PHYSICS for you

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Managing Editor
Mahabir Singh
Editor
Anil Ahlawat

Corporate Office:
Plot 99, Sector 44 Institutional area, Gurgaon -122 003 (HR).
Tel : 0124-6601200 e-mail : info@mtg.in website : www.mtg.in
Regd. Office:
406, Taj Apartment, Near Safdarjung Hospital, New Delhi - 110029.

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Editor : Anil Ahlawat

Akshat Chug

AIR **2**

Cracking the JEE Advanced EXAM

- **MTG : Why did you appear for Engineering Entrance?**

  Akshat : I was always interested in studying Science and Mathematics. I wanted to study in the best institutes and it's a dream come true to be able to do so.

- **MTG : What exams have you appeared for and what are your ranks in these exams?**

  Akshat : JEE Advanced - AIR 2
  JEE Main - AIR 7
  BITSAT - cleared

- **MTG : How many hours in a day did you study to prepare for the examination?**

  Akshat : I studied for 6-7 hours per day.

- **MTG : On which topics and chapters you laid more stress in each subject?**

  Akshat : I have always given equal importance to all the chapters.

- **MTG : How much time does one require for serious preparation for this exam?**

  Akshat : I started studying in class 9 and 10 itself but it was only when I was in class 11 then I decided to seriously pursue engineering as a career therefore it was important that I crack the JEE exam. As far as preparation is concerned, after school hours, I used to attend the classes at my coaching institute for three hours. Once, I returned from the coaching institute, I used to do self-study and every night I would study for at least one hour.

- **MTG : How was the preparation for JEE Advanced different from JEE Main?**

  Akshat : As far as JEE Main preparation is concerned it is more about memorizing the formulae and speed accompanied with accuracy. While one is preparing for JEE Advanced, there has to be clarity of concepts. If you have studied properly, then you need to relax before JEE Advanced.

> "Clarity of mind, a clear strategy and then follow it strictly. This has been the secret of my success."

- **MTG : Any extra coaching?**

  Akshat : I joined Bakliwal Tutorials and it was of great help to me.

- **MTG : Which Books/Magazines you read?**

  Akshat : I studied from all the standard books for JEE Advanced. There are no special text books otherwise. The syllabus is the same; just a slight difference in the approach is needed I love to read mystery novels and I am a big fan of Sherlock Holmes.

- **MTG : In your words what are the components of an ideal preparation plan?**

  Akshat : Regular study is a must. Time Management is required, you should utilise your time properly. After studying for long hours, the mind gets exhausted so some recreational activities are also required.

- **MTG : What role did the following play in your success:**

  (a) Parents

  (b) Teachers

  (c) School

  Akshat : Parents - They have been very supportive and they have never pressurised me for anything.

  Teachers - At my coaching institute, Bakliwal Tutorials teacher's were very supportive. Whenever, I approached them with any kind of problem, they helped me a lot.

  School - I did my class 12th from DPS, Pune and all my teachers in school were very supportive and were always ready to help me out.

- **MTG : Your family background?**

  Akshat : I belong to Jaipur. My family shifted to Pune when my father got transferred here 6 years back. My father Mr. Vikas Chugh is Vice President, Western Reigon in Tata Sky and my mother is a home maker. My sister is in class 8^{th}.

- **MTG : What mistake you think you shouldn't have made?**

  Akshat : I think I should have studied more in the beginning.

- **MTG : Was this your first attempt?**

  Akshat : Yes, it was my first attempt.

- **MTG : What do you think is the secret of your success?**

  Akshat : Clarity of mind, a clear strategy and then follow it strictly. This has been the secret of my success.

- **MTG : How did you de-stress yourself during the preparation? What are your hobbies? How often could you pursue them?**

  Akshat : Yes, it is important to take break but you should not loose your focus in doing so. My hobbies were reading and watching TV shows such as 'Arrow' and 'Flash' but was not able to do so during my JEE Advanced preparation.

- **MTG : What do you feel is lacking in our education/ examination system? Is the examination system fair to the student?**

  Akshat : Too much of mugging up is there in our educational/examination system.

- **MTG : Had you not been selected then what would have been your future plan?**

  Akshat : I would love to join BITS.

- **MTG : What advice would you like to give our readers who are JEE aspirants?**

  Akshat : Plan out your strategy. Work more on your weak points. Practice and remain focussed. Try to strike a balance between your hobbies and studies.

All the Best!☺☺

NEET | JEE ESSENTIALS

Class XI

Maximize your chance of success, and high rank in NEET, JEE (Main and Advanced) by reading this column. This specially designed column is updated year after year by a panel of highly qualified teaching experts well-tuned to the requirements of these Entrance Tests.

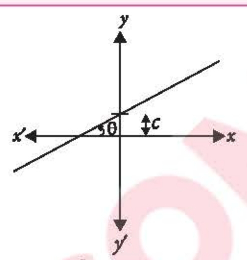
Unit 1 Mathematical Tools and Measurements

ESSENTIALS IN 2-D COORDINATE GEOMETRY

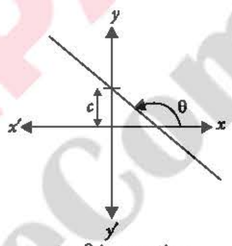
Straight Line

- Slope-intercept form : $y = mx + c$

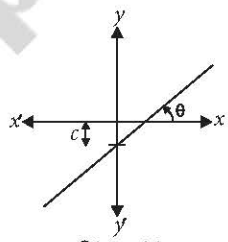
Cases of Slope-Intercept Form



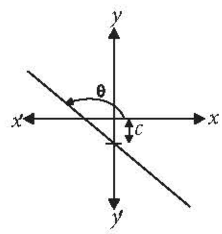
$m = \tan\theta$ is positive
$c = y$-intercept is positive



$m = \tan\theta$ is negative
$c = y$-intercept is positive



$m = \tan\theta$ is positive
$c = y$-intercept is negative

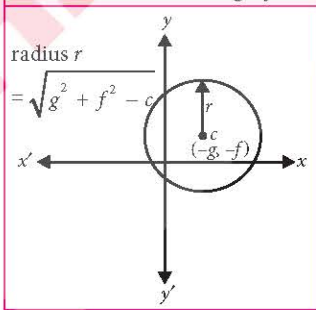


$m = \tan\theta$ is negative
$c = y$-intercept is negative

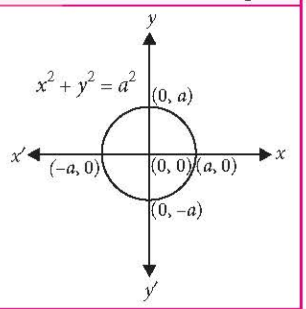
Conic Section

- **Circle** General Equation: $x^2 + y^2 + 2gx + 2fy + c = 0$

Circle with centre $(-g, -f)$

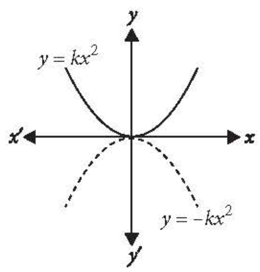


Circle with centre at origin

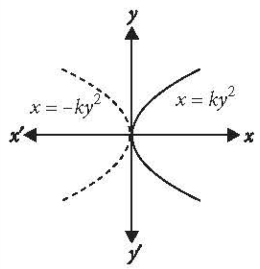


- **Parabola**

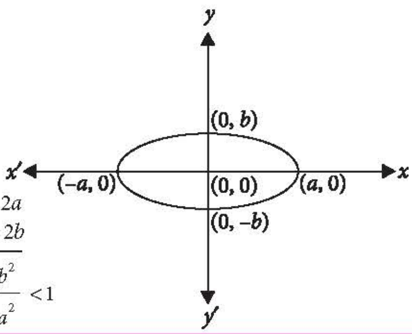
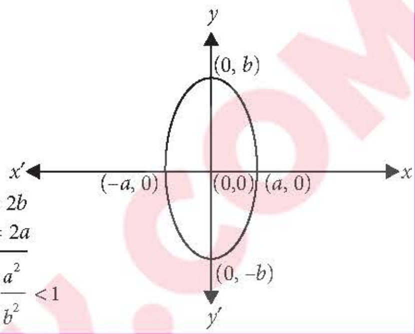
$y = \pm kx^2$ Passing through origin



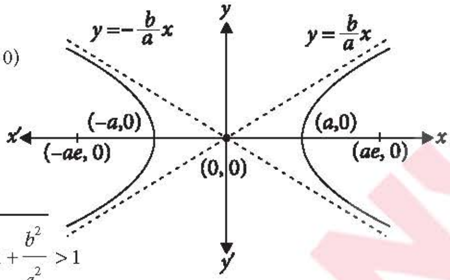
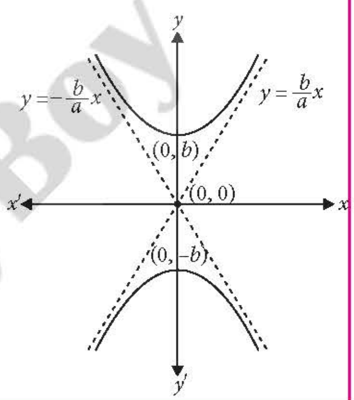
$x = \pm ky^2$ Passing through origin



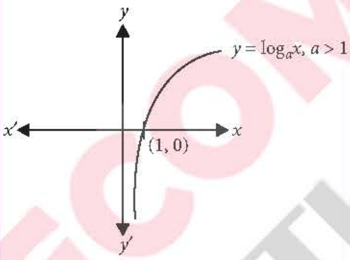
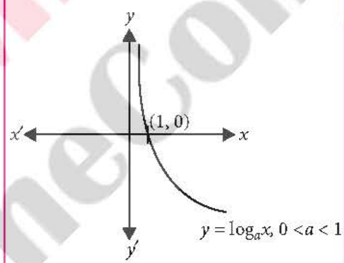
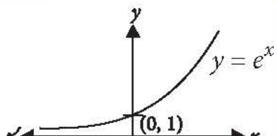
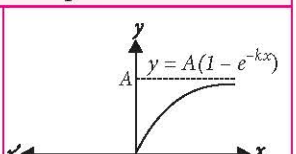
- **Ellipse**

Horizontal Ellipse	Vertical Ellipse
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ (a^2>b^2)$ Centre: (0, 0) Foci: ($\pm\, ae$, 0) Vertices: ($\pm\, a$, 0) Length of major axis = 2a Length of minor axis = 2b Eccentricity, $e=\sqrt{1-\frac{b^2}{a^2}}<1$ 	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ (a^2<b^2)$ Centre: (0, 0) Foci: (0, $\pm\, be$) Vertices: (0, $\pm\, b$) Length of major axis = 2b Length of minor axis = 2a Eccentricity, $e=\sqrt{1-\frac{a^2}{b^2}}<1$ 

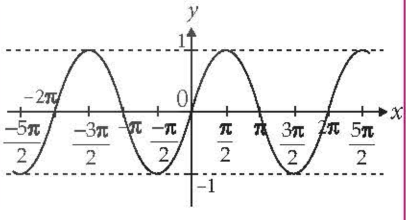
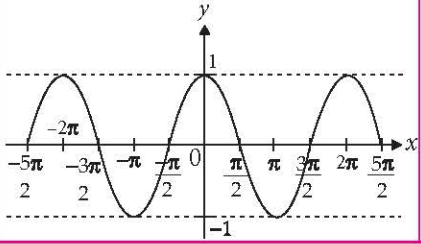
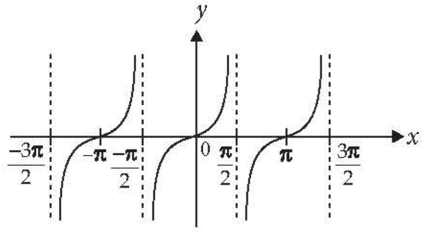
- **Hyperbola**

Transverse Hyperbola	Conjugate Hyperbola
$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1\ (a,\ b>0)$ Centre: (0, 0) Foci: ($\pm\, ae$, 0) Vertices: ($\pm\, a$, 0) Eccentricity, $e=\sqrt{1+\frac{b^2}{a^2}}>1$ 	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1\ (a,\ b>0)$ Centre: (0, 0) Foci: (0, $\pm\, be$) Vertices: (0, $\pm\, b$) Eccentricity, $e=\sqrt{1+\frac{a^2}{b^2}}>1$ 

- **Logarithmic and Exponential Functions**

Logarithmic Graphs		Exponential Graphs	
 $x=a^y\ (a>0)\ a\neq 1\ (a>1)$	 $x=a^y\ (a>0)\ a\neq 1\ (0<a<1)$	 $\frac{dy}{dx}=e^x>0$ $\forall\, x\in R$ Exponentially increasing	 $\frac{dy}{dx}=kAe^{-kx}>0$ $\forall\, x\in R$ Exponentially increasing

- **Trigonometric Functions**

$f(x)$ = sin x	$f(x)$ = cos x	$f(x)$ = tan x
		

ESSENTIALS IN TRIGONOMETRY

Basic Trigonometric Results

- $\sin^2\theta + \cos^2\theta = 1$
- $1 + \tan^2\theta = \sec^2\theta$
- $1 + \cot^2\theta = \text{cosec}^2\theta$
- $\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta = \cos^2\theta - \sin^2\theta$
- $\sin 2\theta = 2\sin\theta\cos\theta$
- $\cos(A + B) = \cos A\cos B - \sin A\sin B$
- $\cos(A - B) = \cos A\cos B + \sin A\sin B$
- $\sin(A + B) = \sin A\cos B + \cos A\sin B$
- $\sin(A - B) = \sin A\cos B - \cos A\sin B$
- $\sin C + \sin D = 2\sin\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$
- $\sin C - \sin D = 2\cos\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$
- $\cos C + \cos D = 2\cos\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$
- $\cos C - \cos D = -2\sin\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$
- $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$
- $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$
- $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

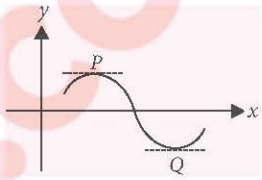
ESSENTIALS IN CALCULUS

Differentiation

- $\dfrac{d}{dx}x^n = nx^{n-1}$
- $\dfrac{d}{dx}\log_e x = \dfrac{1}{x}$
- $\dfrac{d}{dx}\sin x = \cos x$
- $\dfrac{d}{dx}\cos x = -\sin x$
- $\dfrac{d}{dx}\tan x = \sec^2 x$
- $\dfrac{d}{dx}\cot x = -\text{cosec}^2 x$
- $\dfrac{d}{dx}\sec x = \sec x\tan x$
- $\dfrac{d}{dx}\text{cosec}\,x = -\text{cosec}\,x\cot x$
- $\dfrac{d}{dx}e^x = e^x$
- $\dfrac{d}{dx}a^x = a^x\log_e a$
- $\dfrac{d}{dx}e^{ax} = ae^{ax}$
- $\dfrac{d}{dx}a^{bx} = ba^{bx}\log_e a$
- $\dfrac{d}{dx}\dfrac{f_1(x)}{f_2(x)} = \dfrac{f_2(x)\dfrac{d}{dx}f_1(x) - f_1(x)\dfrac{d}{dx}f_2(x)}{(f_2(x))^2}$
- $\dfrac{d}{dx}(f_1(x)f_2(x)) = f_1(x)\dfrac{d}{dx}f_2(x) + f_2(x)\dfrac{d}{dx}f_1(x)$
- $\dfrac{d}{dx}(ax+b) = a$
- $\dfrac{d}{dx}(ax+b)^n = an(ax+b)^{n-1}$

Maxima and Minima

Let $y = f(x)$ be a function. Then first draw the graph of $f(x)$ as shown.



From the graph we see that at maxima or minima, slope dy/dx of the graph at P and Q respectively is zero.

$\therefore \quad \dfrac{dy}{dx} = 0$ at maximum or minimum values of y.

Put $\dfrac{dy}{dx} = 0$ and solve for x. (We may get different values of x.)

At all those values of x for which $\dfrac{d^2y}{dx^2}$ is negative, we have maximum value of y.

Similarly, at all those values of x for which $\dfrac{d^2y}{dx^2}$ is positive, we have minimum value of y.

Illustration 1. Find maximum or minimum values of the functions $y = 25x^2 + 5 - 10x$

Sol.: For maximum and minimum value, we can put

$$\frac{dy}{dx} = 0$$

or $\dfrac{dy}{dx} = 50x - 10 - 0 = 0 \quad \therefore \quad x = \dfrac{1}{5}$

Further $\dfrac{d^2y}{dx^2} = 50 > 0$ at $x = \dfrac{1}{5}$. Therefore, y has minimum value at $x = \dfrac{1}{5}$. Substituting $x = \dfrac{1}{5}$ in given equation, we get

$$y_{\min} = 25\left(\frac{1}{5}\right)^2 + 5 - 10\left(\frac{1}{5}\right) = 4$$

Integration

- $\int x^n dx = \dfrac{x^{n+1}}{n+1} + c \qquad (n \neq -1)$
- $\int \dfrac{dx}{x} = \log_e |x| + c$
- $\int \sin x\, dx = -\cos x + c$
- $\int \cos x\, dx = \sin x + c$
- $\int \sec x\tan x\, dx = \sec x + c$

- $\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + c$
- $\int \sec x \, dx = \log_e |\sec x + \tan x| + c$
- $\int \operatorname{cosec} x \, dx = \log_e |\operatorname{cosec} x - \cot x| + c$
- $\int (ax + b) \, dx = \dfrac{ax^2}{2} + bx + c$

- **Area bounded by a region in the graph can be done as**

  $\text{Area} = \int_{x_1}^{x_2} y \, dx$

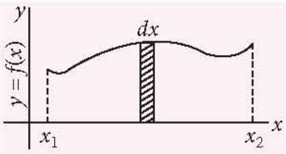


Illustration 2. Integrate the following functions

(a) $\int (5x^2 + 3x - 2) dx$ (b) $\int \left(4 \sin x - \dfrac{2}{x}\right) dx$

Sol.:

(a) $\int (5x^2 + 3x - 2) dx = 5\int x^2 dx + 3\int x \, dx - 2\int dx$

$$= \frac{5x^3}{3} + \frac{3x^2}{2} - 2x + c$$

(b) $\int \left(4 \sin x - \dfrac{2}{x}\right) dx = 4\int \sin x \, dx - 2\int \dfrac{dx}{x}$

$= -4 \cos x - 2 \log_e x + c$

ESSENTIALS IN VECTOR ALGEBRA

- **Triangle Law of Vector Addition**

  If two vectors are represented in both magnitude and direction by the two sides of a triangle in same order, the resultant $\vec{R}$ is represented by the third side of a triangle both in magnitude and direction but in opposite order.

- **Parallelogram Law of Vector Addition**

  For two coinitial vectors acting on a particle represented in magnitude and direction by the two adjacent sides of a parallelogram, the diagonal of the parallelogram so formed will be the resultant.

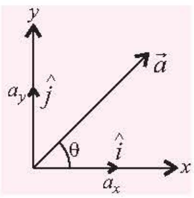
- **Resolution of a Vector**

  Consider a vector $\vec{a}$ in x-y plane making an angle θ with x-axis

  x component of $\vec{a} = a_x = a \cos \theta$

  y component of $\vec{a} = a_y = a \sin \theta$

  and $\vec{a} = a_x \hat{i} + a_y \hat{j}$



  Magnitude of $\vec{a}$ is

  $|\vec{a}| = \sqrt{a_x^2 + a_y^2} = a$

  Here $\tan \theta = \dfrac{a_y}{a_x} \Rightarrow \theta = \tan^{-1}\left(\dfrac{a_y}{a_x}\right)$

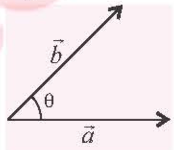
- **Dot or Scalar Product**

  Consider two vectors $\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$ and $\vec{b} = b_x \hat{i} + b_y \hat{j} + b_z \hat{k}$ and angle between $\vec{a}$ and $\vec{b}$ is θ, Their scalar product is given by $\vec{a} \cdot \vec{b} = |\vec{a}| \, |\vec{b}| \cos \theta$,

  where $|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$, $|\vec{b}| = \sqrt{b_x^2 + b_y^2 + b_z^2}$

  Angle θ can be determined as

  $\cos \theta = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$



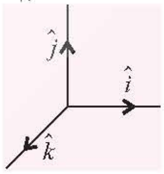
  For three unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$ along three axes,

  $\hat{i} \cdot \hat{i} = 1 \times 1 \cos 0° = 1$ ($\because |\hat{i}| = 1 = |\hat{j}| = |\hat{k}|$)

  $\therefore \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = \hat{i} \cdot \hat{i} = 1$

  and $\hat{i} \cdot \hat{j} = 1 \times 1 \times \cos 90° = 0$

  $\therefore \hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$



  In component form,

  $\vec{a} \cdot \vec{b} = (a_x \hat{i} + a_y \hat{j} + a_z \hat{k}) \cdot (b_x \hat{i} + b_y \hat{j} + b_z \hat{k})$

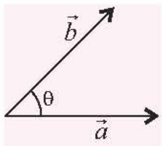
  $\Rightarrow \vec{a} \cdot \vec{b} = (a_x b_x + a_y b_y + a_z b_z)$

- **Cross or Vector Product**

  Consider two vectors $\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$ and $\vec{b} = b_x \hat{i} + b_y \hat{j} + b_z \hat{k}$ and angle between $\vec{a}$ and $\vec{b}$ is θ.

  So vector product of two vectors is

  $(\vec{a} \times \vec{b}) = |a||b| \sin \theta \, \hat{n}$



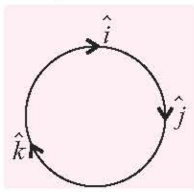
  - $(\vec{a} \times \vec{b})$ is a vector quantity and $\hat{n}$ is the perpendicular to the plane carrying vectors $\vec{a}$ and $\vec{b}$
  - For three unit vectors $\hat{i}, \hat{j}, \hat{k}$ along three axes.

    $\hat{i} \times \hat{j} = 1 \times 1 \times \sin 90° \hat{k} = \hat{k}$;

    Where $|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$

    $\hat{j} \times \hat{k} = 1 \times 1 \times \sin 90° \hat{i} = \hat{i}$

    $\hat{k} \times \hat{i} = 1 \times 1 \times \sin 90° \hat{j} = \hat{j}$



  and $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$

  In short, for cyclic order, $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}, \hat{k} \times \hat{i} = \hat{j}$

  For anti cyclic order, $\hat{i} \times \hat{k} = -\hat{j}$, $\hat{j} \times \hat{i} = -\hat{k}, \hat{k} \times \hat{j} = -\hat{i}$,

  - Calculation of $(\vec{a} \times \vec{b})$ in component form

    $\vec{a} \times \vec{b} = (a_x \hat{i} + a_y \hat{j} + a_z \hat{k}) \times (b_x \hat{i} + b_y \hat{j} + b_z \hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = (a_y b_z - a_z b_y)\hat{i} - (a_x b_z - a_z b_x)\hat{j} \\ (a_x b_y - a_y b_x)\hat{k}$$

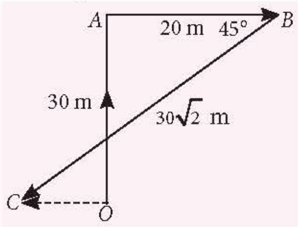
Illustration 3. A person moves 30 m north and then 20 m towards east and finally $30\sqrt{2}$ m in south-west direction. The displacement of the person from the origin will be

(a) 10 m along north (b) 10 m along south
(c) 10 m along west (d) Zero

Sol.: (c) From figure, $\overline{OA} = 0\hat{i} + 30\hat{j}, \overline{AB} = 20\hat{i} + 0\hat{j}$

$\overline{BC} = -30\sqrt{2}\cos45°\hat{i} - 30\sqrt{2}\sin45°\hat{j}$

$= -30\hat{i} - 30\hat{j}$



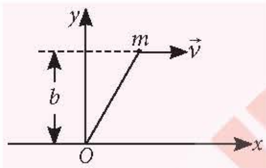
$\therefore$ Net displacement,

$\overline{OC} = \overline{OA} + \overline{OB} + \overline{BC}$

$= -10\hat{i} + 0\hat{j}$

$|\overline{OC}| = 10$ m along west

Illustration 4. If a particle of mass m is moving with constant velocity v parallel to x-axis in x-y plane as shown in figure. Its angular momentum $\vec{L} = \vec{r} \times \vec{p}$ with respect to origin at any time t will be



(a) $mvb\,\hat{k}$ (b) $-mvb\,\hat{k}$
(c) $mvb\,\hat{i}$ (d) $mv\,\hat{i}$

Sol.: (b) We know that angular momentum $\vec{L} = \vec{r} \times \vec{p}$ in terms of component becomes

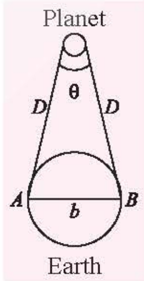
$$\vec{L} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix}$$

As motion is in x-y plane ($z = 0$ and $p_z = 0$), so $\vec{L} = \hat{k}(xp_y - yp_x)$

$\therefore \quad \vec{L} = \hat{k}[vt \times 0 - b\,mv] = -mvb\,\hat{k}$

MEASUREMENT OF PHYSICAL QUANTITIES

- **Fundamental Quantities** : Quantities which are independent of all other quantities and do not require any other physical quantity for their definition are called fundamental or base quantities and the units in which base quantities are measured are called fundamental units.
- **Derived Quantities** : The quantities that can be expressed as combinations of the base quantities are called derived quantities and the units for measuring them is also the combinations of fundamental units are called derived units.
- **Different Systems of Units**
  - CGS system
  - FPS system
  - MKS system
- **Measurement of Length**
  - **Large scale measurements**
    1 astronomical units (AU) = 1.496×10^{11} m
    1 light year (ly) = 9.46×10^{15} m
    1 parsec (pc) = 3.08×10^{16} m
  - **Small scale measurements**
    1 fermi (fm) = 10^{-15} m ($\approx$ size of nucleus)
    1 X-ray unit (xμ) = 10^{-13} m ($\approx$ size of an atom)
    1 angstrom (Å) = 10^{-10} m
    1 micron (μ) = 10^{-6} m
  - **Parallax Method :** To measure the distance of far away planet,
    $\therefore \theta = \dfrac{b}{D}$ or $D = \dfrac{b}{\theta}$



- **Measurement of Mass**
  - **Large scale units**
    1 Quintal = 100 kg
    1 Metric tonne = 1000 kg
    1 Chandrasekhar unit = 1.4 times solar mass = 2.8×10^{30} kg
  - **Small scale units**
    1 a.m.u. = 1.67×10^{-27} kg = $\dfrac{1}{12}$ of mass of carbon 12 (in kg)
    Nuclear mass = 10^{-27} kg
- **Measurement of Time**
  1 second = time interval for 9192631770 vibrations of the radiation corresponding to the transition between the two hyper fine levels of Cs^{133} (g).
  1 year = 3.156×10^7; 1 solar year = 365.25 day

ESSENTIALS IN ERRORS IN MEASUREMENT

- **Errors** : The difference in the true value and measured value of a quantity is called error of the measurement.
- **Absolute, Relative and Percentage Error**
  - **Mean absolute error,**
    $$\Delta a_{\text{mean}} = \frac{|\Delta a_1| + |\Delta a_2| + \ldots + |\Delta a_n|}{n}$$
  - **Relative error** : The ratio of mean absolute error to the mean value of observations is called relative error.
    $$\text{Relative error} = \frac{\Delta a_{\text{mean}}}{a_{\text{mean}}}$$

- **Percentage error :** Percent representation of relative error is called percentage error.

$$\delta a = \frac{\Delta a_{\text{mean}}}{a_{\text{mean}}} \times 100\%$$

- **Combination of Errors**
  - **Error of a sum or a difference**
    If $Z = A \pm B$
    Maximum possible error can be
    $\Delta Z = \Delta A + \Delta B$
  - **Error of a product or a quotient**
    If $Z = AB$ or A/B
    Maximum relative error, $\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$
  - **Error of a measured quantity raised to a power**
    If $Z = A^n$
    then $\frac{\Delta Z}{Z} = n\frac{\Delta A}{A}$

Illustration 5. The length and breadth of a rectangle sheet are 16.2 cm and 10.1 cm, respectively. The area of the sheet in appropriate significant figures and error is
(a) 164 ± 3 cm^2 (b) 163.62 ± 2.6 cm^2
(c) 163.6 ± 2.6 cm^2 (d) 163.62 ± 3 cm^2

Sol.: (a) If Δx is error in a physical quantity, then relative error is calculated as $\frac{\Delta x}{x}$

Given, length $l = (16.2 \pm 0.10)$ cm
Breadth $b = (10.1 \pm 0.1)$ cm
Area $A = l \times b = (16.2 \text{ cm}) \times (10.1 \text{ cm}) = 163.62 \text{ cm}^2$
Rounding off to three significant digits, area $A = 164 \text{ cm}^2$

$$\frac{\Delta A}{A} = \frac{\Delta l}{l} + \frac{\Delta b}{b} = \frac{0.1}{16.2} + \frac{0.1}{10.1} = \frac{1.01 + 1.62}{16.2 \times 10.1} = \frac{2.63}{163.62}$$

$$\Rightarrow \quad \Delta A = A \times \frac{2.63}{163.62} = 163.62 \times \frac{2.63}{163.62} = 2.63 \text{ cm}^2$$

$\Delta A = 3 \text{ cm}^2$ (By rounding off to one significant figure)
Area, $A = A \pm \Delta A = (164 \pm 3) \text{ cm}^2$

Illustration 6. A physical parameter a can be determined by measuring the parameters b, c, d and e using the relation $a = \frac{b^\alpha c^\beta}{d^\gamma e^\delta}$. If the maximum errors in the measurement of b, c, d and e are b_1 %, c_1 %, d_1 % and e_1 %, then the maximum error in the value of a determined by the experiment is
(a) $(b_1 + c_1 + d_1 + e_1)\%$ (b) $(b_1 + c_1 - d_1 - e_1)\%$
(c) $(\alpha b_1 + \beta c_1 - \gamma d_1 - \delta e_1)\%$
(d) $(\alpha b_1 + \beta c_1 + \gamma d_1 + \delta e_1)\%$

Sol.: (d) $a = \frac{b^\alpha c^\beta}{d^\gamma e^\delta}$

So maximum error in a is given by

$$\left(\frac{\Delta a}{a} \times 100\right)_{\max} = \alpha \cdot \frac{\Delta b}{b} \times 100 + \beta \cdot \frac{\Delta c}{c} \times 100 + \gamma \cdot \frac{\Delta d}{d} \times 100 + \delta \cdot \frac{\Delta e}{e} \times 100$$
$$= (\alpha b_1 + \beta c_1 + \gamma d_1 + \delta e_1)\%$$

SIGNIFICANT FIGURES

Rules for counting significant figures

- All non-zero digits are significant.
- A zero between two non zero digits is significant.
- Leading zeros to the left of non zero number are not significant.
- Trailing zeros to right of the number without decimal point are not significant.
- The powers of 10 are not taken as significant figure.
- All zeros to the right of the decimal point are significant.

ESSENTIALS IN DIMENSIONAL ANALYSIS

- The derived quantities can be expressed in terms of fundamental quantities as a product of different powers of the letters M,L,T etc. where M = Mass; L = Length; T = Time.
- **Uses of Dimensional Analysis**
  - **To check the correctness of a given relation**
    According to principle of homogeneity, dimensions of each term on both sides of an equation must be same.
  - **To convert a physical quantity from one to another system of units**
    $Q_1 n_1 = Q_2 n_2$; where Q_1 = unit in 1st system, Q_2 = unit in 2nd system n_1 and n_2 be constant values in 1st and 2nd system.
    $\therefore n_2 = \frac{Q_1 n_1}{Q_2}$, $n_2 = n_1\left[\frac{Q_1}{Q_2}\right]$
  - **To derive a relation among the physical quantities :**
    If we know the dependency of a physical quantity on the other quantities then using dimension analysis relation between them can be derived.
- **Limitations of Dimensional analysis**
  - Dimensional method cannot be used to derive relations other than multiplication and division. Also, it can not be used to derive trigonometric relations.

- We cannot determine the value of constants in a relation.
- The physical quantities depending on more than three quantities cannot be derived by dimensional method.

Illustration 7. Density of a liquid in CGS system is 0.625 g cm^{-3}. What is its magnitude in SI system?

(a) 0.625 (b) 0.0625
(c) 0.000625 (d) 625

Sol.: (b) Since $n_1Q_1 = n_2Q_2$

$\Rightarrow n_1[M_1L_1^{-3}] = n_2[M_2L_2^{-3}]$

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]\times\left[\frac{L_1}{L_2}\right]^{-3} = 0.625\left[\frac{1\text{ g}}{1\text{ kg}}\right]\times\left[\frac{1\text{ cm}}{1\text{ m}}\right]^{-3}$$

$$= 0.625 \times 10^{-3} \times 10^{6} = 625$$

Illustration 8. If the speed v of a particle of mass m as function of time t is given by $v = \omega A\ \sin\left[\left(\sqrt{\frac{k}{m}}\right)t\right]$, where A had dimension of length.

Which of the following statements is correct?

(a) The argument of trigonometric function must be a dimensionless quantity.
(b) Dimensional formula of ω is $[LT^{-1}]$.
(c) Dimensional formula of k is $[MLT^{-2}]$.
(d) Dimensional formula of $\sqrt{\frac{k}{m}}$ is [T].

Sol.: (a) Plane angle is dimensionless.

$[\omega A] = [v] = LT^{-1} \Rightarrow [\omega] = T^{-1}$

$$\left[\sqrt{\frac{k}{m}}\right] = \frac{1}{[t]} = [T^{-1}]$$

VERNIER CALLIPERS AND SCREW GUAGE

- **Vernier constant** : It is the difference between values of one main scale division and one vernier scale division of vernier callipers. Let n vernier scale divisions (VSD) coincide with $(n - 1)$ main scale divisions (MSD)

  $\therefore\ n$ VSD = $(n - 1)$ MSD

  $$1\text{ VSD} = \left(\frac{n-1}{n}\right)\text{MSD}$$

  Vernier constant, VC = 1 MSD – 1 VSD

  $$= 1\text{ MSD} - \left(\frac{n-1}{n}\right)\text{MSD} = \frac{1}{n}\text{ MSD}$$

  $$\text{VC} = \frac{\text{Value of one main scale division}}{\text{Total number of divisions on vernier scale}}$$

- **Reading of Vernier callipers** : Place the body between the jaws and the zero of vernier scale lies ahead of N^{th} division of main scale. Then

  Main scale reading (MSR) = N

  If n^{th} division of vernier scale coincides with any division of main scale, then

  Vernier scale reading (VSR) = $n \times$ (VC)

  Total reading = MSR + VSR = $N + n \times$ (VC)

- $$\text{Pitch of the screw} = \frac{\text{Distance moved on linear scale}}{\text{Number of rotations}}$$

- **Least count of the screw gauge**

  $$= \frac{\text{Pitch of the screw}}{\text{Total number of divisions on the circular scale}}$$

- **Reading of a Screw Gauge :** Place a wire between A and B, the edge of the cap lies ahead of N^{th} division of linear scale. Then

  Linear scale reading (LSR) = N

  If n^{th} division of circular scale lies over reference line, then

  Circular scale reading (CSR) = $n \times$ (LC)

  Total reading = LSR + CSR = $N + n \times$ (LC)

Illustration 9. The pitch of a screw gauge is 1 mm and there are 100 divisions on circular scale. When faces A and B are just touching each other without putting anything between the studs 32nd division of the circular scale coincides with the reference line. When a glass plate is placed between the studs, the linear scale reads 4 divisions and the circular scale reads 16 divisions. Find the thickness of the glass plate. Zero, of linear scale is not hidden from circular scale when A and B touches each other.

Sol.: Least count L.C.

$$= \frac{\text{Pitch}}{\text{Number of divisions on circular scale}} = \frac{1}{100}\text{ mm} = 0.01\text{ mm}$$

As zero is not hidden from circular scale when A and B touches each other. Hence, the screw gauge has positive error.

$e = +\,n$ (L.C.) = 32 × 0.01 = 0.32 mm

Linear scale reading = 4 × (1 mm) = 4 mm

Circular scale reading = 16 × (0.01 mm) = 0.16 mm

$\therefore$ Measured reading = (4 + 0.16) mm = 4.16 mm

$\therefore$ Absolute reading = Measured reading – e

= (4.16 – 0.32) mm = 3.84 mm

Therefore, thickness of the glass plate is 3.84 mm.

SPEED PRACTICE

1. The fundamental unit which has same power in the dimensional formula of surface tension and coefficient of viscosity is
(a) mass (b) length
(c) time (d) none of these

2. Which of the following systems of units is not based on units of mass, length and time alone?
(a) SI (b) MKS (c) FPS (d) CGS

3. The radius of a circle is 1.22 m. Area enclosed by it upto correct significant figures is
(a) $4.6778\ m^2$ (b) $4.677\ m^2$
(c) $4.67782\ m^2$ (d) $4.68\ m^2$

4. If voltage $V = (200 \pm 8)$ V and current $I = (20 \pm 0.5)$ A, the percentage error in resistance R is
(a) 5.2% (b) 25% (c) 3% (d) 6.5%

5. The pitch of a screw gauge is 1 mm and there are 100 divisions on the circular scale. In measuring the diameter of a sphere there are six divisions on the linear scale and forty divisions on circular scale coincides with the reference line. The diameter of the sphere is
(a) 6.0 mm (b) 5.0 mm (c) 5.4 mm (d) 6.4 mm

6. Three concurrent forces of the same magnitude are in equilibrium. The angle between the forces and name of the triangle formed by the forces as sides will be
(a) 60° and equilateral triangle
(b) 90°, 45°, 45° and right angled triangle
(c) 120°, 30°, 30° and an isosceles triangle
(d) 120° and an obtuse angled triangle.

7. The resultant $\vec{P}$ and $\vec{Q}$ is perpendicular to $\vec{P}$. what is the angle between $\vec{P}$ and $\vec{Q}$?
(a) $\cos^{-1}\left(\frac{P}{Q}\right)$ (b) $\cos^{-1}\left(-\frac{P}{Q}\right)$
(c) $\sin^{-1}\left(\frac{P}{Q}\right)$ (d) $\sin^{-1}\left(-\frac{P}{Q}\right)$

8. N-divisions on the main scale of a vernier callipers coincide with $N+1$ divisions on the vernier scale. If each division on the main scale is of a units, the least count of the instrument is
(a) $\frac{N}{N+1}$ (b) $\frac{N}{a+1}$
(c) $\frac{a}{N+1}$ (d) $\frac{a}{N}$

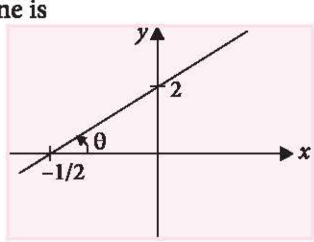
9. The charge flown through a circuit in the time internal between t and $t + dt$ is given by $dq = e^{-t/\tau}\, dt$, where τ is a constant. Find the total charge flown through the circuit between $t = 0$ to $t = \tau$.
(a) $\tau\left(1-\frac{1}{e}\right)$ (b) $\tau(e-1)$
(c) $\tau\left(\frac{e+1}{e}\right)$ (d) τ

10. The position vector of a particle is determined by the expression $\vec{r} = 3t^2\hat{i} + 4t^2\hat{j} + 7\hat{k}$. The distance travelled in first 10 s is
(a) 500 m (b) 300 m
(c) 150 m (d) 100 m

11. The height reached in time t by a particle thrown upward with a speed u is given by $h = ut - \frac{1}{2}gt^2$. The time taken in reaching the maximum height will be
(a) $\frac{u}{4g}$ (b) $\frac{u}{g}$ (c) $\frac{h}{u}$ (d) $\left(\frac{1}{2}\right)\frac{h}{u}$

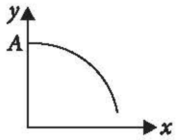
12. If $|\vec{V}_1 + \vec{V}_2| = |\vec{V}_1 - \vec{V}_2|$ and $\vec{V}_2$ is finite, then
(a) $\vec{V}_1$ is parallel to $\vec{V}_2$
(b) $\vec{V}_1 = \vec{V}_2$
(c) $\vec{V}_1$ and $\vec{V}_2$ are mutually perpendicular
(d) $|\vec{V}_1| = |\vec{V}_2|$

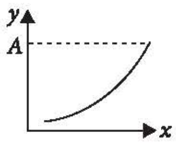
13. Choose the correct statement for the given graph.
(i) slope of the line is 4
(ii) Equation of the line is
$2x + \frac{y}{2} = -1$
(iii) $\left(\frac{dy}{dx}\right) = 4$
(iv) none of these

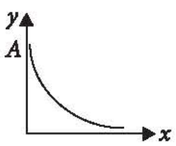


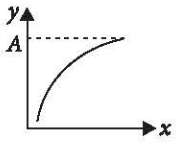
(a) (i) and (ii) (b) (ii) only
(c) (iv) (d) (i) and (iii)

14. For the equation $y = Ae^{-kx}$ graphical representation will be

(a) 

(b) 

(c) 

(d) 

15. In an experiment to determine the acceleration due to gravity g, the formula used for the time period of a periodic motion is $T = 2\pi\sqrt{\frac{7(R-r)}{5g}}$. The value of R and r are measured to be (60 ± 1) mm and (10 ± 1) mm, respectively. In five successive measurements, the time period is found to be 0.52 s, 0.56 s, 0.57 s, 0.54 s and 0.59 s. The least count of the watch used for the measurement of time period is 0.01 s. Which of the following statement is true?
(a) The error in the measurement of r is 9%
(b) The error in the measurement of T is 4.57%
(c) The error in the measurement of T is 2%
(d) The error in the determined value of g is 11%

16. A physical quantity of the dimensions of length that can be formed out of c, G and $\frac{e^2}{4\pi\varepsilon_0}$ is [c is velocity of light, G is universal constant of gravitation and e is charge]

(a) $c^2\left[G\frac{e^2}{4\pi\varepsilon_0}\right]^{1/2}$ (b) $\frac{1}{c^2}\left[\frac{e^2}{G\,4\pi\varepsilon_0}\right]^{1/2}$

(c) $\frac{1}{c}G\frac{e^2}{4\pi\varepsilon_0}$ (d) $\frac{1}{c^2}\left[G\frac{e^2}{4\pi\varepsilon_0}\right]^{1/2}$

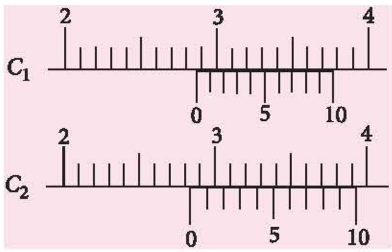
[NEET 2017]

17. Time (T), velocity (C) and angular momentum (h) are chosen as fundamental quantities instead of mass, length and time. In terms of these, the dimensions of mass would be
(a) $[M] = [TC^{-2}h]$
(b) $[M] = [T^{-1}C^{-2}h^{-1}]$
(c) $[M] = [T^{-1}C^{-2}h]$
(d) $[M] = [T^{-1}C^{2}h]$ **[JEE Main Online 2017]**

18. A physical quantity P is described by the relation $P = a^{1/2}\,b^2\,c^3\,d^{-4}$. If the relative errors in the measurement of a, b, c and d respectively, are 2%, 1%, 3% and 5%, then the relative error in P will be
(a) 25% (b) 12% (c) 8% (d) 32%
[JEE Main Online 2017]

19. The following observations were taken for determining surface tension T of water by capillary method. Diameter of capillary, $D = 1.25 \times 10^{-2}$ m and rise of water, $h = 1.45 \times 10^{-2}$ m. Using $g = 9.80$ m s^{-2} and the simplified relation, $T = \frac{rhg}{2} \times 10^3$ N m^{-1}, the possible error in surface tension is closest to
(a) 0.15% (b) 1.5% (c) 2.4% (d) 10%
[JEE Main Online 2017]

20. There are two Vernier calipers both of which have 1 cm divided into 10 equal divisions on the main scale. The Vernier scale of one of the calipers (C_1) has 10 equal divisions that correspond to 9 main scale divisions. The Vernier scale of the other caliper (C_2) has 10 equal divisions that correspond to 11 main scale divisions. The readings of the two calipers are shown in the figure. The measured values (in cm) by calipers C_1 and C_2 respectively are



(a) 2.87 and 2.86 (b) 2.85 and 2.82
(c) 2.87 and 2.87 (d) 2.87 and 2.83

[JEE Advanced 2016]

SOLUTIONS

1. **(a) :** Since [surface tension] = $[ML^0T^{-1}]$
[coefficient of viscosity] = $[ML^{-1}T^{-1}]$
Mass has the same power in both dimensional formula.

2. **(a) :** International system (SI) is not based on units of mass, length and time alone.

3. **(d):** Area $= \pi r^2 = \frac{22}{7} \times (1.22)^2 = 4.67782\ \text{m}^2$.
As per rule, the area will have three significant figures. Rounding off, we get
$A = 4.68\ \text{m}^2$

4. **(d):** $V = (200 \pm 8)$ V
$I = (20 \pm 0.5)$ A

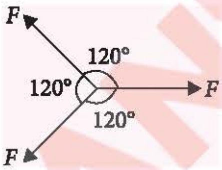
$$R = \frac{V}{I} = \frac{200}{20} = 10\ \Omega$$

$$\frac{\Delta R}{R} = \pm\left(\frac{\Delta V}{V} + \frac{\Delta I}{I}\right) = \pm\left(\frac{8}{200} + \frac{0.5}{20}\right) = \pm\frac{13}{200}$$

$$\frac{\Delta R}{R} \times 100 = \pm\frac{13}{200} \times 100 = \pm 6.5\%.$$

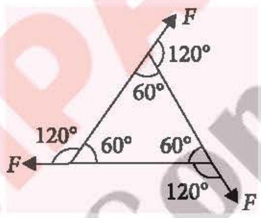
5. **(d):** Least count, L.C. $= \frac{1}{100} = 0.01$ mm
Linear scale reading = 6 (pitch) = 6 mm
Circular scale reading = n(L.C.) = $40 \times 0.01 = 0.4$ mm
$\therefore$ Total reading = (6 + 0.4) = 6.4 mm

6. **(a):** In N forces of equal magnitude works on a single point and their resultant is zero, angle between any two forces is given

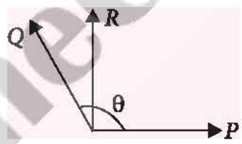


$$\theta = \frac{360°}{N} = \frac{360°}{3} = 120°$$

The angle between the forces and the triangle formed by the forces is shown in the figure.



7. **(b):**



$$\Rightarrow \tan 90° = \frac{Q\sin\theta}{P + Q\cos\theta}$$

$\Rightarrow P + Q\cos\theta = 0$

$\cos\theta = \frac{-P}{Q}$ $\therefore$ $\theta = \cos^{-1}\left(\frac{-P}{Q}\right)$

8. **(c):** $(N + 1)$ divisions on the vermier scale
$= N$ divisions on main scale

$\therefore$ 1 division on vernier scale $= \frac{N}{N+1}$ divisions on main scale.
Given, each division on the main scale is of a units.

$\therefore$ 1 division on vernier scale $= \left(\frac{N}{N+1}\right) a$ units $= d$ (say)

Least count = 1 main scale division
− 1 vernier scale division

$$= a - d = a - \left(\frac{N}{N+1}\right)a = \frac{a}{N+1}$$

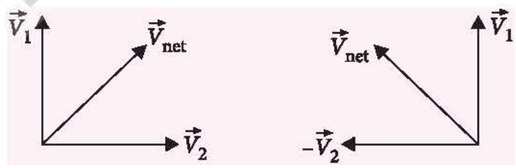
9. **(a):** The total charge flown is the sum of all the dq for t varying from $t = 0$ to $t = \tau$. Thus, the total charge flown is

$$Q = \int_0^\tau e^{-t/\tau}dt = \left[\frac{e^{-t/\tau}}{-1/\tau}\right]_0^\tau = \tau\left(1 - \frac{1}{e}\right).$$

10. **(a):** $\vec{r} = 3t^2\hat{i} + 4t^2\hat{j} + 7\hat{k}$
at $t = 0$, $\vec{r}_1 = 7\hat{k}$
at $t = 10s$, $\vec{r}_2 = 300\hat{i} + 400\hat{j} + 7\hat{k}$,
$\overrightarrow{\Delta r} = \vec{r}_2 - \vec{r}_1 = 300\hat{i} + 400\hat{j}$
$|\overrightarrow{\Delta r}| = |\vec{r}_2 - \vec{r}_1| = \sqrt{(300)^2 + (400)^2} = 500$ m

11. **(b):** Given, $h = ut - \frac{1}{2}gt^2$
or $\frac{dh}{dt} = u - gt$
For, maximum height, $\frac{dh}{dt} = 0$
or $u - gt = 0$ or $t = \frac{u}{g}$

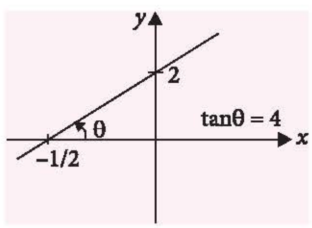
12. **(c):**



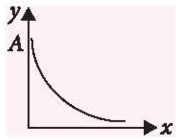
According to problem $|\vec{V}_1 + \vec{V}_2| = |\vec{V}_1 - \vec{V}_2|$
$\Rightarrow$ $|\vec{V}_{net}| = |\vec{V}_{net}|$
So, $\vec{V}_1$ and $\vec{V}_2$ will be mutually perpendicular.

13. **(d):** Here slope is +4 and y-intercept is 2. For graph as shown in the figure, equation will be $y = 4x + 2$

Slope of line $= \left(\frac{dy}{dx}\right)$ $\therefore$ $\frac{dy}{dx} = 4$,



14. (c) : $y = Ae^{-kx}$ represents exponentially decreasing graph. Value of y decreases exponentially from A to 0. The graph is shown in the figure.



From the graph and the equation, the value of $y = A$ at $x = 0$ and $y \to 0$ as $x \to \infty$.

15. (d): As $T = \sum_{i=1}^{n} \frac{T_i}{n}$

$$\therefore\ T = \frac{0.52\,\text{s} + 0.56\,\text{s} + 0.57\,\text{s} + 0.54\,\text{s} + 0.59\,\text{s}}{5}$$

$$= \frac{2.78\,\text{s}}{5} = 0.556\,\text{s} = 0.56\,\text{s}$$

($\because$ least count of watch is 0.01 s)

Percentage error in $T = \frac{\Delta T}{T} \times 100\%$

where $\Delta T = \dfrac{\sum_{i=1}^{n} |T_i - T|}{n}$

$$= \frac{|0.52 - 0.56| + |0.56 - 0.56| + |0.57 - 0.56| + |0.54 - 0.56| + |0.59 - 0.56|}{5}$$

$$= \frac{0.04 + 0.00 + 0.01 + 0.02 + 0.03}{5} = \frac{0.10}{5} = 0.02\,\text{s}$$

$\therefore$ Percentage error in $T = \frac{0.02}{0.56} \times 100\% = 3.57\%$

Given, $r = (10 \pm 1)$ mm, $R = (60 \pm 1)$ mm

$\therefore$ % error in measurement of r

$$= \frac{1}{10} \times 100\% = 10\%$$

As $T = 2\pi\sqrt{\frac{7(R-r)}{5g}}$ (given)

$$\therefore\ g = \frac{7(R-r)}{5} \times \frac{4\pi^2}{T^2}$$

$$\Rightarrow\ \frac{\Delta g}{g} = \frac{\Delta(R-r)}{R-r} + \frac{2\Delta T}{T}$$

$\therefore$ Percentage error in $g = \frac{\Delta g}{g} \times 100\%$

$$= \frac{\Delta(R-r)}{R-r} \times 100\% + 2 \times \frac{\Delta T}{T} \times 100\%$$

$$= \frac{2\text{ mm}}{50\text{ mm}} \times 100\% + 2 \times 3.57\%$$

($\because \Delta(R - r) = \Delta R + \Delta r$)

$= 4\% + 7.14\% \approx 11\%$

16. (d): Dimensions of

$$\frac{e^2}{4\pi\varepsilon_o} = [F \times d^2] = [\text{ML}^3\text{T}^{-2}]$$

Dimensions of $G = [\text{M}^{-1}\text{L}^3\text{T}^{-2}]$,

Dimensions of $c = [\text{LT}^{-1}]$

$$l \propto \left(\frac{e^2}{4\pi\varepsilon_o}\right)^p G^q c^r$$

$\therefore\ [\text{L}^1] = [\text{ML}^3\text{T}^{-2}]^p\,[\text{M}^{-1}\text{L}^3\text{T}^{-2}]^q\,[\text{LT}^{-1}]^r$

On comparing both sides and solving, we get

$$p = \frac{1}{2},\quad q = \frac{1}{2} \text{ and } r = -2$$

$$\therefore\ [l] = \frac{1}{c^2}\left[\frac{Ge^2}{4\pi\varepsilon_o}\right]^{1/2}$$

17. (c) : Let $m = kT^xC^yh^z$

where k is a dimensionless constant.

$\therefore\ [\text{ML}^0\text{T}^0] = [\text{T}]^x\,[\text{LT}^{-1}]^y\,[\text{ML}^2\text{T}^{-1}]^z$

$[\text{ML}^0\text{T}^0] = [\text{M}^z\text{L}^{y+2z}\text{T}^{x-y-z}]$

$\Rightarrow\ z = 1,\ y + 2z = 0$ and $x - y - z = 0$

Solving, we get, $x = -1$, $y = -2$, $z = 1$; on putting values we get

$\therefore\ [\text{M}] = [\text{T}^{-1}\text{C}^{-2}\text{h}]$

18. (d): Here, $P = a^{1/2}\,b^2\,c^3\,d^{-4}$

$$\frac{\Delta P}{P} = \frac{1}{2}\frac{\Delta a}{a} + 2\frac{\Delta b}{b} + 3\frac{\Delta c}{c} + 4\frac{\Delta d}{d}$$

or $\left(\frac{\Delta P}{P} \times 100\right)\%$

$$= \left(\frac{1}{2}\frac{\Delta a}{a} + 2\frac{\Delta b}{b} + 3\frac{\Delta c}{c} + 4\frac{\Delta d}{d}\right) \times 100\%$$

$\therefore$ Relative error in P

$$= \left(\frac{1}{2} \times 2 + 2 \times 1 + 3 \times 3 + 4 \times 5\right)\% = 32\%$$

19. (b): Surface tension is given by

$$T = \frac{rhg}{2} \times 10^3\ \text{N m}^{-1} = \frac{Dhg}{4} \times 10^3\,\text{N m}^{-1}$$

Possible error in the surface tension is

$$\frac{\Delta T}{T}\times 100=\frac{\Delta D}{D}\times 100+\frac{\Delta h}{h}\times 100+0$$

$$=\left(\frac{0.01\times 10^{-2}}{1.25\times 10^{-2}}+\frac{0.01\times 10^{-2}}{1.45\times 10^{-2}}\right)\times 100$$

(Permissible error in D and h is the place value of the last digit.)

$$\frac{\Delta T}{T}\times 100=\left(\frac{100}{125}+\frac{100}{145}\right)$$

$$\frac{\Delta T}{T}\times 100=0.8+0.689=1.489\approx 1.5\%$$

20. (d): For Vernier calipers C_1,

Smallest division on the main scale $=\frac{1\text{ cm}}{10}=1\text{ mm}$

As 10 V.S.D. = 9 M.S.D.

or $1\text{ V.S.D.}=\frac{9}{10}\text{ M.S.D.}$

$\therefore$ Vernier constant = 1 M.S.D. – 1 V.S.D.

$$=1\text{ M.S.D.}-\frac{9}{10}\text{ M.S.D.}$$

$$=\frac{1}{10}\text{ M.S.D.}=\frac{1}{10}\times 1\text{ mm}=0.1\text{ mm}=0.01\text{ cm}$$

$\therefore$ Reading = Main scale reading + Vernier divisions coinciding × Vernier constant

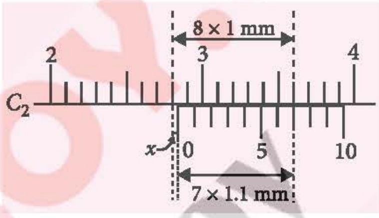
$= 2.8\text{ cm} + 7\times 0.01\text{ cm} = 2.87\text{ cm}$

For Vernier calipers C_2,

Smallest division on the main scale $=\frac{1\text{ cm}}{10}=1\text{ mm}$

Now, 10 V.S.D. = 11 M.S.D.

or $1\text{ V.S.D.}=\frac{11}{10}\text{ M.S.D}=\frac{11}{10}\times 1\text{ mm}=1.1\text{ mm}$



From figure, reading = 2.8 cm + x

where $x = (8\times 1)\text{ mm} - (7\times 1.1)\text{ mm}$

$= 0.3\text{ mm} = 0.03\text{ cm}$

$\therefore$ Reading = 2.8 cm + 0.03 cm = 2.83 cm

EXAM PREP 2018

CLASS XI

Useful for Medical/Engg. Entrance Exams

CHAPTERWISE MCQs FOR PRACTICE

MOTION IN A PLANE

1. What are the components of a vector $\vec{A} = 2\hat{i} + 3\hat{j}$ along the directions of $(\hat{i} + \hat{j})$ and $(\hat{i} - \hat{j})$?

(a) $\left(2, \frac{1}{2}\right)$ (b) $\left(\frac{5}{2}, \frac{-1}{2}\right)$

(c) $\left(\frac{5}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ (d) $\left(\frac{-5}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

2. A man rows a boat with a speed of 18 km h^{-1} in the north-west direction. The shoreline makes an angle of 15° south of west. The component of the velocity of the boat along the shoreline and perpendicular to the shoreline are respectively

(a) 9 km h^{-1}, 12 km h^{-1} (b) 12 km h^{-1}, 9 km h^{-1}

(c) 9 km h^{-1}, 15.5 km h^{-1} (d) 15 km h^{-1}, 9.5 km h^{-1}

3. Two projectiles A and B thrown with speeds in the ratio $1:\sqrt{2}$ acquired the same heights. If A is thrown at an angle of 45° with the horizontal, the angle of projection of B will be

(a) 0° (b) 60° (c) 30° (d) 45°

4. If the position vector of a particle is given by $\vec{r} = (4\cos 2t)\hat{i} + (4\sin 2t)\hat{j} + (6t)\hat{k}$ m.
Find its acceleration at $t = \pi/4$.

(a) $-8\hat{j}$ m s^{-2} (b) $-16\hat{j}$ m s^{-2}

(c) $12\hat{j}$ m s^{-2} (d) $4\hat{j}$ m s^{-2}

5. For a particle in uniform circular motion, the acceleration $\vec{a}$ at a point $P(R, \theta)$ on the circle of radius R is (Here v is the speed of the particle. θ is acute angle and measured from the x-axis.)

(a) $\frac{v^2}{R}\hat{i} + \frac{v^2}{R}\hat{j}$

(b) $-\frac{v^2}{R}\cos\theta\hat{i} + \frac{v^2}{R}\sin\theta\hat{j}$

(c) $-\frac{v^2}{R}\sin\theta\hat{i} + \frac{v^2}{R}\cos\theta\hat{j}$

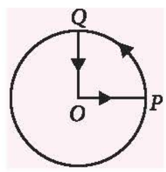
(d) $-\frac{v^2}{R}\cos\theta\hat{i} - \frac{v^2}{R}\sin\theta\hat{j}$

6. A cricket ball thrown across a field is at heights h_1 and h_2 from the point of projection at times t_1 and t_2 respectively after the throw. The ball is caught by a fielder at the same height as that of projection. The time of flight of the ball in this journey is

(a) $\frac{h_1t_2^2 - h_2t_1^2}{h_1t_2 - h_2t_1}$ (b) $\frac{h_1t_1^2 + h_2t_2^2}{h_2t_1 + h_1t_2}$

(c) $\frac{h_1t_2^2 + h_2t_1^2}{h_1t_2 + h_2t_1}$ (d) $\frac{h_1t_1^2 - h_2t_2^2}{h_1t_1 - h_2t_2}$

7. A cyclist starts from the centre O of a circular park of radius 1 km, reaches the edge P of the park, then cycles along the circumference and returns to the centre along QO as shown in the figure. If the round trip takes 10 min, the net displacement and average speed of the cyclist (in m and km h^{-1}) are



(a) 0, 1 (b) $\frac{\pi+4}{2}$, 0

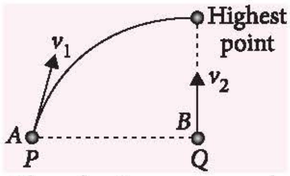
(c) 21.4, $\frac{\pi+4}{2}$ (d) 0, 21.4

8. A particle is projected from the ground with an initial speed of 15 m s^{-1} at an angle 60° with horizontal. The average velocity of the particle between its point of projection and highest point of trajectory is

(a) $15\sqrt{7}$m s^{-1} (b) $9\sqrt{3}$m s^{-1}

(c) $\frac{15\sqrt{7}}{4}$ m s^{-1} (d) $\frac{15\sqrt{7}}{2}$ m s^{-1}

9. A projectile A is thrown thrown at an angle of 30° to the horizontal from point P. At the same time, another projectile B is thrown with velocity v_2 upwards from the point Q vertically below the highest point.



For B to collide with A, $\frac{v_2}{v_1}$ should be

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) 4

10. The resultant of two vectors $\vec{P}$ and $\vec{Q}$ is $\vec{R}$. If the magnitude of $\vec{Q}$ is doubled, the new resultant becomes perpendicular to $\vec{P}$. Then the magnitude of $\vec{R}$ is

(a) $P + Q$ (b) Q

(c) P (d) $\frac{P+Q}{2}$

11. A ball is projected from the ground at a speed of 10 m s^{-1} making an angle of 30° with the horizontal. Another ball is simultaneously released from a point on the vertical line along the maximum height of the projectile. This ball collides with first ball at the maximum height of projectile. The initial height of the second ball is (g = 10 m s^{-2})

(a) 6.25 m (b) 2.5 m

(c) 3.75 m (d) 5 m

12. Let $\vec{r}_1(t) = 3t\,\hat{i} + 4t^2\,\hat{j}$ and $\vec{r}_2(t) = 4t^2\,\hat{i} + 3t\,\hat{j}$ represent the positions of particles 1 and 2 respectively as function of time t; $\vec{r}_1(t)$ and $\vec{r}_2(t)$ are in m and t in s. The relative speed of the two particles at the instant t = 1 s, will be

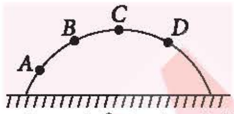
(a) 1 m s^{-1} (b) $3\sqrt{2}$ m s^{-1}

(c) $5\sqrt{2}$ m s^{-1} (d) $7\sqrt{2}$ m s^{-1}

13. The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 m s^{-1} can go without hitting the ceiling ?

(a) 108 m (b) 120 m (c) 150 m (d) 162 m

14. A stone is projected from ground. Its path is as shown in figure. At which point its speed is decreasing at fastest rate?



(a) A (b) B

(c) C (d) D

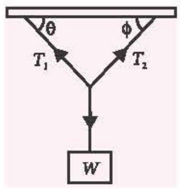
15. A particle is moving uniformly in a circular path of radius r. When it moves through an angular displacement θ, then the magnitude of the corresponding linear displacement will be

(a) $2r\cos\left(\frac{\theta}{2}\right)$ (b) $2r\cot\left(\frac{\theta}{2}\right)$

(c) $2r\tan\left(\frac{\theta}{2}\right)$ (d) $2r\sin\left(\frac{\theta}{2}\right)$

LAWS OF MOTION

16. A weight W hangs from a rope that is tied to two other ropes that are fastened to the ceiling as shown in figure. The upper ropes make angles θ and ϕ with the horizontal. Now, the values of T_1 and T_2 are



(a) $\frac{W\sin\phi}{\sin(\theta+\phi)}, \frac{W\sin\theta}{\sin(\theta+\phi)}$

(b) $\frac{W\sin\phi}{\cos(\theta+\phi)}, \frac{W\sin\theta}{\cos(\theta+\phi)}$

(c) $\frac{W\cos\phi}{\sin(\theta+\phi)}, \frac{W\cos\theta}{\sin(\theta+\phi)}$

(d) $\frac{W\cos\phi}{\tan(\theta+\phi)}, \frac{W\cos\theta}{\tan(\theta+\phi)}$

17. A mass of 5 kg is suspended in equilibrium, by two light inextensible strings S_1 and S_2 which make angle of 30° and 45° respectively with the horizontal. Then (Take g = 10 m s^{-2})

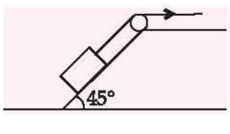
(a) tension in both the strings is same

(b) tension in S_1 is more than that in S_2

(c) tension in S_1 is less than that in S_2

(d) sum of tension in both is equal to 50 N

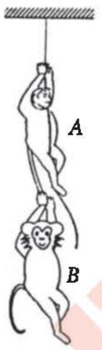
18. A block of mass 200 kg is being pulled up by men on an inclined plane at angle of 45° as shown in figure. The coefficient of static friction is 0.5. Each man can only apply a maximum force of 500 N. Find the minimum number of men required for the block to just start moving up the plane.



(a) 10 (b) 15 (c) 5 (d) 3

19. Which one of the following motions on a smooth plane surface does not involve force?
(a) Accelerated motion in a straight line.
(b) Retarded motion in a straight line.
(c) Motion with constant momentum along a straight line.
(d) Motion along a straight line with varying velocity.

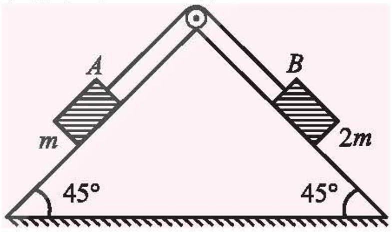
20. The monkey B shown in figure is holding on to the tail of monkey A which is climbing up a rope. The masses of the monkeys A and B are 5 kg and 2 kg respectively. If A can tolerate a tension of 30 N in its tail, what force should it apply on the rope in order to carry the monkey B with it? (Take $g = 10$ m s^{-2}).



(a) Between 70 N and 105 N
(b) Between 50 N and 69 N
(c) Between 30 N and 50 N
(d) Between 30 N and 116 N

21. A gramophone record is revolving with an angular velocity ω. A coin is placed at a distance r from the centre of the record. The coefficient of static friction is μ. The coin will revolve with the record if
(a) $r = \mu g\omega^2$ (b) $r < \frac{\omega^2}{\mu g}$
(c) $r \le \frac{\mu g}{\omega^2}$ (d) $r \ge \frac{\mu g}{\omega^2}$

22. Block A of mass m and block B of mass $2m$ are placed on a fixed triangular wedge by means of a massless, inextensible string and a frictionless pulley as shown in the figure. The wedge is inclined at 45° to the horizontal on both the sides. If the coefficient of friction between the block A and the wedge is 2/3 and that between the block B and the wedge is 1/3. Both A and B are released from rest, the acceleration of A will be



(a) 0.1 (b) zero
(c) 0.2 (d) 0.6

23. A body of 100 kg is placed on a truck. The coefficient of static friction between the body and the truck is 0.2. The truck suddenly decreases its speed from 90 km h^{-1} to 36 km h^{-1} in 5 s. Then
(a) the block does not move.
(b) the block slips forward and hits the driver's cabin
(c) block shifts backward
(d) nothing can be said about the block.

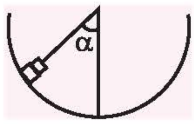
24. A boy stands on a weighing machine inside a lift. When the lift is going down with acceleration $g/4$, the machine shows a reading 30 kg. When the lift goes upwards with acceleration $g/4$, the reading would be
(a) 18 kgf (b) 37.5 kgf
(c) 50 kgf (d) 67.5 kgf

25. A body of mass m is travelling with a velocity u. When a constant retarding force F is applied, it comes to rest after travelling a distance s_1. If the initial velocity is $2u$, with the same force F, the distance travelled before it comes to rest is s_2. Then
(a) $s_2 = 2s_1$ (b) $s_2 = \frac{s_1}{2}$
(c) $s_2 = s_1$ (d) $s_2 = 4s_1$

26. A person sitting in an open car moving at constant velocity throws a ball vertically up into air. The ball falls
(a) Outside the car
(b) In the car ahead of the person
(c) In the car to the side of the person
(d) Exactly in the hand which threw it up

27. The rear side of a truck is open and a box of mass 20 kg is placed on the truck 4 m away from the open end. If μ is 0.15 and $g = 10$ m s^{-2} and the truck starts from rest with an acceleration of 2 m s^{-2} on a straight road, then the box will fall off the truck when it is at a distance of x metre from the starting point. The value of x is
(a) 4 m (b) 8 m (c) 16 m (d) 32 m.

28. An insect crawls up a hemispherical surface very slowly as shown in the figure. The coefficient of friction between the insect and the surface is 1/3. If the line joining the centre of the hemispherical surface to the insect makes an angle α with the vertical, the maximum possible value of α is

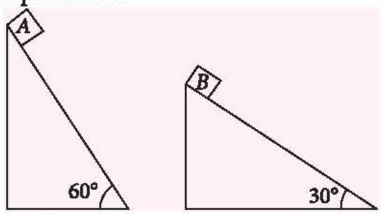


(a) $\cot\alpha = 3$ (b) $\tan\alpha = 3$
(c) $\sec\alpha = 3$ (d) $\mathrm{cosec}\,\alpha = 3$

29. A weightless thread can bear tension upto 3.7 kg wt. A stone of mass 500 g is tied to it and revolves in a circular path of radius 4 m in vertical plane. If g = 10 m s^{-2}, then what will be the maximum angular velocity of the stone?

(a) 2 rad s^{-1} (b) 4 rad s^{-1}
(c) 6 rad s^{-1} (d) 10 rad s^{-1}

30. Two fixed frictionless inclined planes making an angle 30° and 60° with the vertical are shown in the figure. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B?



(a) 4.9 m s^{-2} in vertical direction
(b) 4.9 m s^{-2} in horizontal direction
(c) 9.8 m s^{-2} in vertical direction
(d) zero

SOLUTIONS

1. (c) : $\vec{A}=2\hat{i}+3\hat{j} = \lambda(\hat{i}+\hat{j})+u(\hat{i}-\hat{j})$

$2\hat{i}+3\hat{j} = (\lambda+u)\hat{i}+(\lambda-u)\hat{j}$

$\Rightarrow \lambda + u = 2$ and $\lambda - u = 3$

$\Rightarrow \lambda = \frac{5}{2}$ and $u = \frac{-1}{2}$

Now, unit vector along $\hat{i}+\hat{j}$ is $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$ and unit vector along $\hat{i}-\hat{j} = \frac{\hat{i}-\hat{j}}{\sqrt{2}}$

Thus, $2\hat{i}+3\hat{j} = \frac{5}{2}(\hat{i}+\hat{j}) - \frac{1}{2}(\hat{i}-\hat{j})$

$$=\left(\frac{5}{\sqrt{2}}\right)\cdot\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)-\frac{1}{\sqrt{2}}\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right)$$

As, the components of $2\hat{i}+3\hat{j}$ along $\hat{i}+\hat{j}$ and $\hat{i}-\hat{j}$ directions are $\left(\frac{5}{\sqrt{2}},\frac{-1}{\sqrt{2}}\right)$.

2. (c)

3. (c) : For projectile A maximum height,

$$H_A = \frac{u_A^2 \sin^2 45°}{2g}$$

For projectile B maximum height,

$$H_B = \frac{u_B^2 \sin^2\theta}{2g}$$

As per question, $H_A = H_B$

$$\frac{u_A^2 \sin^2 45°}{2g} = \frac{u_B^2 \sin^2\theta}{2g} \quad \text{or} \quad \frac{\sin^2\theta}{\sin^2 45°} = \frac{u_A^2}{u_B^2}$$

$$\sin^2\theta = \left(\frac{u_A}{u_B}\right)^2 \sin^2 45°$$

$$\sin^2\theta = \left(\frac{1}{\sqrt{2}}\right)^2\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{4} \quad \left(\because \frac{u_A}{u_B} = \frac{1}{\sqrt{2}}\right)$$

$$\sin\theta = \frac{1}{2} \quad \text{or} \quad \theta = \sin^{-1}\left(\frac{1}{2}\right) = 30°$$

4. (b): Position, $\vec{r} = (4\cos 2t)\hat{i} + (4\sin 2t)\hat{j} + 6t\hat{k}$

Velocity,

$$\vec{v} = \frac{d\vec{r}}{dt} = [4(-\sin 2t)(2)]\hat{i} + [4(\cos 2t)\cdot(2)]\hat{j} + 6\hat{k}$$

$$= (-8\sin 2t)\hat{i} + (8\cos 2t)\hat{j} + 6\hat{k}$$

Acceleration,

$$\vec{a} = \frac{d\vec{v}}{dt} = [-8(\cos 2t)(2)]\hat{i} + [8(-\sin 2t)(2)]\hat{j}$$

$$= (-16\cos 2t)\hat{i} + (-16\sin 2t)\hat{j}$$

When $t = \pi/4$

$$\vec{a} = (-16\cos\pi/2)\hat{i} + (-16\sin\pi/2)\hat{j}$$

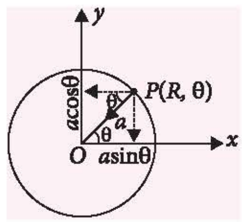
$$= (-16\times 0)\hat{i} + (-16\times 1)\hat{j} = -16\hat{j} \text{ m s}^{-2}$$

5. (d): For a particle in uniform circular motion,

acceleration, $a = \frac{v^2}{R}$

towards the centre

From figure,



$$\vec{a} = -a\cos\theta\hat{i} - a\sin\theta\hat{j} = -\frac{v^2}{R}\cos\theta\hat{i} - \frac{v^2}{R}\sin\theta\hat{j}$$

6. (a) : Let a cricket ball be thrown with velocity u at an angle θ with the horizontal.

As per question, $h_1 = u\sin\theta t_1 - \frac{1}{2}gt_1^2$

or $u\sin\theta t_1 = h_1 + \frac{1}{2}gt_1^2$ ... (i)

and $h_2 = u\sin\theta t_2 - \frac{1}{2}gt_2^2$

or $u\sin\theta t_2 = h_2 + \frac{1}{2}gt_2^2$ ... (ii)

Divide eqn. (i) by eqn. (ii), we get

$$\frac{t_1}{t_2} = \frac{h_1 + \frac{1}{2}gt_1^2}{h_2 + \frac{1}{2}gt_2^2};\quad h_2t_1 + \frac{1}{2}gt_2^2t_1 = h_1t_2 + \frac{1}{2}gt_1^2t_2$$

$$h_1t_2 - h_2t_1 = \frac{1}{2}g(t_1t_2^2 - t_1^2t_2) \quad \text{... (iii)}$$

Time of flight, $T = \frac{2u\sin\theta}{g}$

$$T = \frac{2}{g}\left[\frac{h_1 + \frac{1}{2}gt_1^2}{t_1}\right] \quad \text{(Using (i))}$$

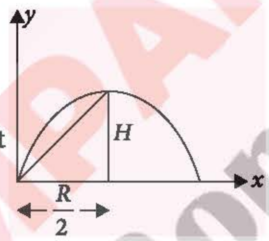
$$= \frac{2}{g}\frac{h_1}{t_1} + t_1 = \frac{h_1}{t_1}\left(\frac{t_1t_2^2 - t_1^2t_2}{h_1t_2 - h_2t_1}\right) + t_1 \quad \text{(Using (iii))}$$

$$= \frac{h_1t_2^2 - h_1t_1t_2}{h_1t_2 - h_2t_1} + t_1 = \frac{h_1t_2^2 - h_2t_1^2}{h_1t_2 - h_2t_1}$$

7. (d)

8. (c): Average velocity $= \frac{\text{Displacement}}{\text{Time}}$

$$v_{av} = \frac{\sqrt{H^2 + \frac{R^2}{4}}}{T/2} \quad \text{...(i)}$$



Here, H = maximum height

$$= \frac{v^2\sin^2\theta}{2g}$$

$R = \text{range} = \frac{v^2\sin 2\theta}{g}$ and $T = \text{time of flight} = \frac{2v\sin\theta}{g}$

Putting these values in eqn. (i) we get

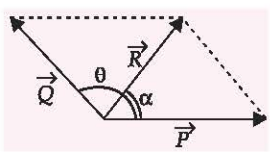
$$v_{av} = \frac{v}{2}\sqrt{1+3\cos^2\theta}$$

$$v_{av} = \frac{15}{2}\sqrt{1+3\times\cos^2 60°} = \frac{15}{2}\sqrt{1+\frac{3}{4}} = \frac{15\sqrt{7}}{4}\ \text{m s}^{-1}$$

9. (c): Both the projectiles will collide in the air if vertical component of velocity of projectile A is equal to the velocity of projectile B.

$$v_1\sin 30° = v_2 \Rightarrow \frac{v_2}{v_1} = \frac{1}{2}$$

10. (b): $\vec{P} + \vec{Q} = \vec{R}$



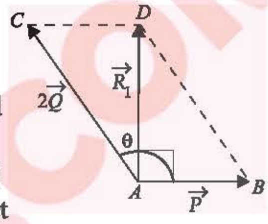
$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}$$

and $R^2 = P^2 + Q^2 + 2PQ\cos\theta$ ...(i)

When $\vec{Q}$ is doubled, resultant $\vec{R}_1$ is perpendicular to $\vec{P}$

$\therefore\ R_1^2 = P^2 + 4Q^2 + 4PQ\cos\theta$ ...(ii)

From right angled triangle BAD



$4Q^2 = R_1^2 + P^2$,

$R_1^2 = 4Q^2 - P^2$

Substituting in (ii) and solving, we get

$P^2 + 2PQ\cos\theta = 0$ ...(iii)

Substituting (iii) in (i), we get

$R^2 = Q^2$ or $R = Q$

11. (b): Maximum height of projectile, $H = \frac{u^2\sin^2\theta}{2g}$

$$\therefore\ H = \frac{(10)^2\times\sin^2 30°}{2\times 10} = \frac{5}{4} = 1.25\ \text{m}$$

Time for attaining maximum height, $t = \frac{u\sin\theta}{g}$

$$\therefore\ t = \frac{10\times\sin 30°}{10} = 0.5\ \text{s}$$

$\therefore$ Distance of vertical fall in 0.5 s, $h = \frac{1}{2}gt^2$

or $h = \frac{1}{2}\times 10\times(0.5)^2 = 1.25$ m

$\therefore$ Height of second ball = 1.25 + 1.25 = 2.5 m

12. (c): Here, $\vec{r}_1(t) = 3t\hat{i} + 4t^2\hat{j}$, $\vec{r}_2(t) = 4t^2\hat{i} + 3t\hat{j}$

Velocity, $\vec{v}_1(t) = \frac{d\vec{r}_1}{dt} = \frac{d}{dt}(3t\hat{i} + 4t^2\hat{j}) = 3\hat{i} + 8t\hat{j}$

$$\vec{v}_2(t) = \frac{d\vec{r}_2}{dt} = \frac{d}{dt}(4t^2\hat{i} + 3t\hat{j}) = 8t\hat{i} + 3\hat{j}$$

The relative speed of particle 1 with respect to particle 2 is

$$\vec{v}_{12} = \vec{v}_1 - \vec{v}_2 = (3\hat{i} + 8t\hat{j}) - (8t\hat{i} + 3\hat{j})$$

$$= (3-8t)\hat{i} + (8t-3)\hat{j}$$

At $t = 1$ s, $\vec{v}_{12} = (3-8)\hat{i} + (8-3)\hat{j} = -5\hat{i} + 5\hat{j}$

$$|\vec{v}_{12}| = \sqrt{(-5)^2 + (5)^2} = \sqrt{25+25} = 5\sqrt{2}\ \text{m s}^{-1}$$

13. (c): $h_{\max} = \frac{u^2\sin^2\theta}{2g} \Rightarrow 25 = \frac{40^2\times\sin^2\theta}{2g}$

$$\sin^2\theta = \frac{50g}{40^2} = \frac{50\times 10}{40\times 40} = \frac{5}{16};\ \sin\theta = \frac{\sqrt{5}}{4}$$

$$\cos\theta = \sqrt{1-\sin^2\theta} = \frac{\sqrt{11}}{4}$$

$$R = \frac{u^2\sin 2\theta}{g} = \frac{2u^2\sin\theta\cos\theta}{g} = \frac{2(40)^2\left(\frac{\sqrt{5}}{4}\right)\left(\frac{\sqrt{11}}{4}\right)}{10}$$