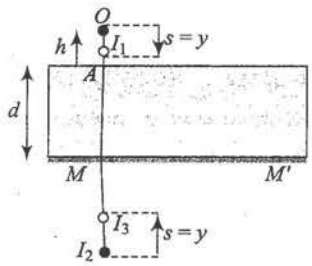


Fig. 1.131

Now, image I_1 will act as object for mirror MM'. As a plane mirror forms image at same distance behind the mirror as the object is in front of it, the image of I_1 formed by the mirror MM' will be I_2 such that $I_1M = I_2M = h + (d/\mu)$.

Now, this image I_2 will act as object for water again and water will produce image I_3 such that $I_2I_3 = y = d\left(1 - \frac{1}{\mu}\right)$. So, the distance of image I_3 from the surface of water AC will be

$$AI_3 = AM + MI_2 - I_2I_3 = d + \left[h + \frac{d}{\mu}\right] - d\left[1 - \frac{1}{\mu}\right]$$

$$AI_3 = h + 2\frac{d}{\mu} = h + \frac{3}{2}d$$

Alternative solution:

As shown in Fig. 1.132, water will form the image of bottom, i.e., mirror MM' at a depth (d/μ) from its surface. So, the distance of object O from virtual mirror mm' will be $h + (d/\mu)$.

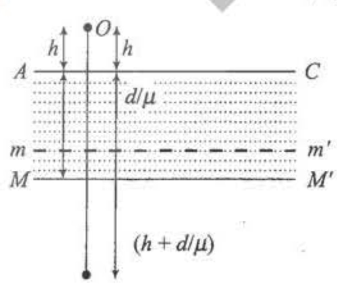


Fig. 1.132

Now, as a plane mirror forms image behind the mirror at the same distance as the object is in front of it, the distance of image I from mm' will be $h + (d/\mu)$. Also, as the distance of virtual mirror from the surface of water is (d/μ), the distance of image I from the surface of water will be

$$\left[h + \frac{d}{\mu}\right] + \frac{d}{\mu} = h + \frac{2d}{\mu} = h + \frac{3}{2}d \qquad \left[\text{as } \mu = \frac{4}{3}\right]$$

Illustration 1.56 **A concave mirror of focal length 20 cm is placed inside water with its shining surface upwards and principal axis vertical as shown in Fig. 1.133. Rays are incident parallel to the principal axis of concave mirror. Find the position of final image.**

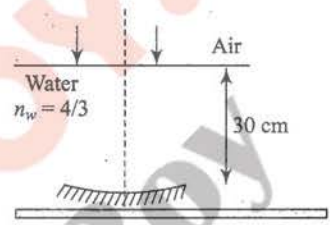


Fig. 1.133

Sol. The incident rays will pass undeviated through the water surface and strike the mirror parallel to its principal axis. Therefore, for the mirror, object is at ∞. Its image A (in figure Fig. 1.134) will be formed at focus which is 20 cm from the mirror. Now, for the interface between water and air. For observer at air, the image formed by mirror will act as an object for observer with real depth $d = 30 - 20 = 10$ cm.

Apparent depth observed by observer,

$$d' = \frac{d}{\left(\frac{n_w}{n_a}\right)} = \frac{10}{\left(\frac{4/3}{1}\right)} = 7.5 \text{ cm}$$

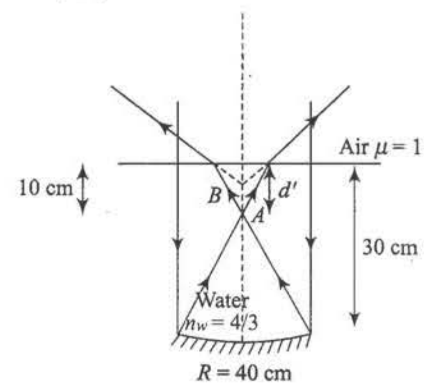


Fig. 1.134

REFRACTION IN A MEDIUM WITH VARIABLE REFRACTIVE INDEX

So far, we have assumed that the refractive index of the slab is a constant. This need not be necessarily true. An example of such a situation is the atmosphere. The atmosphere becomes thinner as we go up. Hence, the refractive index of air is highest close to surface of earth and decreases as we move upward. How do we analyse such a problem?

Divide the medium into different layers. This indicates that each layer has different refractive index having value according to a given expression. If μ is a function of x, then a differential layer will be a thin layer of thickness dx. If μ is a function of y, then a differential layer will be a thin layer of thickness dy.

At a given layer, let the angle of incidence be i. Relate this angle of incidence to the initial condition and the refractive index at that point using the relation

$$\mu \sin(i) = \text{constant} \tag{i}$$

From the given ray diagram (Fig. 1.135), draw a tangent to the path taken by the ray of light. Geometrically, relate the slope of this tangent to the angle of incidence.

$$\tan\theta = \frac{dy}{dx} = \tan(90 - i) \tag{ii}$$

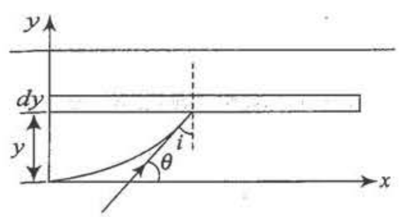


Fig. 1.135

Substitute for i from Eq. (i) and determine dy/dx as a function of x and y.

Integrate and obtain an expression for y as a function of x.

Let us discuss a case where refractive index is changing in y-direction.

As 1: $\mu = f(y)$

Consider a slab with a medium where the refractive index varies from μ_1 to μ_2 in relation with y. Assume that the outside medium is air. Let light be incident on the glass slab at an angle θ. The light ray strikes the glass slab at the point A. The normal at this point is NN' as shown in Fig. 1.136. Once the ray enters the medium, the refractive index varies continuously as if there are infinite number of successive glass slabs. Therefore, the angle of incidence also varies continuously and the path of the ray is curved. The ray finally reaches the second surface at the point B before exiting the slab, once again refracting at the exit point.

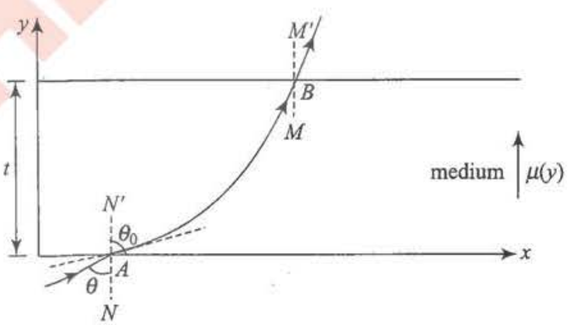


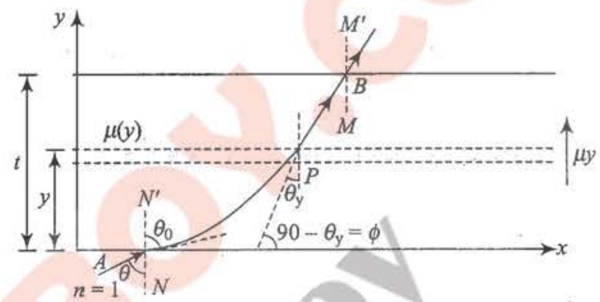
Fig. 1.136

How can we derive a mathematical expression for the equation of the ray in the medium?

Consider refraction at some height y (Fig. 1.137). Here, the angle of incidence is θ_y and the refractive index of the medium is μ_y.

The product of the refractive index and the sine of the angle of incidence is constant. Therefore, at the height y we can say that

$$\mu_y \sin\theta_y = \text{constant.} \tag{i}$$



Path of a ray of light being refracted in a slab with refractive index as a function of y and y ranges from 0 to 1.

Fig. 1.137

Furthermore, applying the law of refraction at point A which is the interface between air and the medium, we have

$$1 \times \sin(\theta) = \mu_y \sin\theta_y \tag{ii}$$

Therefore, $\sin\theta_y = \dfrac{\sin\theta}{\mu_y}$

$$\cos\theta_y = \frac{\sqrt{\mu_y^2 - \sin^2\theta}}{\mu_y}$$

Hence, $\cot\theta_y = \dfrac{\sqrt{\mu_y^2 - \sin^2\theta}}{\sin\theta}$

Now, the slope of the curve at the point P is given by

$$\tan\phi = \frac{dy}{dx} \tag{iii}$$

But $\phi = 90 - \theta_y$

Therefore,
$$\frac{dy}{dx} = \cot(\theta_y) = \frac{\sqrt{\mu_y^2 - \sin^2\theta}}{\sin\theta} \tag{iv}$$

$$\frac{\sin\theta\, dy}{\sqrt{\mu_y^2 - \sin^2\theta}} = dx$$

Integrating both side with proper limits.

If the functional dependence of "μ_y" is known, we can easily solve for the equation of the curve.

$$\sin\theta \int_{y_1}^{y_1} \frac{dy}{\sqrt{\mu_y^2 - \sin^2\theta}} = \int_{x_1}^{x_2} dx$$

Illustration 1.57 **A ray of light is incident on a glass slab at grazing incidence. The refractive index of the material of the slab is given by $\mu = \sqrt{(1+y)}$. If the thickness of the slab is $d = 2$ m, determine the equation of the trajectory of the ray inside the slab and the coordinates of the point where the ray exits from the slab. Take the origin to be at the point of entry of the ray.**

Sol. From the equation, we have $\frac{dy}{dx} = \cot(\theta_y) = \frac{\sqrt{\mu_y^2 - \sin^2\theta}}{\sin\theta}$

Here, $\mu = \sqrt{(1+y)}$ and $\theta = 90°$. Therefore, $\frac{dy}{dt} = y^{1/2}$

Integrating with the boundary condition that $y = 0$ at $x = 0$, we get $y = x^2/4$ to be the equation of the path of the ray through the slab. The ray will obviously exit at the point $(2\sqrt{2}\text{ m}, 2\text{ m})$.

Illustration 1.58 **Due to a vertical temperature gradient in the atmosphere, the index of refraction varies. Suppose index of refraction varies as $n = n_0\sqrt{1+ay}$, where n_0 is the index of refraction at the surface and $a = 2.0 \times 10^{-6}\text{ m}^{-1}$. A person of height h = 2.0 m stands on a level surface. Beyond what distance will he not see the runway?**

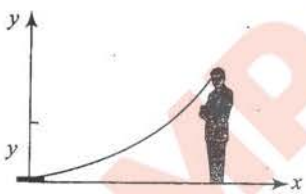


Fig. 1.138

Sol. As refractive index is changing along y-direction, we can assume a number of thin layers of air placed parallel to x-axis. Let O be the distant object just visible to the man. Consider a layer of air at a distance y from the ground. Let P be a point on the trajectory of the ray. From Fig. 1.139, $\theta = 90 - i$.

The slope of tangent at point P is $\tan\theta = dy/dx = \cot i$.

From Snell's law, $n \sin i$ = constant

At the surface, $n = n_0$ and $i = 90°$.

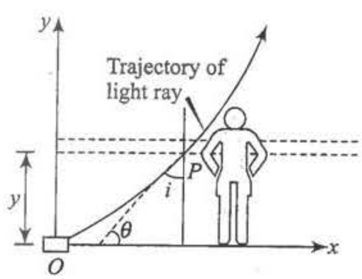


Fig. 1.139

$$n_0 \sin 90° = n \sin i = (n_0\sqrt{1+ay}) \sin i$$

$$\sin i = \frac{1}{\sqrt{1+ay}} \Rightarrow \cot i = \frac{dy}{dx} = \sqrt{ay}$$

$$\int_0^y \frac{dy}{\sqrt{ay}} = \int_0^x dx \Rightarrow x = 2\sqrt{\frac{y}{a}}$$

On substituting $y = 2.0$ m and $a = 2 \times 10^{-6}\text{ m}^{-1}$, we have

$$x_{max} = 2\sqrt{\frac{2}{2\times10^{-6}}} = 2000\text{ m}$$

MEASUREMENT OF REFRACTIVE INDEX OF A LIQUID BY A TRAVELLING MICROSCOPE

Using the fact that $\mu = \frac{\text{Real depth}}{\text{Apparent depth}}$

We may determine the refractive index of a glass slab. (Fig. 1.140)

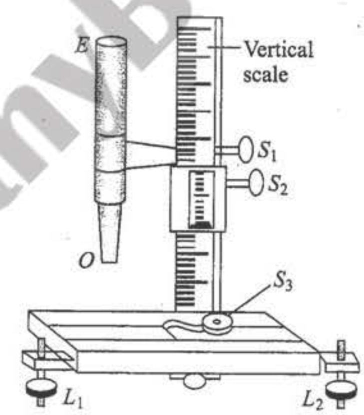


Fig. 1.140

The traveling microscope is first focused on a fine mark on a piece of paper and the reading on the scale is noted as Y_1. Then, the glass slab is placed on the paper.

The microscope is now shifted up and is focused again on the mark so that it is distinctly visible. The reading of the microscope is noted as Y_2.

Now, a little lycopodium powder is scattered on the upper surface of the glass block. The microscope is focused on the powder and the third reading of the scale is noted as Y_3.

The real thickness of the glass slab is $Y_3 - Y_1$ and apparent thickness of the glass slab is $Y_3 - Y_2$. Thus,

$$\mu = \frac{Y_3 - Y_1}{Y_3 - Y_2}$$

The same method can also be used to find the refractive index of a liquid. Any mark on the bottom of a glass vessel is first focused and then liquid is poured into it and the same mark is again focused. Finally, a little lycopodium powder is sprinkled on the surface of the liquid and is focused as before.

Concept Application Exercise 1.3

1. Identify the True and False statements.
   (i) A glass slab cannot deviate the light.
   (ii) A glass slab can produce lateral displacement.
   (iii) The shift produced by a slab depends on the converging and diverging nature of beam.
   (iv) Apparent shift in case of a slab always occurs in the direction of light ray travelling.
   (v) The shift produced by a slab can never exceed its thickness.
2. In figure, a point source S is placed at a height h above the plane mirror in a medium of refractive index μ.
   (i) Find the number of images seen for normal view.
   (ii) Find the distance between the images.

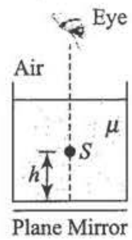


Fig. 1.141

3. A beam of width t is incident at 45° on an air–water boundary. The width of the beam in water is ______.

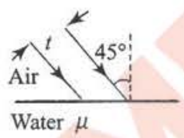


Fig. 1.142

4. A fish is 60 cm under water (μ = 4/3). A bird directly overhead looks at the fish. If the bird is at a distance of 120 cm from the water surface, (i) what is the apparent position of the fish as seen by the bird and (ii) what is the apparent position of the bird as seen by the fish?
5. A converging set of rays, traveling from water to air, is incident on a plane interface. In the absence of the interface, the rays would have converged to a point O, 60 cm above the interface. However, due to refraction the rays will bend. At what distance above the interface will the rays actually converge?

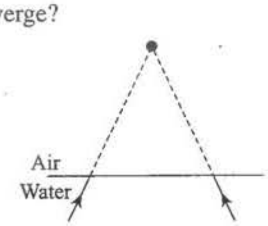


Fig. 1.143

6. A tank contains three layers of immiscible liquids. The first layer is of water with refractive index 4/3 and thickness 8 cm. The second layer is of oil with refractive index 3/2 and thickness 9 cm while the third layer is of glycerine with refractive index 2 and thickness 4 cm. Find the apparent depth of the bottom of the container.

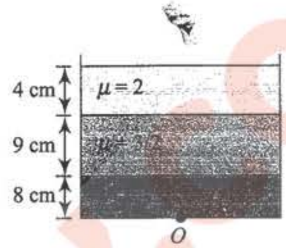


Fig. 1.144

7. A convergent beam is incident on two slabs placed in contact as shown in Fig. 1.145. Where will the rays finally converge?

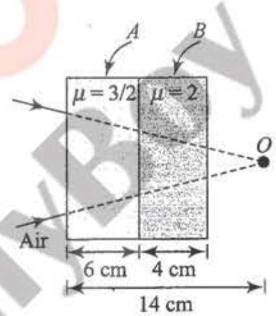


Fig. 1.145

8. A slab of water is on the top of a glass slab of refractive index 2. At what angle to the normal must a ray be incident on the top surface of the glass slab so that it is reflected from the bottom surface as shown in Fig. 1.146?

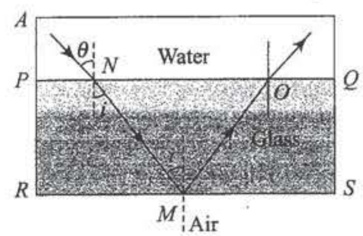


Fig. 1.146

9. A ray of light travels from a liquid of refractive index μ to air. If the incident beam is rotating at a rate ω, what is the angular speed of the refracted beam at the instant the angle of incidence is 30°?
   (Given $\mu = \sqrt{2}, \omega = 1/\sqrt{6}$ rad/sec.)
10. What should be the value of refractive index n of a glass rod placed in air, so that the light entering through the flat surface of the rod does not cross the curved surface of the rod?

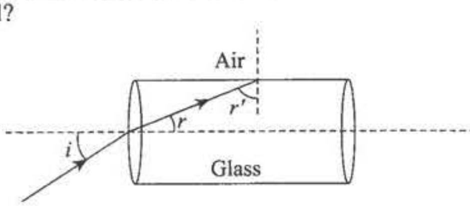


Fig. 1.147

11. An object is placed on the principle axis of a concave mirror of focal length 10 cm at a distance of 21 cm from it. A glass slab is placed between the mirror and the object as shown in Fig. 1.148.

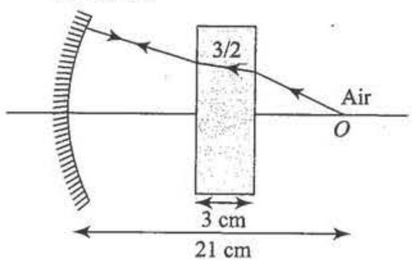


Fig. 1.148

Find the distance of final image formed by the mirror.

12. The image of an object kept at a distance of 30 cm in front of a concave mirror is found to coincide with itself. If a glass slab (μ = 1.5) of thickness 3 cm is introduced between the mirror and the object, then

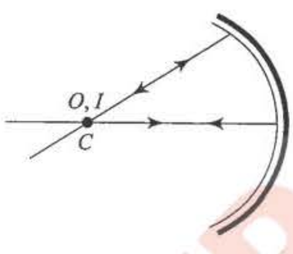


Fig. 1.149

a. Identify, in which direction the mirror should be displaced so that the final image may again coincide with the object itself.

b. Find the magnitude of displacement.

13. In Fig. 1.150, a fish watcher watches a fish through a 3.0 cm thick glass wall of a fish tank. The watcher is in level with the fish; the index of refraction of the glass is 8/5 and that of the water is 4/3.

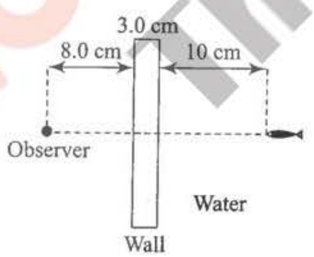


Fig. 1.150

a. To the fish, how far away does the watcher appear to be?

b. To the watcher, how far away does the fish appear to be?

14. A observer can see through a pin hole, the top of a thin rod of height h, placed as shown in Fig. 1.151. The beaker's height is $3h$ and its radius is h. When the beaker is filled with a liquid upto a height $2h$, he can see the lower end of the rod. Find the refractive index of the liquid.

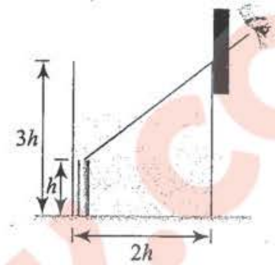


Fig. 1.151

15. A vessel contains a slab of glass 8 cm thick and of refractive index 1.6. Over the slab, the vessel is filled by oil of refractive index μ upto height 4.5 cm and then by another liquid, i.e., water of refractive index 4/3 and height 6 cm as shown in Fig. 1.152. An observer looking down from above observes that a mark at the bottom of glass slab appears to be raised up to a position 6 cm from bottom of the slab. Find refractive index of oil (μ).

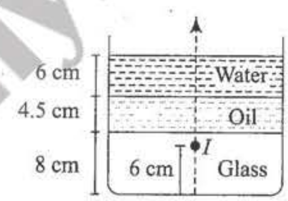


Fig. 1.152

16. An object O is placed at 8 cm in front of a glass slab, whose one face is silvered as shown in Fig. 1.153. The thickness of the slab is 6 cm. If the image formed 10 cm behind the silvered face, find the refractive index of glass.

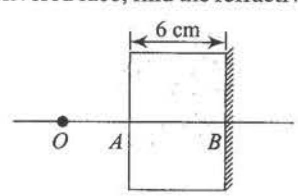


Fig. 1.153

17. x-y plane separates two media, $z \geq 0$ contains a medium of refractive index 1 and $z \leq 0$ contains a medium of refractive index 2. A ray of light is incident from first medium along a vector $\hat{i} + \hat{j} - \hat{k}$. Find the unit vector along the refracted ray.

18. The n transparent slabs of refractive index 1.5 each having thicknesses 1 cm, 2 cm, ... to n cm are arranged one over another. A point object is seen through this combination with near perpendicular light. If the shift of object by the combination is 1 cm, then find the value of n.

19. A concave mirror with its optic axis vertical and mirror facing upward is placed at the bottom of the water tank. The radius of curvature of the mirror is 40 cm and refractive index for water μ = 4/3. The tank is 20 cm deep and if a bird is flying over the tank at a height of 60 cm above the surface of water, find the position of image of the bird.

20. Consider the situation shown in Fig. 1.154. A plane mirror is fixed at a height h above the bottom of a beaker containing water (refractive index μ) upto a height d. Find the position of the image of the bottom formed by the mirror.

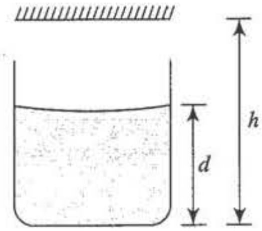


Fig. 1.154

21. A concave mirror of radius 40 cm lies on a horizontal table and water is filled in it up to a height of 5.00 cm. A small dust particles floats on the water surface at a point P vertically above the point of contact of the mirror with the table. Locate the image of the dust particle as seen from a point directly above it. The refractive index of water is 1.33.

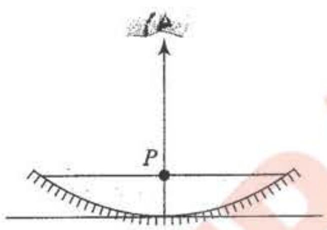


Fig. 1.155

22. A concave mirror of radius R is kept on a horizontal table. Water (refractive index = μ) is poured into it upto a height h. Where should an object be placed so that its image is formed on itself ?

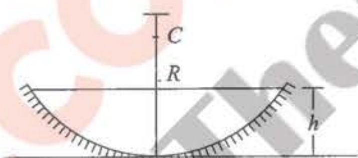


Fig. 1.156

23. The refractive index of an anisotropic medium varies as $\mu = \mu_0 = \sqrt{(x+1)}$, where $0 \le x \le a$. A ray of light is incident at the origin just along y-axis (shown in figure). Find the equation of ray in the medium.

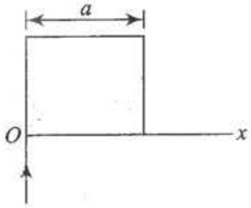


Fig. 1.157

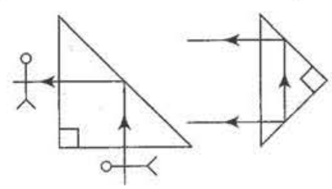
PRISM

A prism is a transparent medium whose refracting surfaces are not parallel but are inclined to each other (Fig. 1.158).

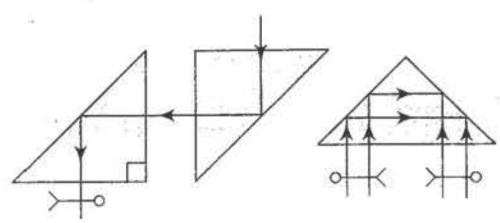
Fig. 1.158

Prisms have the property to change the direction of light. Fig. 1.159 (a) to (e) present some of the useful applications.

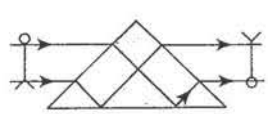
The right-angled prism in figure (a) turns the light through 90°. A poro prism shown in figure (b) is a right-angled prism, it turns light through 180°. Figure (c) shows two right-angled prisms used in binoculars. Figures (d) and (e) illustrate image formation with and without deviation; in both the cases the image is laterally inverted.



(a) **(b)**



(c) **(d)**



(e)

Fig. 1.159

Basic Terms

(i) Angle of prism or reflecting angle (A):
The angle between the faces on which light is incident and from which it emerges (Fig. 1.160).

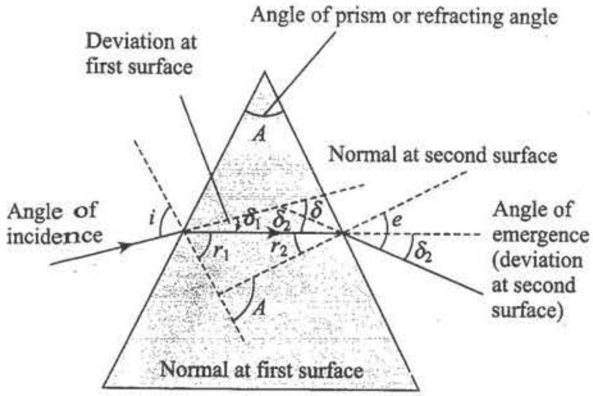


Fig. 1.160

(ii) Angle of deviation (δ):

It is the angle between the emergent and the incident ray. It other words, it is the angle through which incident ray turns in passing through a prism.

$$\delta = (i - r_1) + (e - r_2) \quad \text{or} \quad \delta = i + e - (r_1 + r_2)$$

or $$\delta = i + e - A$$

Condition of No Emergence

A ray of light incident on a prism of angle A and refractive index μ will not emerge out of a prism (whatever may be the angle of incidence) if $A > 2\theta_c$, where θ_c is the critical angle, i.e., $\mu > 1/[\sin(A/2)]$. (see Fig. 1.161).

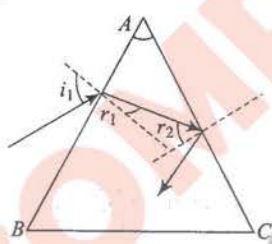


Fig. 1.161

Illustration 1.59 **What should be the minimum value of refractive index of a prism, refracting angle A, so that there is no emergent ray irrespective of the angle of incidence?**

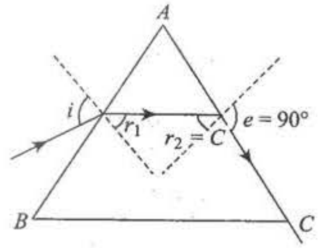


Fig. 1.162

Sol. If the ray just emerges from face AC,

$$e = 90^\circ \quad \text{and} \quad r_2 = C \qquad \text{(i)}$$

From Snell's law at face AB, we have

$$1 \sin i = n \sin r_1 \qquad \text{(ii)}$$

$$A = r_1 + r_2 = r_1 + C \qquad \text{(iii)}$$

From Eq. (ii), n is minimum when r_1 is maximum, i.e., $r_2 = C$. In this case, $i = 90^\circ$

From Eq. (iii), $A = 2C$ or $C = A/2$

As $$\sin C = \frac{1}{n} \Rightarrow \sin\frac{A}{2} = \frac{1}{n}$$

$$n = \operatorname{cosec}\frac{A}{2}$$

Condition of Grazing Emergence

By the condition of grazing emergence, we mean the angle of incidence i at which the angle of emergence becomes 90° (see Fig. 1.163).

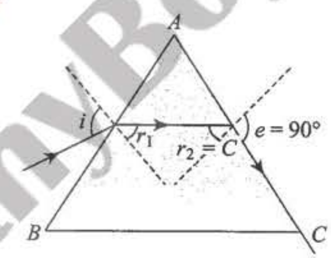


Fig. 1.163

Consider a prism with refracting angle A and refractive index n. The ray grazes face AC.

So, $$e = 90^\circ, r_2 = C \qquad \text{(i)}$$

and $$A = r_1 + r_2 = r_1 + C \qquad \text{(ii)}$$

Also, $\sin C = 1/n$

From Snell's law at face AB, $1 \sin i = n \sin r_1$

$$\sin i = n \sin (A - C)$$
$$= n[\sin A \cos C - \cos A \sin C]$$
$$= n\left[\sin A\sqrt{1 - \sin^2 C} - \cos A \sin C\right]$$
$$= \sqrt{n^2 - 1}\sin A - \cos A$$
$$i = \sin^{-1}\left[\sqrt{n^2 - 1}\sin A - \cos A\right]$$

Condition of Maximum Deviation

Maximum deviation occurs when the angle of incidence is 90° (Fig. 1.164).

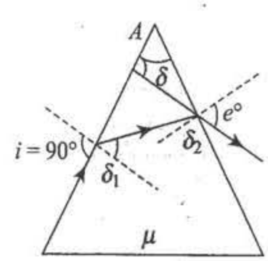


Fig. 1.164

$$\delta_{max} = 90° + e - A$$

where $\quad e = \sin^{-1}[\mu \sin(A - \theta_c)]$

Condition of Minimum Deviation

The minimum deviation occurs when the angle of incidence is equal to the angle of emergence (Fig. 1.165),

i.e, $\quad i = e;\ \delta_{min} = 2i - A.$

Using Snell's law, we get

$$\mu = \frac{\sin[(\delta_{min} + A)/2]}{\sin[A/2]}$$

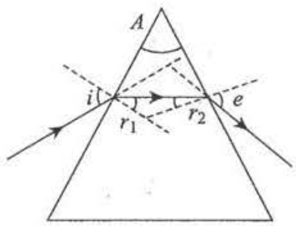


Fig. 1.165

Note that in the condition of minimum deviation the light ray passes through the prism symmetrically, i.e., the light ray in the prism becomes parallel to its base.

(i) Variation of δ versus i (shown in diagram).
For each δ (except δ_{min}), there are two values of angle of incidence. If i and e are interchanged, then we get the same value of δ because of reversibility principle of light.

(ii) There is one and only one angle of incidence for which the angle of deviation is minimum.

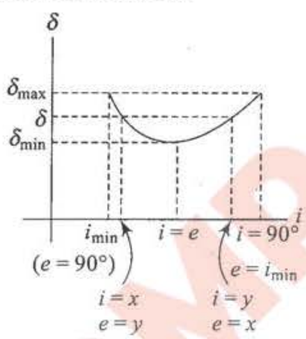


Fig. 1.166

Illustration 1.60 **An isosceles prism has one of the refracting surfaces silvered. A ray of light is incident normally on the refracting face AB. After two reflections, the ray emerges from the base of the prism perpendicular to it. Find the angle of the prism.**

Sol. The incident ray passes without deviation from face AB. It suffers reflections at P and Q. From Fig. 1.167, incident ray and normal at Q are parallel; therefore,

$$a = 2A \tag{i}$$

Also,
$$2\alpha + A = 180° \tag{ii}$$

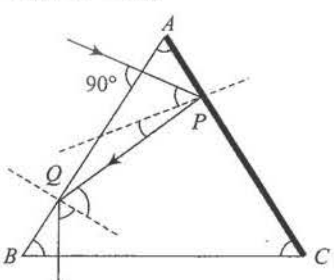


Fig. 1.167

On solving Eqs. (i) and (ii), we get $A = 36°$, $\alpha = 72°$

Illustration 1.61 **Figure 1.168 shows a triangular prism of refracting angle 90°. A ray of light incident at face AB at an angle θ_1 refracts at point Q with an angle of refraction 90°.**

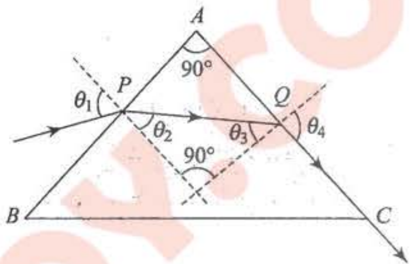


Fig. 1.168

a. What is the refractive index of the prism in terms of θ_1?
b. What is the maximum value that the refractive index can have?
What happens to the light at Q if the incident angle at Q is
c. increased slightly, and
d. decreased slightly?

Sol. a. Let the ray be incident at an angle θ_1 at the face AB. It refracts at an angle θ_2 and is incident at an angle θ_3 at face AC. Finally, the ray comes out at an angle $\theta_4 = 90°$.
From Fig. 1.169, the normals at faces AB and AC make an angle of 90° with each other, $\theta_3 = 90° - \theta_2$

$$\sin\theta_3 = \sin(90° - \theta_2) = \cos\theta_2 = \sqrt{1 - \sin^2\theta_2} \tag{i}$$

From Snell's law at face AC, $n \sin\theta_3 = 1$

$$n\sqrt{1 - \sin^2\theta_2} = 1 \tag{ii}$$

From Snell's law at face AB, $1 \sin\theta_1 = n \sin\theta_2$

$$\sin\theta_2 = \frac{\sin\theta_1}{n} \tag{iii}$$

From Eqs. (ii) and (iii), we have

$$n\sqrt{1 - \frac{\sin^2\theta_1}{n^2}} = 1 \tag{iv}$$

On squaring Eq. (iv) and solving for n, we get

$$n = \sqrt{1 + \sin^2\theta_1}$$

b. The greatest possible value of $\sin^2\theta_1$ is 1, hence the greatest possible value of n is $n_{max} = \sqrt{2} = 1.41$.

c. For a given n, if θ_1 is increased the angle of refraction θ_2 increases. As $\theta_3 = 90° - \theta_2$ the angle θ decreases, i.e., the angle of incidence at face AC is less than the critical angle for total reflection; hence light emerges into air.

d. If the angle of incidence is decreased, the angle of refraction θ_2 decreases. So, the angle θ_3 increases. The angle of incidence at the second surface is greater than the critical angle; so light is reflected at Q.

Note: ***In the condition of minimum deviation, the light ray passes through the prism symmetrically, i.e., the light ray in the prism becomes parallel to its base.***

Illustration 1.62 **A prism has refracting angle equal to $\pi/2$. It is given that γ is the angle of minimum deviation and β is the deviation of the ray entering at grazing incidence. Prove that $\sin\gamma = \sin^2\beta$.**

Sol. Applying condition of minimum deviation,

$$\mu = \frac{\sin\frac{(A+\gamma)}{2}}{\sin\frac{A}{2}} = \frac{\sin\frac{A}{2}\cos\frac{\gamma}{2} + \cos\frac{A}{2}\sin\frac{\gamma}{2}}{\sin\frac{A}{2}}$$

$$\Rightarrow \quad \mu = \cos\frac{\gamma}{2} + \cot\frac{A}{2}\sin\frac{\gamma}{2}$$

Using $A = 90°$, $\mu = \cos\frac{\gamma}{2} + \cot 45° \sin\frac{\gamma}{2}$

$$\Rightarrow \quad \cos\frac{\gamma}{2} + \sin\frac{\gamma}{2} = \mu$$

Squaring, $\cos^2\frac{r}{2} + \sin^2\frac{\gamma}{2} + \gamma = \mu^2 \Rightarrow \sin\gamma = \mu^2 - 1$ (i)

Deviation at grazing incidence,

$$\beta = \delta_1 + \delta_2$$

$$\beta = \left(\frac{\pi}{2} - C\right) + (e - r_2)$$

$$\Rightarrow \quad \beta = \left(\frac{\pi}{2} - C\right) + \left[e - \left(\frac{\pi}{2} - C\right)\right]$$

$$\Rightarrow \quad \beta = e$$

or $\quad \sin\beta = \sin e = \mu \sin r_2 = \mu \sin\left(\frac{\pi}{2} - C\right)$

$\Rightarrow \quad \sin\beta = \mu\cos C$ (ii)

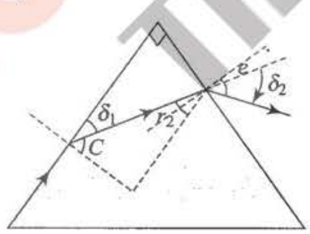


Fig. 1.169

Squaring Eq. (ii),

$$\sin^2\beta = \mu^2\cos^2 C \quad \Rightarrow \quad \sin^2\beta = \mu^2(1 - \sin^2 C)$$

Using $\quad \sin C = \frac{1}{\mu}$, $\sin^2\beta = \mu^2\left(1 - \frac{1}{\mu^2}\right)$

$\Rightarrow \quad \sin^2\beta = \mu^2 - 1$ (iii)

From Eqs. (i) and (iii),

$$\sin\gamma = \sin^2\beta$$

Illustration 1.63 **A rectangular block of refractive index μ is placed on a printed page lying on a horizontal surface as shown in Fig. 1.170. Find the minimum value of μ so that the letter L on the page is not visible from any of the vertical sides.**

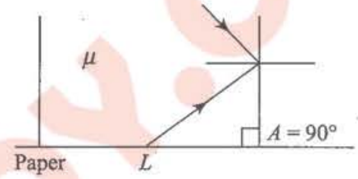


Fig. 1.170

Sol. The letter L will not be visible from the vertical sides if the light ray does not enter through it.

We can apply the condition of no emergence for a prism of angle $A = 90°$.

That is, $\quad \mu > \frac{1}{\sin\frac{A}{2}}$ or $\mu > \frac{1}{\sin\frac{90°}{2}}$ or $\mu > \sqrt{2}$

Illustration 1.64 **The cross section of a glass prism has the form of an isosceles triangle. One of the refracting faces is silvered. A ray of light falling normally on the other refracting face, being reflected twice, emerges through the base of the prism perpendicular to it. Find the angles of the prism.**

Sol. The incident ray BC at normal incidence is reflected at silvered face, along DE and at E it again suffers reflection along EF. Since the ray emerges normally from the base, therefore the ray EF must fall normally on the base and emerges along EG.

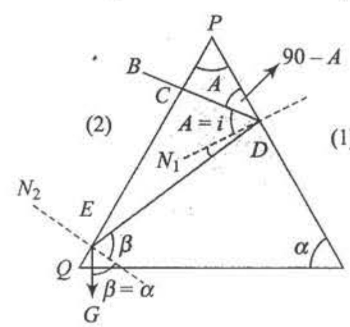


Fig. 1.171

We find $\quad i = A$. Also, $\beta = a$.

Since $\quad EN_2 \parallel CD$, $\beta = 2i$ (alternate angles)

$\therefore \quad \alpha = 2A \quad (\beta = \alpha, i = A)$ (i)

Also, $\quad 2\alpha + A = 180°$ ($\because$ Sum of angles of a triangle = 180°) (ii)

Solving Eqs. (i) and (ii), we get $A = 36°$, $\alpha = 72°$.