PHYSICS for you

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Class XI

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Unit 4 SYSTEM OF PARTICLES AND ROTATIONAL MOTION

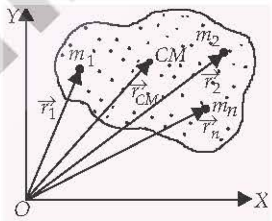
CENTRE OF MASS

- For a system of particles or a body, centre of mass is an imaginary point at which its total mass is supposed to be concentrated.
- The position of centre of mass of a rigid body depends on two factors :
  - The geometrical shape of the body.
  - The distribution of mass in the body.
- Position of centre of mass of n point masses m_1, m_2, ..., m_n whose position vectors from origin O are $\vec{r}_1, \vec{r}_2, ..., \vec{r}_n$ is given by,

$$\vec{r}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + ... + m_n\vec{r}_n}{m_1 + m_2 + ... + m_n};$$

$$\vec{r}_{CM} = \frac{\sum_{i=1}^{n} m_i\vec{r}_i}{\sum_{i=1}^{n} m_i}$$

$$\vec{r}_{CM} = \frac{1}{M}\sum_{i=1}^{n} m_i\vec{r}_i$$



where $M \left(= \sum_{i=1}^{n} m_i\right)$ is the total mass of the system.

Since $\vec{r}_{CM} = x_{CM}\hat{i} + y_{CM}\hat{j} + z_{CM}\hat{k}$, therefore,

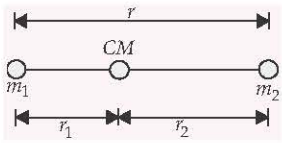
$$x_{CM} = \left(\frac{m_1x_1 + m_2x_2 + ...}{m_1 + m_2 + ...}\right) = \frac{1}{M}\sum m_i x_i,$$

$$y_{CM} = \left(\frac{m_1y_1 + m_2y_2 + ...}{m_1 + m_2 + ...}\right) = \frac{1}{M}\sum m_i y_i,$$

$$z_{CM} = \left(\frac{m_1z_1 + m_2z_2 + ...}{m_1 + m_2 + ...}\right) = \frac{1}{M}\sum m_i z_i$$

- **Position of centre of mass of two particles :**
  The distance of centre of mass (r) from any of the particle is inversely proportional to the mass of the particle (m)

i.e., $r \propto \dfrac{1}{m}$



or $\dfrac{r_1}{r_2} = \dfrac{m_2}{m_1}$

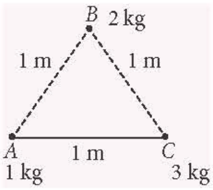
or $m_1r_1 = m_2r_2$

or $r_1 = \left(\dfrac{m_2}{m_2 + m_1}\right)r$ and $r_2 = \left(\dfrac{m_1}{m_1 + m_2}\right)r$

Here, r_1 = distance of centre of mass from m_1
and r_2 = distance of centre of mass from m_2

- From this discussion, we see that $r_1 = r_2 = \dfrac{1}{2}r$ if $m_1 = m_2$, *i.e.*, centre of mass lies midway between the two particles of equal masses.

Illustration 1 : Consider a system of 3 particles of masses 1 kg, 2 kg and 3 kg are kept at the vertices of an equilateral triangle of side 1 m as shown in figure. Find the coordinates of the centre of mass.



Soln.: Taking particle A as origin and x-axis along AC, we write down the coordinates of the particles.

Particles	x-coordinate	y-coordinate
A	0 cm	0 cm
B	$\frac{1}{2}$ cm	$\frac{\sqrt{3}}{2}$ cm
C	1 cm	0 cm

$$x_{CM} = \frac{m_A x_A + m_B x_B + m_C x_C}{m_A + m_B + m_C}$$

$$= \frac{1\,\text{kg}(0\,\text{m}) + 2\text{kg}\left(\frac{1}{2}\,\text{m}\right) + 3\,\text{kg}(1\,\text{m})}{1\,\text{kg} + 2\,\text{kg} + 3\,\text{kg}} = \frac{2}{3}\,\text{m}$$

$$y_{CM} = \frac{m_A y_A + m_B y_B + m_C y_C}{m_A + m_B + m_C}$$

$$= \frac{(1\,\text{kg})(0\,\text{m}) + (2\,\text{kg})\cdot\left(\frac{\sqrt{3}}{2}\,\text{m}\right) + 3\,\text{kg}(0\,\text{m})}{1\,\text{kg} + 2\,\text{kg} + 3\,\text{kg}}$$

$$= \frac{\sqrt{3}}{6}\,\text{m}$$

Hence the coordinates of centre of mass for the choice of axes is $\left(\frac{2}{3}\,\text{m}, \frac{\sqrt{3}}{6}\,\text{m}\right)$.

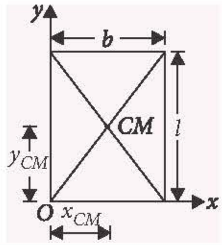
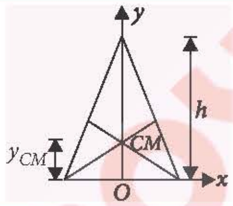
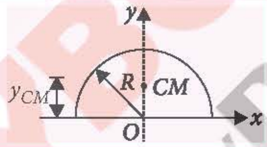
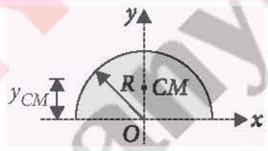
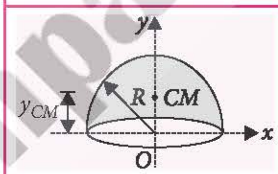
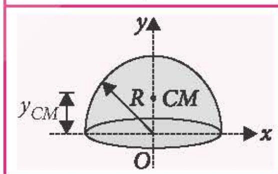
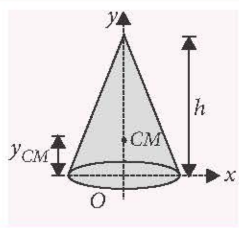
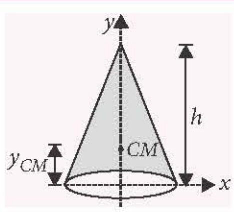
- **Centre of mass of a continuous mass distribution:** Consider a small mass element dm at position $\vec{r}$ as a point mass and replacing the summation by integration.

  $$\vec{R}_{CM} = \frac{1}{M}\int \vec{r}\, dm$$

  Consequently, $x_{CM} = \frac{1}{M}\int x\, dm$, $y_{CM} = \frac{1}{M}\int y\, dm$

  and $z_{CM} = \frac{1}{M}\int z\, dm$

- **Centre of mass of some well known rigid bodies :**

Different Body	Position of Centre of Mass
	Rectangular plate $x_{CM} = \frac{b}{2}$, $y_{CM} = \frac{l}{2}$
	Triangular plate At the centroid, $y_{CM} = \frac{h}{3}$, $x_{CM} = 0$
	Semi-circular ring $y_{CM} = \frac{2R}{\pi}$, $x_{CM} = 0$
	Semi-circular disc $y_{CM} = \frac{4R}{3\pi}$, $x_{CM} = 0$
	Hemispherical shell $y_{CM} = \frac{R}{2}$, $x_{CM} = 0$
	Solid hemisphere $y_{CM} = \frac{3R}{8}$, $x_{CM} = 0$
	Circular cone (solid) $y_{CM} = \frac{h}{4}$, $x_{CM} = 0$
	Circular cone (hollow) $y_{CM} = \frac{h}{3}$, $x_{CM} = 0$

- For a lamina type (two-dimensional) body with uniform negligible thickness, the formulae for finding the position of centre of mass are as follows:

$$\vec{r}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots}{m_1 + m_2 + \ldots}$$

$$= \frac{\rho A_1 t\vec{r}_1 + \rho A_2 t\vec{r}_2 + \ldots}{\rho A_1 t + \rho A_2 t + \ldots} \qquad (\because m = \rho A t)$$

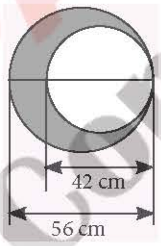
$$= \frac{A_1\vec{r}_1 + A_2\vec{r}_2 + \ldots}{A_1 + A_2 + \ldots}$$

Here, A stands for area and ρ for density.

- If some mass is removed from a rigid body, then the position of centre of mass of the remaining portion is obtained using.
  - $\vec{r}_{CM} = \dfrac{m_1\vec{r}_1 - m_2\vec{r}_2}{m_1 - m_2} = \dfrac{A_1\vec{r}_1 - A_2\vec{r}_2}{A_1 - A_2}$
  - $x_{CM} = \dfrac{m_1x_1 - m_2x_2}{m_1 - m_2} = \dfrac{A_1x_1 - A_2x_2}{A_1 - A_2}$
  - $y_{CM} = \dfrac{m_1y_1 - m_2y_2}{m_1 - m_2} = \dfrac{A_1y_1 - A_2y_2}{A_1 - A_2}$
  - $z_{CM} = \dfrac{m_1z_1 - m_2z_2}{m_1 - m_2} = \dfrac{A_1z_1 - A_2z_2}{A_1 - A_2}$

Here, m_1, $\vec{r}_1$, x_1, y_1 and z_1 are the values for the whole mass while m_2, A_2, $\vec{r}_2$, x_2, y_2 and z_2 are the values for the mass which has been removed.

Illustration 2 : A circular plate of uniform thickness has a diameter of 56 cm. A circular portion of diameter 42 cm is removed from one edge of the plate as shown in the figure. Find the position of the centre of mass of the remaining portion.



Soln.: Let us assume the centre of mass of the circular plate be origin. Let r_1 be the distance of the centre of mass of remaining portion from centre of the bigger circle. Remaining area,

$$A_1 = \frac{\pi[(56)^2 - (42)^2]}{4}$$

$$\therefore \quad A_1r_1 = A_2r_2 \quad \text{or} \quad r_1 = \left(\frac{A_2}{A_1}\right)r_2$$

Here r_2 is the distance of the centre of mass of removed portion from centre of the bigger circle and A_2 is the area of removed portion.

$$\text{or} \quad r_1 = \frac{\pi(42)^2 \times 4}{\pi[(56)^2 - (42)^2] \times 4} \times \frac{7}{1}$$

$$\text{or} \quad r_1 = \frac{42 \times 42 \times 7}{98 \times 14} = 9 \text{ cm}$$

$\therefore$ $r_1 = 9$ cm = Required distance of centre of mass of remaining part.

Motion of Centre of Mass

- For a system of particles, position of centre of mass is

$$\vec{R}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \ldots\ldots}{m_1 + m_2 + m_3 + \ldots\ldots}$$

- Velocity of centre of mass

$$\vec{v}_{CM} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots\ldots}{m_1 + m_2 + \ldots\ldots} \qquad \left(\because \frac{d\vec{r}}{dt} = \vec{v}\right)$$

- Acceleration of centre of mass

$$\vec{a}_{CM} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + \ldots.}{m_1 + m_2 + \ldots.} \qquad \left(\because \vec{a} = \frac{d\vec{v}}{dt}\right)$$

- Linear momentum of a system of particles is equal to the product of mass of the system with the velocity of its centre of mass.

PURE ROTATIONAL MOTION

- A body is said to be in pure rotational motion if the perpendicular distance of each particle remains constant from a fixed line or point and do not move parallel to the line, and that line is known as axis of rotation.
- Angular displacement, $\theta = \dfrac{s}{r}$

  where s = length of arc traced by the particle.
  r = distance of particle from the axis of rotation.
- Angular velocity, $\omega = \dfrac{d\theta}{dt}$
- Angular acceleration, $\alpha = \dfrac{d\omega}{dt}$
- All the parameters θ, ω and α are same for all the particles. Axis of rotation is perpendicular to the plane of rotation of particles.
- If α = constant, then

  $\omega = \omega_0 + \alpha t$ where ω_0 = initial angular speed, t = time interval

  $\theta = \omega_0 t + \dfrac{1}{2}\alpha t^2$

  $\omega^2 = \omega_0^2 + 2\alpha\theta$

  These equations are known as equations of rotational motion.

Illustration 3 : A wheel rotates with an angular acceleration given by $\alpha = 4at^3 - 3bt^2$, where t is the time and a and b are constants. If the wheel has initial angular speed ω_0, find the equations for the (i) angular speed, and (ii) angular displacement.

Soln.: (i) As $\alpha = \frac{d\omega}{dt} \Rightarrow d\omega = \alpha dt$

$$\int_{\omega_0}^{\omega} d\omega = \int_0^t \alpha\, dt = \int_0^t (4at^3 - 3bt^2)\, dt$$

$$\omega = \omega_0 + at^4 - bt^3$$

(ii) Further, $\omega = \frac{d\theta}{dt} \Rightarrow d\theta = \omega\, dt$

$$\int_0^{\theta} d\theta = \int_0^t \omega\, dt = \int_0^t (\omega_0 + at^4 - bt^3)\, dt$$

$$\theta = \omega_0 t + \frac{at^5}{5} - \frac{bt^4}{4}$$

COMBINED ROTATION AND TRANSLATION

- If a body rotates about an axis with an angular frequency ω with respect to the axis of rotation, linear velocity of any particle in the body at a distance r from the axis of rotation is equal to

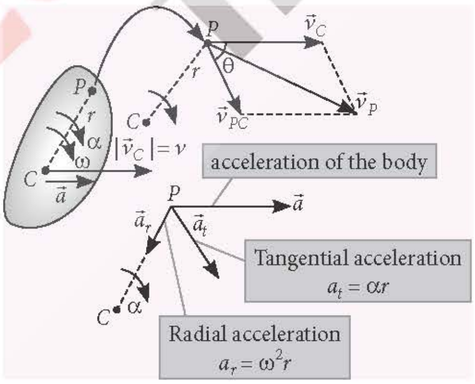
$$\vec{v} = \vec{\omega} \times \vec{r}$$

If the axis of rotation also moves with velocity $\vec{v}_0$, then the net velocity of the particle relative to stationary frame will be

$$\vec{v} = \vec{\omega} \times \vec{r} + \vec{v}_0$$

- Take a body which is moving with velocity $\vec{v}$ and also rotating about centre of mass with angular velocity ω. Let us analyse a point P on the body.
  - The velocity of point P in the body is

$$|\vec{v}_P = |\vec{v}_{PC} + \vec{v}_C| = \sqrt{v_{PC}^2 + v_C^2 + 2v_{PC} \cdot v_C \cdot \cos\theta}$$



We have $v_{PC} = r\,\omega$ and $v_C = v$. Thus,

$$v_P = \sqrt{(r\omega)^2 + v^2 + 2(r\omega)v \cdot \cos\theta}$$

- In same way we can write the acceleration of point P

$$\vec{a}_P = \vec{a}_{PC} + \vec{a}_C$$

$\vec{a}_{PC}$ has both tangential as well as radial acceleration components. Hence we can express $\vec{a}_{PC}$ as

$$\vec{a}_{PC} = (\vec{a}_{PC})_{\text{tangential}} + (\vec{a}_{PC})_{\text{radial}} = \vec{a}_t + \vec{a}_r$$

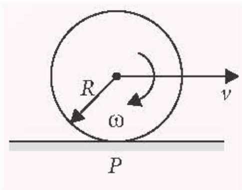
$\vec{a}_t = r\,\alpha$ (acts perpendicular to line CP)

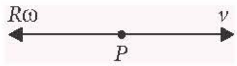
$\vec{a}_r = \omega^2 r$ (acts along the line PC)

Hence net acceleration of point P can be express as

$$\vec{a}_P = (\vec{a}_t + \vec{a}_r) + \vec{a} \qquad (\because \vec{a}_C = a)$$

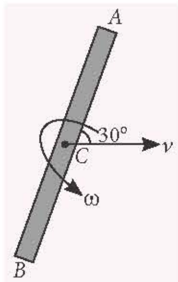
- **Rolling :** A body in combined motion said to be rolling over a surface if the surfaces in contact do not slide relative to each other. It means that the relative velocity between the points of contact is zero.
- Condition of pure rolling is $v = R\omega$. In this case bottommost point of the spherical body is at rest. It has no slipping with its contact point on ground. Because ground point is also at rest.





  - If $v > R\omega$, then net velocity of point P is in the direction of v. This is called forward slipping.
  - If $v < R\omega$, then net velocity of point P is in opposite direction of v. This is called backward slipping.

Illustration 4 : A uniform rod of length l is spinning with an angular velocity $\omega = 2v/l$ while its centre of mass moves with a velocity v as shown in figure. Find the velocity of the end A of the rod.



Soln.: Velocity of end A,

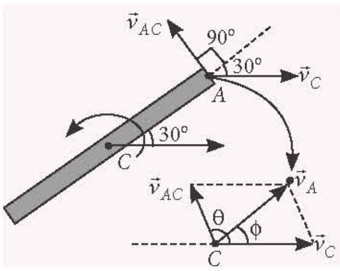
$$\vec{v}_A = \vec{v}_{AC} + \vec{v}_C$$

Hence the velocity of A is

$$v_A = \sqrt{v_{AC}^2 + v_C^2 + 2v_{AC}v_C\cos\theta}$$

We know, $v_C = v$, $v_{AC} = \frac{l}{2}\,\omega$ and $\theta = 90° + 30° = 120°$

Hence, $v_A = \sqrt{v^2 + \frac{l^2\omega^2}{4} + lv\,\omega\cos 120°}$



But, $\omega = \frac{2v}{l}$, $v_{AC} = \frac{l}{2}\omega = \frac{l}{2}\left(\frac{2v}{l}\right) = v$

$\therefore \quad v_A = v$

Let $\vec{v}_A$ make angle ϕ with the direction of $\vec{v}_C$. Then

$$\phi = \tan^{-1}\left(\frac{v\sin\theta}{v_{AC}\cos\theta + v_C}\right)$$

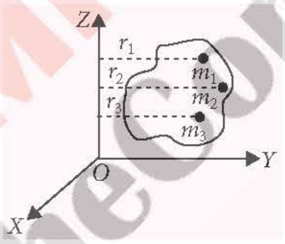
We get, $\phi = \tan^{-1}\left(\frac{v\sin 120°}{v\cos 120° + v}\right) = \frac{\pi}{3}$

MOMENT OF INERTIA

- The moment of inertia of a rigid body about a given axis is the sum of the product of the masses of its constituent particles and the square of their respective distances from the axis of rotation.

$$I = \sum_{i=1}^{n} m_i r_i^2$$

- The radius of gyration k of a body about an axis of rotation is defined as the root mean square distances of the particles from the axis of rotation and its square when multiplied with the mass of the body gives moment of inertia of the body about the axis.

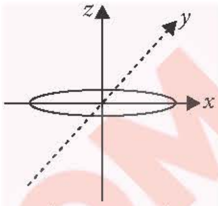


$$k = \sqrt{\frac{r_1^2 + r_2^2 + ... + r_n^2}{n}}$$

= root mean square distance of a particles from axis OZ.

- Two important theorems on moment of inertia :
  - **Perpendicular axes theorem** (Only applicable to plane lamina *i.e.*, 2-D objects only):

$I_z = I_x + I_y$ (Object is in x-y plane)



Body is in x-y plane

where I_z = moment of inertia of the body about z-axis.

I_x = moment of inertia of the body about x-axis.

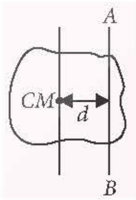
I_y = moment of inertia of the body about y-axis.

$I_y = I_x + I_z$ (Object is in x-z plane)

$I_x = I_y + I_z$ (Object is in y-z plane)

  - **Parallel axes theorem** (Applicable to any type of object) :

$I_{AB} = I_{CM} + Md^2$



where,

I_{CM} = Moment of inertia of the object about an axis passing through centre of mass and parallel to axis AB

I_{AB} = Moment of inertia of the object about axis AB

M = Total mass of object

d = Perpendicular distance between axis about which moment of inertia is to be calculated and the one passing through the centre of mass.

- **Some important points about rolling motion :**
  - Kinetic energy of a rolling body = translational kinetic energy (K_T) + rotational kinetic energy (K_R)

$$= \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}Mv^2\left[1 + \frac{k^2}{R^2}\right]$$

  - When a body rolls down an inclined plane of inclination θ without slipping its velocity at the bottom of incline is given by

$$v = \sqrt{\frac{2gh}{1 + \frac{k^2}{R^2}}}$$

where h is the height of the incline.

  - When a body rolls down on an inclined plane without slipping, its acceleration down the inclined plane is given by

$$a = \frac{g\sin\theta}{1 + \frac{k^2}{R^2}}$$

  - When a body rolls down on an inclined plane without slipping, time taken by the body to reach the bottom is given by

$$t = \sqrt{\frac{2l\left(1+\frac{k^2}{R^2}\right)}{g\sin\theta}}$$

where l is the length of the inclined plane.

- **Moment of inertia of some important cases :**

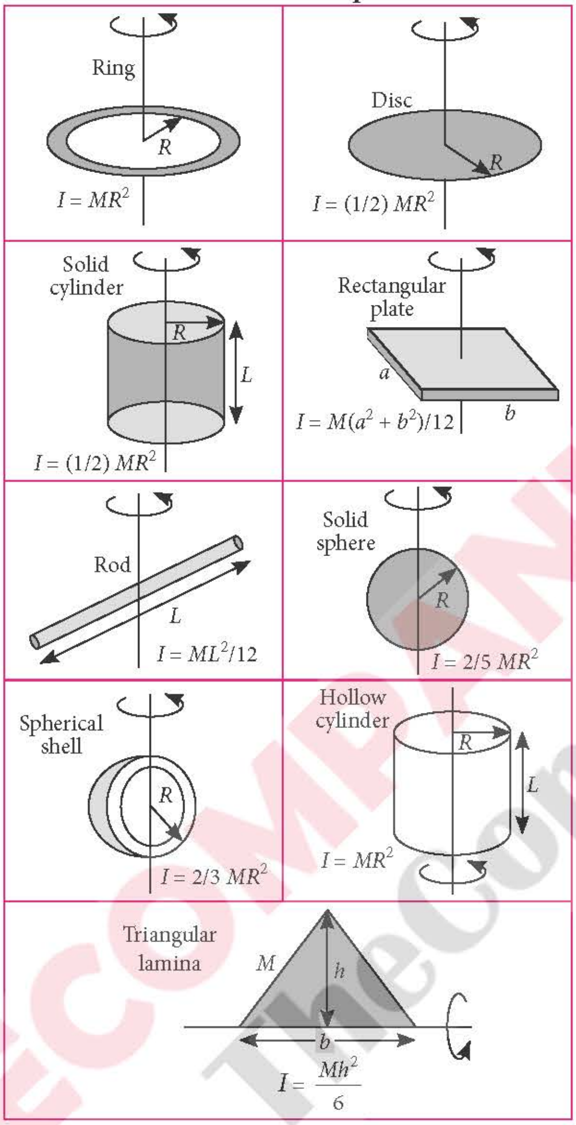


Illustration 5 : A circular disc A of radius r is made from an iron plate of thickness t and another circular disc B of radius $4r$ is made from an iron plate thickness $t/4$. What is the relation between the moments of inertia I_A and I_B?

Soln.: $I_{Disc} = \frac{1}{2}MR^2$

Let ρ be the density of iron, then $I_A = \frac{1}{2}(\rho)[\pi r^2 t](r^2)$ as radius of plate A is r and thickness is t.

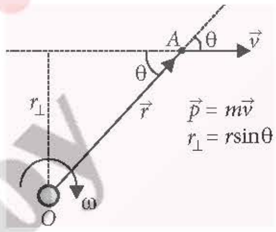
$$\therefore \quad I_A = \frac{\rho\pi r^4 t}{2}.$$

Now, $I_B = \frac{1}{2}(\rho)\left(\pi(4r)^2\frac{t}{4}\right)(4r)^2$ as radius of plate B is $4r$ and thickness is $t/4$.

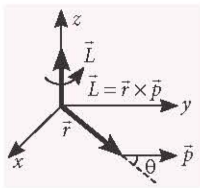
$$\Rightarrow \quad I_B = \left[\frac{\rho\pi r^4 t}{2}\right]\left(\frac{16\times16}{4}\right) \quad \text{or} \quad I_B = 64\,I_A \;\therefore\; I_A < I_B.$$

ANGULAR MOMENTUM

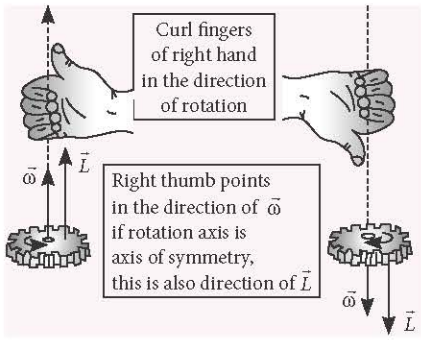
- Angular momentum of particle A about point O will be, $\vec{L} = \vec{r}\times\vec{p} = \vec{r}\times(m\vec{v}) = m(\vec{r}\times\vec{v})$
- Magnitude of $\vec{L}$ is $L = mvr \sin\theta = mvr_\perp$ where θ is the angle between $\vec{r}$ and $\vec{p}$.



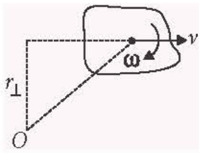
- Direction of $\vec{L}$ will be given by right hand screw rule. For the given figure, direction of $\vec{L}$ is perpendicular to paper inwards.
- If a body is rotating about a fixed axis, we can write angular momentum of the body $L = I\omega$
  I = Moment of inertia, about fixed axis,
  ω = angular velocity of rotation
- Direction of angular momentum
  - We define angular momentum even if the particle is not moving in a circular path. Even a particle moving in a straight line has angular momentum about any axis displaced from the path of the particle.



  - For rotation about an axis of symmetry, $\vec{\omega}$ and $\vec{L}$ are parallel and along the axis, respectively. The directions of both vectors are given by the right hand rule.



- Angular momentum of a rigid body in rotation plus translation about a general axis :

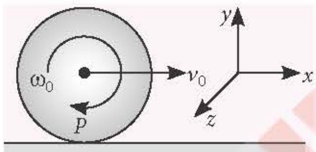


- There will be two terms :
  (a) $I_{CM}\,\omega$ (b) $mv_{CM}r_\perp = mvr_\perp$
- From right hand screw rule, we can see that $I_{CM}\,\omega$ and $mvr_\perp$ both terms are perpendicular to the paper in inward direction. Hence, they are added or $L_{\text{Total}} = I_{CM}\,\omega + mvr_\perp$

Illustration 6 : A sphere of mass M and radius R rolls without slipping on a rough surface with centre of mass has constant speed v_0. Find the angular momentum of the sphere about the point of contact.

Soln.: Since $\vec{L}_P = \vec{L}_{CM} + \vec{r} \times \vec{p}_{CM}$

$= I_{CM}\omega_0(-\hat{k}) + Mv_0R(-\hat{k})$



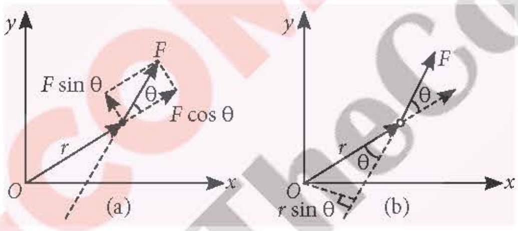
Since sphere is in pure rolling motion, hence

$\omega_0 = v_0/R$

$$\Rightarrow \vec{L}_P = \left[\frac{2}{5}MR^2\left(\frac{v_0}{R}\right)(-\hat{k}) + Mv_0R(-\hat{k})\right] = \frac{7}{5}Mv_0R(-\hat{k})$$

TORQUE

- The turning ability of a force about an axis is called its torque about that axis.
- Consider force $\vec{F}$ which acts through a point whose position vector is $\vec{r}$, as shown in figure.



- Its torque about the origin is defined as $\vec{\tau} = \vec{r} \times \vec{F}$.
- The direction of the torque can be obtained from the right hand rule and its magnitude is given by $\tau = rF\sin\theta$, where θ is the angle between the vector $\vec{r}$ and $\vec{F}$.

  $\tau = r(F\sin\theta) = rF_\perp$

  where $F_\perp$ is the component of F perpendicular to r.

  $\tau = (r\sin\theta)F = r_\perp F$

  where $r_\perp$ is the perpendicular distance from the origin to the line of action of the force. It is also called the lever arm.

- Torque is also the rate of change of angular momentum

  $$\vec{\tau} = \frac{d\vec{L}}{dt} = I\frac{d\vec{\omega}}{dt} = I\vec{\alpha}$$

- If a torque τ applied on a body rotates it through an angle $\Delta\theta$, the work done by the torque is

  $\Delta W = \tau\Delta\theta$

  or work done = torque × angular displacement
  Power of a torque is given as

  $$P = \frac{\Delta W}{\Delta t} = \frac{\tau\Delta\theta}{\Delta t} = \tau\omega$$

  i.e., Power of a torque = torque × angular velocity

- **Law of conservation of angular momentum :** If no external torque acts on a system, total angular momentum of the system remains unchanged. In the absence of any external torque,

  $L = I\omega$ = constant

  or $I_1\omega_1 = I_2\omega_2$ or $I_1 \cdot \frac{2\pi}{T_1} = I_2 \cdot \frac{2\pi}{T_2}$

Illustration 7 : A turntable turns about a fixed vertical axis, making one revolution in 10 s. The moment of inertia of the turntable about the axis is 1200 kg m^2. A man of 80 kg, initially standing at the centre of the turntable, runs out along the radius. What is the angular velocity of the turntable when the man is 2 m from the centre?

Soln.: Let I_0 be the initial moment of inertia of the system (man + table)

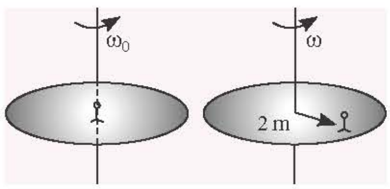
$I_0 = I_{\text{man}} + I_{\text{table}} = 0 + 1200 = 1200$ kg m^2

($I_{\text{man}} = 0$ as the man is at the axis)

I = final moment of inertia of the system

$= I_{\text{man}} + I_{\text{table}} = mr^2 + 1200$

$= 80(2)^2 + 1200 = 1520$ kg m^2



As there is no external torque about the axis, we can conserve angular momentum.

By conservation of angular momentum : $I_0\omega_0 = I\omega$

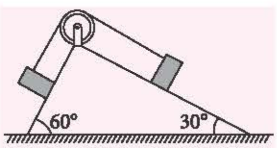
Now $\omega_0 = 2\pi/T_0 = 2\pi/10 = \pi/5$ rad s^{-1}

$$\Rightarrow \quad \omega = \frac{I_0\omega_0}{I} = \frac{1200\times\pi}{1520\times 5} = 0.49 \text{ rad s}^{-1}$$

SPEED PRACTICE

1. A man of mass 80 kg is riding on a small cart of mass 40 kg which is moving along a level floor at a speed of 2 m s^{-1}. He is running on the cart, so that his velocity relative to the cart is 3 m s^{-1} in the direction opposite to the motion of cart. What is the speed of the centre of mass of the system?
(a) 1.5 m s^{-1} (b) 1 m s^{-1}
(c) 3 m s^{-1} (d) zero

2. Two blocks of equal mass are tied with a light string, which passes over a massless pulley as shown in figure. The magnitude of acceleration of centre of mass of both the blocks is (neglect friction everywhere)



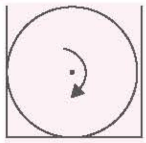
(a) $\left(\frac{\sqrt{3}-1}{4\sqrt{2}}\right)g$ (b) $(\sqrt{3}-1)g$
(c) $\frac{g}{2}$ (d) $\left(\frac{\sqrt{3}-1}{\sqrt{2}}\right)g$

3. The position of a particle is given by $\vec{r} = \hat{i} + 2\hat{j} - \hat{k}$ and its linear momentum is given by $\vec{p} = 3\hat{i} + 4\hat{j} - 2\hat{k}$. Then its angular momentum, about the origin is perpendicular to
(a) yz-plane (b) z-axis
(c) y-axis (d) x-axis

4. A uniform rod of mass m and length l makes a constant angle θ with an axis of rotation which passes through one end of the rod. Its moment of inertia about this axis is
(a) $\frac{ml^2}{3}$ (b) $\frac{ml^2}{3}\sin\theta$
(c) $\frac{ml^2}{3}\sin^2\theta$ (d) $\frac{ml^2}{3}\cos^2\theta$

5. A ring of radius R is rotating with an angular speed ω_0 about a horizontal axis. It is placed on a rough horizontal table. The coefficient of kinetic friction is μ_k. The time after which it starts rolling is
(a) $\frac{\omega_0\mu_k R}{2g}$ (b) $\frac{\omega_0 g}{2\mu_k R}$
(c) $\frac{2\omega_0 R}{\mu_k g}$ (d) $\frac{\omega_0 R}{2\mu_k g}$

6. A solid sphere of mass 5 kg and radius 1 m after rotating with angular speed $\omega_0 = 40$ rad s^{-1} is placed between two smooth walls on a rough ground. Distance between the walls is slightly greater than the diameter of the sphere. Coefficient of friction between the sphere and the ground is $\mu = 0.1$. Sphere will stop rotating after



(a) 8 s (b) 12 s (c) 20 s (d) 16 s

7. A uniform rod AB of mass m and length l is at rest on a smooth horizontal surface. An impulse J is applied to the end B perpendicular to the rod in horizontal direction. Speed of particle P at a distance $l/6$ from the center towards A of the rod after time $t = \frac{\pi ml}{12J}$ is
(a) $2\frac{J}{m}$ (b) $\frac{J}{\sqrt{2}m}$
(c) $\frac{J}{m}$ (d) $\sqrt{2}\frac{J}{m}$

8. A billiard ball is hit by a cue at a height h above that center. It acquires a linear velocity v_0. Mass of the ball is m and radius is r. The angular velocity ω_0 acquired by the ball is
(a) $\frac{2v_0 h}{5r^2}$ (b) $\frac{5v_0 h}{2r^2}$
(c) $\frac{2v_0 r^2}{5h}$ (d) $\frac{5v_0 r^2}{2h}$

9. A child is sitting at one end of a long trolley moving with a uniform speed v on a smooth horizontal track. If the child starts running towards the other end of the trolley with a speed u (w.r.t. trolley), the speed of the centre of mass of the system will
(a) $u + v$ (b) $v - u$
(c) v (d) none

10. A body of mass m slides down an inclined plane and reaches the bottom with a velocity v. If the same mass were in the form of a ring which rolls

down this incline, the velocity of the ring at the bottom would be

(a) v (b) $v/\sqrt{2}$

(c) $\sqrt{2}v$ (d) $\sqrt{2/5}\,v$

11. A circular platform is mounted on a frictionless vertical axle. Its radius R = 2 m and its moment of inertia about the axle is 200 kg m^2. It is initially at rest. A 50 kg man stands on the edge of the platform and begins to walk along the edge at the speed of 1 m s^{-1} relative to the ground. Time taken by the man to complete one revolution is

(a) π s (b) $\frac{3\pi}{2}$ s (c) 2π s (d) $\frac{\pi}{2}$ s

12. In a gravity free space, a man of mass M standing at a height h above the floor, throws a ball of mass m straight down with a speed u. When the ball reaches the floor, the distance of the man above the floor will be

(a) $h(1 + m/M)$
(b) $h(2 - m/M)$
(c) $2h$
(d) a function of m, M, h and u

13. A rope is wound around a hollow cylinder of mass 3 kg and radius 40 cm. What is the angular acceleration of the cylinder if the rope is pulled with a force of 30 N?

(a) 0.25 rad s^{-2} (b) 25 rad s^{-2}
(c) 5 m s^{-2} (d) 25 m s^{-2}

[NEET 2017]

14. Two discs of same moment of inertia rotating about their regular axis passing through centre and perpendicular to the plane of disc with angular velocities ω_1 and ω_2. They are brought into contact face to face coinciding the axis of rotation. The expression for loss of energy during this process is

(a) $\frac{1}{4}I(\omega_1 - \omega_2)^2$ (b) $I(\omega_1 - \omega_2)^2$

(c) $\frac{1}{8}I(\omega_1 - \omega_2)^2$ (d) $\frac{1}{2}I(\omega_1 + \omega_2)^2$

[NEET 2017]

15. Two rotating bodies A and B of masses m and $2m$ with moments of inertia I_A and I_B ($I_B > I_A$) have equal kinetic energy of rotation. If L_A and L_B be their angular momenta respectively, then

(a) $L_A = \frac{L_B}{2}$ (b) $L_A = 2L_B$

(c) $L_B > L_A$ (d) $L_A > L_B$

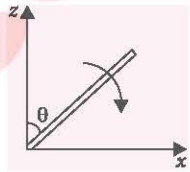
[NEET Phase-II 2016]

16. A solid sphere of mass m and radius R is rotating about its diameter. A solid cylinder of the same mass and same radius is also rotating about its geometrical axis with an angular speed twice that of the sphere. The ratio of their kinetic energies of rotation ($E_{\text{sphere}} / E_{\text{cylinder}}$) will be

(a) 2 : 3 (b) 1 : 5 (c) 1 : 4 (d) 3 : 1

[NEET Phase-II 2016]

17. A slender uniform rod of mass M and length l is pivoted at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical is



(a) $\frac{3g}{2l}\sin\theta$ (b) $\frac{2g}{3l}\sin\theta$

(c) $\frac{3g}{2l}\cos\theta$ (d) $\frac{2g}{3l}\cos\theta$

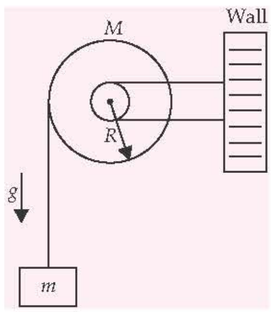
[JEE Main 2017]

18. The moment of inertia of a uniform cylinder of length l and radius R about its perpendicular bisector is I. What is the ratio l/R such that the moment of inertia is minimum ?

(a) $\sqrt{\frac{3}{2}}$ (b) $\frac{\sqrt{3}}{2}$ (c) 1 (d) $\frac{3}{\sqrt{2}}$

[JEE Main 2017]

19. A uniform disc of radius R and mass M is free to rotate only about its axis. A string is wrapped over its rim and a body of mass m is tied to the free end of the string as shown in the figure. The body is released from rest. Then the acceleration of the body is



(a) $\frac{2mg}{2m+M}$ (b) $\frac{2Mg}{2m+M}$

(c) $\frac{2Mg}{2M+m}$ (d) $\frac{2mg}{2M+m}$

[JEE Main Online 2017]

20. In a physical balance working on the principle of moments, when 5 mg weight is placed on the left pan, the beam becomes horizontal. Both the empty pans of the balance are of equal mass. Which of the following statements is correct?

(a) Left arm is shorter than the right arm
(b) Left arm is longer than the right arm
(c) Every object that is weighed using this balance appears lighter than its actual weight
(d) Both the arms are of same length

[JEE Main Online 2017]

SOLUTIONS

1. (d): Velocity of man with respect to ground is 1 m s^{-1} in opposite direction of motion of the car.

Hence, $v_{CM} = \dfrac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \dfrac{40\times 2 - 80\times 1}{40+80} = 0$

2. (a) : Acceleration of system,

$a = \dfrac{mg\sin 60° - mg\sin 30°}{2m}$ or $a = \left(\dfrac{\sqrt{3}-1}{4}\right)g$

Here, m = mass of each block

Now, $\vec{a}_{CM} = \dfrac{m\vec{a}_1 + m\vec{a}_2}{2m}$

Here, $\vec{a}_1$ and $\vec{a}_2$ are $\left(\dfrac{\sqrt{3}-1}{4}\right)g$ at right angles.

Hence, $|\vec{a}_{CM}| = \dfrac{\sqrt{2}}{2}a = \left(\dfrac{\sqrt{3}-1}{4\sqrt{2}}\right)g$

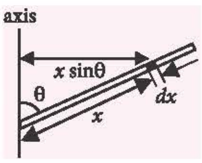
3. (d): $\vec{L} = \vec{r}\times\vec{p} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & -1\\ 3 & 4 & -2\end{vmatrix}$

$= \hat{i}(-4+4) - \hat{j}(-2+3) + \hat{k}(4-6) = 0\hat{i} - 1\hat{j} - 2\hat{k}$

$\vec{L}$ has components along $-y$ axis and $-z$ axis but it has no component along the x-axis. The angular momentum is in yz plane, *i.e.*, perpendicular to x-axis.

4. (c) : Mass of the element $= \left(\dfrac{m}{l}\right)dx$

Moment of inertia of the element about the axis



$= \left(\dfrac{m}{l}dx\right)(x\sin\theta)^2 = \dfrac{m}{l}\sin^2\theta\, x^2 dx$

$\therefore\ I = \dfrac{m}{l}\sin^2\theta\int_0^l x^2 dx = \dfrac{ml^2}{3}\sin^2\theta$

5. (d): Acceleration produced in the centre of mass due to friction,

$a = \dfrac{f}{M} = \dfrac{\mu_k Mg}{M} = \mu_k g$ ... (i)

where M is the mass of the ring

Angular retardation produced by the torque due to friction,

$\alpha = -\dfrac{\tau}{I} = -\dfrac{fR}{I} = -\dfrac{\mu_k MgR}{I}$ ... (ii)

As $v = u + at$ $(\because u = 0)$

$\therefore$ $v = 0 + \mu_k gt$ (Using (i))

As $\omega = \omega_0 + \alpha t$

$\therefore$ $\omega = \omega_0 - \dfrac{\mu_k MgR}{I}t$ (Using (ii))

For rolling without slipping

$v = R\omega$

$\therefore$ $\dfrac{v}{R} = \omega_0 - \dfrac{\mu_k MgR}{I}t$

$\dfrac{\mu_k gt}{R} = \omega_0 - \dfrac{\mu_k MgR}{I}t \Rightarrow \dfrac{\mu_k gt}{R}\left[1 + \dfrac{MR^2}{I}\right] = \omega_0$

$\dfrac{\mu_k gt}{R} = \dfrac{\omega_0}{1+\dfrac{MR^2}{I}} \Rightarrow t = \dfrac{R\omega_0}{\mu_k g\left(1+\dfrac{MR^2}{I}\right)}$

For ring, $I = MR^2$

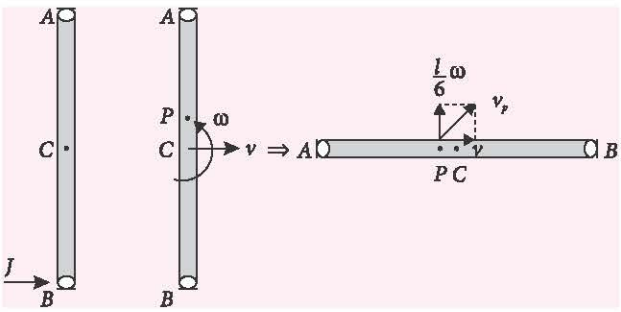
$\therefore$ $t = \dfrac{R\omega_0}{\mu_k g\left(1+\dfrac{MR^2}{MR^2}\right)} = \dfrac{R\omega_0}{2\mu_k g}$

6. (d): Angular retardation, $\alpha = -\dfrac{\tau}{I} = -\dfrac{(\mu mgR)}{\frac{2}{5}mR^2}$

$\therefore$ $\alpha = -\dfrac{5\mu g}{2R} = -\dfrac{5\times 0.1\times 10}{2\times 1} = -2.5\ \text{rad s}^{-2}$

Now, $0 = \omega_0 + \alpha t \Rightarrow t = \dfrac{40}{2.5} = 16$ s

7. (d): Let v and ω be the linear and angular speeds of the rod after applying an impulse J at B.



$\because$ Impulse = Change in momentum

We have, $mv = J$ or $v = \frac{J}{m}$ ...(i)

Due to impulse at end B, the rod will also rotate,

$\therefore \quad I\omega = J \cdot \frac{l}{2}$

$\frac{ml^2}{12} \cdot \omega = J \cdot \frac{l}{2}$ or $\omega = \frac{6J}{ml}$ ...(ii)

After the given time $t = \frac{\pi ml}{12J}$, the rod will rotate an angle

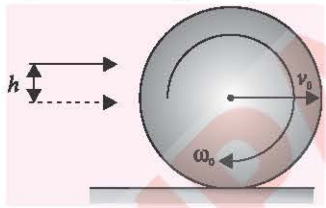
$\theta = \omega t = \left(\frac{6J}{ml}\right)\left(\frac{\pi ml}{12J}\right) = \frac{\pi}{2}$ (from eqn. (ii))

As, shown in figure, the particle at P will have velocity $\vec{v}_P$ due to both rotational and translational motion of rod. Due to rotation,

$\frac{l}{6} \cdot \omega = \left(\frac{l}{6}\right)\left(\frac{6J}{ml}\right) = \frac{J}{m} = v$ (using eqn. (i))

$\therefore \quad |\vec{v}_P| = \sqrt{v^2 + v^2} = \sqrt{2}v = \sqrt{2}\frac{J}{m}$

8. **(b):** Let J be linear impulse imparted to the ball. Applying; Impulse = Change in momentum



We have $J = mv_0$ ...(i)

$J \cdot h = I\omega_0 = \frac{2}{5}mr^2\omega_0$ ...(ii)

From equations (i) and (ii), we get $\omega_0 = \frac{5}{2}\frac{v_0 h}{r^2}$

9. **(c):** Velocity of centre of mass will remain unaffected as no external force acts on system.

10. **(b):** Kinetic energy of translation of mass m sliding down the incline with velocity, $v = \frac{1}{2}mv^2.$

For pure rolling, $v_{CM} = r\omega$

Total kinetic energy of the ring rolling down the incline with velocity v_{CM}

$= \frac{1}{2}mv_{CM}^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv_{CM}^2 + \frac{1}{2}(mr^2)\omega^2 = mv_{CM}^2$

Since both the bodies have mass m, they will gain same amount of kinetic energy.

Thus, $mv_{CM}^2 = \frac{1}{2}mv^2$ or $v_{CM} = v/\sqrt{2}$

11. **(c):** As the system is initially at rest, therefore, initial angular momentum, $L_i = 0$.

According to the principle of conservation of angular momentum,

final angular momentum, $L_f = 0$.

$\therefore$ Angular momentum of platform = Angular momentum of man in opposite direction.

i.e., $mvR = I\omega$

or $\omega = \frac{mvR}{I} = \frac{50 \times 1 \times 2}{200} = \frac{1}{2}$ rad s^{-1}

Angular velocity of man relative to platform is

$\omega_r = \omega + \frac{v}{R} = \frac{1}{2} + \frac{1}{2} = 1$ rad s^{-1}

Time taken by the man to complete one revolution is $T = \frac{2\pi}{\omega_r} = \frac{2\pi}{1} = 2\pi$ s

12. **(a):** In gravity free space, displacement of centre of mass of man and ball system should not move. If displacement of the ball be h then the displacement of man in upward direction.

$mh = Mh' \Rightarrow h' = \frac{mh}{M}$ ...(i)

Hence, the position of man from ground

$H = h + h' = h\left[1 + \frac{m}{M}\right]$

13. **(b):** $m = 3$ kg, $r = 40$ cm $= 40 \times 10^{-2}$ m $= 0.4$ m, $F = 30$ N

Moment of inertia of hollow cylinder about its axis

$= mr^2 = 3 \text{ kg} \times (0.4)^2 \text{ m}^2 = 0.48 \text{ kg m}^2$

The torque is given by, $\tau = I\alpha$

where, I = moment of inertia,

α = angular acceleration

In the given case, $\tau = rF$, as the force is acting perpendicularly to the radial vector.

$\therefore \quad \alpha = \frac{\tau}{I} = \frac{Fr}{mr^2} = \frac{F}{mr} = \frac{30}{3 \times 40 \times 10^{-2}} = \frac{30 \times 100}{3 \times 40}$

$\alpha = 25$ rad s^{-2}

14. **(a):** Initial angular momentum $= I\omega_1 + I\omega_2$

Let ω be angular speed of the combined system.

Final angular momentum $= 2I\omega$

$\therefore$ According to conservation of angular momentum

$I\omega_1 + I\omega_2 = 2I\omega$ or $\omega = \frac{\omega_1 + \omega_2}{2}$

Initial rotational kinetic energy

$E_i = \frac{1}{2}I(\omega_1^2 + \omega_2^2)$

Final rotational kinetic energy

$$E_f = \frac{1}{2}(2I)\omega^2 = \frac{1}{2}(2I)\left(\frac{\omega_1+\omega_2}{2}\right)^2 = \frac{1}{4}I(\omega_1+\omega_2)^2$$

$\therefore$ Loss of energy $\Delta E = E_i - E_f$

$$= \frac{I}{2}(\omega_1^2+\omega_2^2) - \frac{I}{4}(\omega_1^2+\omega_2^2+2\omega_1\omega_2)$$

$$= \frac{I}{4}\left[\omega_1^2+\omega_2^2-2\omega_1\omega_2\right] = \frac{I}{4}(\omega_1-\omega_2)^2$$

15. (c) : Here, $m_A = m$, $m_B = 2\,m$

Both bodies A and B have equal kinetic energy of rotation

$$K_A = K_B \Rightarrow \frac{1}{2}I_A\omega_A^2 = \frac{1}{2}\,I_B\omega_B^2$$

$$\Rightarrow \frac{\omega_A^2}{\omega_B^2} = \frac{I_B}{I_A} \quad \text{...(i)}$$

Ratio of angular momenta,

$$\frac{L_A}{L_B} = \frac{I_A\,\omega_A}{I_B\,\omega_B} = \frac{I_A}{I_B}\times\sqrt{\frac{I_B}{I_A}} \quad \text{[Using eqn. (i)]}$$

$$= \sqrt{\frac{I_A}{I_B}} < 1 \quad (\because I_B > I_A)$$

$\therefore \quad L_B > L_A$

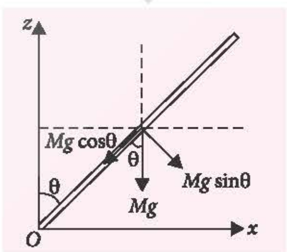
16. (b) : $$\frac{E_{\text{sphere}}}{E_{\text{cylinder}}} = \frac{\frac{1}{2}I_s\omega_s^2}{\frac{1}{2}I_c\omega_c^2} = \frac{I_s\omega_s^2}{I_c\omega_c^2}$$

Here, $I_s = \frac{2}{5}mR^2$, $I_c = \frac{1}{2}mR^2$ and $\omega_c = 2\omega_s$

$$\therefore \quad \frac{E_{\text{sphere}}}{E_{\text{cylinder}}} = \frac{\frac{2}{5}mR^2\times\omega_s^2}{\frac{1}{2}mR^2\times(2\omega_s)^2} = \frac{4}{5}\times\frac{1}{4} = \frac{1}{5}$$

17. (a) : The torque of the weight Mg of the rod about the pivot O is given by

$$\tau = Mg\sin\theta\times\left(\frac{l}{2}\right) \quad \text{...(i)}$$



($Mg\cos\theta$ is passing through the pivot O. Hence, its contribution to the torque will be zero)

Also,

$$\tau = I\alpha \quad \text{...(ii)}$$

$$\therefore \quad I\alpha = Mg\sin\theta\times\left(\frac{l}{2}\right) \quad \text{(Using (i) and (ii))}$$

Now, moment of inertia of the rod about the pivot O is $I = \frac{1}{3}Ml^2$

$$\therefore \quad \frac{1}{3}Ml^2\alpha = Mg\sin\theta\left(\frac{l}{2}\right) \Rightarrow \alpha = \frac{3}{2}\frac{g}{l}\sin\theta$$

18. (a) : Moment of inertia of a uniform cylinder of length l and radius R about its perpendicular bisector is given by

$$I = \frac{1}{12}ml^2 + \frac{mR^2}{4}$$

$$\text{or} \quad I = \frac{m}{4}\left(\frac{1}{3}l^2 + R^2\right) \quad \text{... (i)}$$

Also, $m = \rho V = \rho\pi R^2 l$ or $R^2 = \frac{m}{\rho\pi l}$

Substitute R^2 in eqn. (i), we get

$$I = \frac{m}{4}\left(\frac{l^2}{3} + \frac{m}{\rho\pi l}\right)$$

For moment of inertia to be maximum or minimum,

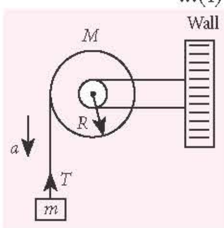
$$\frac{dI}{dl} = 0 \Rightarrow \frac{m}{4}\left(\frac{2l}{3} - \frac{m}{\rho\pi l^2}\right) = 0$$

$$\Rightarrow \frac{2l}{3} - \frac{R^2}{l} = 0 \quad \left(\text{Using } \frac{R^2}{l} = \frac{m}{\rho\pi l^2}\right)$$

$$\Rightarrow \frac{2l}{3} = \frac{R^2}{l} \Rightarrow \frac{l}{R} = \sqrt{\frac{3}{2}}$$

19. (a) : From figure, we conclude

$$mg - T = ma \quad \text{...(i)}$$



Moment of inertia of a uniform disc, $I = \frac{MR^2}{2}$

and an acceleration is,

$a = \alpha R$

$\because \quad RT = I\alpha$

$$\therefore \quad RT = \frac{MR^2}{2}\times\frac{a}{R} \Rightarrow T = \frac{Ma}{2}$$

Putting this value in equation (i)

$$mg - \frac{Ma}{2} = ma \text{ or } mg = a\left(m + \frac{M}{2}\right) \Rightarrow a = \frac{2mg}{M+2m}$$

20. (a)

◈◈

EXAM PREP 2018

CLASS XI

Useful for Medical/Engg. Entrance Exams

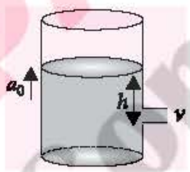
CHAPTERWISE MCQs FOR PRACTICE

MECHANICAL PROPERTIES OF FLUIDS

1. Eight identical droplets of mercury (surface tension $0.55\ \text{N m}^{-1}$) of radius 1 mm. All the droplets merge into one bigger drop. Find the amount of energy evolved.
   (a) 24.62×10^{-5} J (b) 27.65×10^{-6} J
   (c) 24.62×10^{-6} J (d) 27.65×10^{-5} J

2. A spherical soap bubble of radius 2 cm is attached to a spherical bubble of radius 4 cm. Find the radius of the curvature of the common surface.
   (a) 4 cm (b) 3 cm (c) 2 cm (d) 0.4 cm

3. For the area a of the hole is much lesser than the area of the base of a vessel of liquid, velocity of efflux v of the liquid in an accelerating vessel as shown in figure is (a_0 = vertical acceleration)



   (a) $\sqrt{2gh}$ (b) $\sqrt{2(g-a_0)h}$
   (c) $\sqrt{2(g+a_0)h}$ (d) none of these

4. Water from a tap emerges vertically downwards with an initial velocity v_0. Assume pressure is constant throughout the stream of water and the flow is steady, find the distance from the tap at which cross-sectional area of stream is half of the cross-sectional area of stream at the tap.
   (a) $\frac{v_0^2}{2g}$ (b) $\frac{3v_0^2}{2g}$ (c) $\frac{2v_0^2}{g}$ (d) $\frac{5v_0^2}{2g}$

5. The work done in increasing the size of a rectangular soap film with dimensions 8 cm × 3.75 cm to 10 cm × 6 cm is 2×10^{-4} J. The surface tension of the film in N m^{-1} is
   (a) 1.65×10^{-2} (b) 3.3×10^{-2}
   (c) 6.6×10^{-2} (d) 8.25×10^{-2}

6. Water from a tap emerges vertically downward with an initial speed of $1.0\ \text{m s}^{-1}$. The cross-sectional area of the tap is $10^{-4}\ \text{m}^2$. Assume that the pressure is constant throughout the stream of water, and that the flow is steady. What is the cross-sectional area (in m^2) of the stream 0.15 m below the tap?
   (a) 2×10^{-4} (b) 5×10^{-4}
   (c) 5×10^{-5} (d) 2×10^{-5}

7. A large open tank has two holes in the wall. One is a square hole of side L at a depth y from the top and the other is a circular hole of radius R at a depth $4y$ from the top. When the tank is completely filled with water, the quantities of water flowing out per second from both holes are the same. Then, what is the value of R?
   (a) $\frac{L}{\sqrt{2\pi}}$ (b) $\frac{L}{\sqrt{\pi}}$ (c) $\frac{L}{2}$ (d) $\frac{L}{4}$

8. The narrow bores of diameters 3.0 mm and 6.0 mm are joined together to form a U shaped tube open at both ends. If U-tube contains water, what is the difference in its levels in the two limbs of the tube? Surface tension of water is $7.3 \times 10^{-2}\ \text{N m}^{-1}$. Take the angle of contact to be zero, and density of water to be $1.0 \times 10^3\ \text{kg m}^{-3}$ and $g = 9.8\ \text{m s}^{-2}$.
   (a) 0.5×10^{-2} m (b) 0.4×10^{-2} m
   (c) 0.3×10^{-2} m (d) 0.1×10^{-2} m

9. A spherical solid ball of volume V is made of a material of density ρ_1. It is falling through a liquid of density ρ_2 ($\rho_2 < \rho_1$). Assume that the liquid applies a

viscous force on the ball that is proportional to the square of its speed v, *i.e.*, $F_{viscous} = -kv^2$ $(k > 0)$. The terminal speed of the ball is

(a) $\frac{Vg(\rho_1 - \rho_2)}{k}$ (b) $\sqrt{\frac{Vg(\rho_1 - \rho_2)}{k}}$

(c) $\frac{Vg\rho_1}{k}$ (d) $\sqrt{\frac{Vg\rho_1}{k}}$

10. The drop of liquid of density ρ is floating with $\frac{1}{4}$th inside the liquid A of density ρ_1 and remaining in the liquid B of density ρ_2. Then

(a) Upthrust in liquid $A = \frac{\delta_1}{4\delta_2}$ times upthrust in liquid B.

(b) $4\delta = \delta_1 + 3\delta_2$

(c) $3\delta = 4\delta_1 + \delta_2$

(d) Upthrust in liquid $B = \frac{\delta_2}{3\delta_1}$ times upthrust in liquid A.

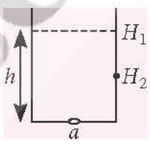
11. Three capillaries of internal radii $2r$, $3r$ and $4r$, all of the same length, are joined end to end. A liquid passes through the combination and the pressure difference across this combination is 20.2 cm of mercury. The pressure difference across the capillary of internal radius $2r$ is

(a) 2 cm of Hg (b) 4 cm of Hg
(c) 8 cm of Hg (d) 16 cm of Hg

12. A frame made of metallic wire enclosing a surface area A is covered with a soap film. If the area of the frame of metallic wire is reduced by 50%, the energy of the soap film will be changed by

(a) 100% (b) 75% (c) 50% (d) 25%

13. A vessel having area of cross-section A contains a liquid upto a height h. At the bottom of the vessel, there is a small hole having area of cross-section a. Then the time taken for the liquid level to fall from height H_1 to H_2 is given by



(a) $\sqrt{2g(H_1 - H_2)}$ (b) $\frac{A}{a}\sqrt{\frac{2}{g}}(\sqrt{H_1} - \sqrt{H_2})$

(c) $\frac{A}{a}\sqrt{\frac{g}{2}}(\sqrt{H_1} - \sqrt{H_2})$ (d) $\sqrt{2gH}$

14. Two soap bubbles of radii a and b combine to form a single bubble of radius c. If P is the external pressure, then the surface tension of the soap solution is

(a) $\frac{P(c^3 + a^3 + b^3)}{4(a^2 + b^2 - c^2)}$ (b) $\frac{P(c^3 - a^3 - b^3)}{4(a^2 + b^2 - c^2)}$

(c) $Pc^3 - 4a^2 - 4b^2$ (d) $Pc^2 - 2a^2 - 3b^2$

15. Water rises in a capillary tube to a height of 2.0 cm. In another capillary tube whose radius is one-third of it, how much the water will rise?

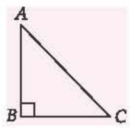
(a) 5 cm (b) 3 cm (c) 6 cm (d) 9 cm

THERMAL PROPERTIES OF MATTER

16. A body cools from 50°C to 40° C in 5 minutes. The temperature of surroundings is 20°C. Find the temperature of body after another 5 minutes.

(a) 33.3°C (b) 43.3°C
(c) 35.3°C (d) 30.3°C

17. Three rods of length L of identical cross-sectional area made from the same metal form the sides of an isosceles triangle ABC right angled at B. The points A and B are maintained at temperatures T and $\sqrt{2}T$ respectively in the steady state. Assuming that only heat conduction takes place, temperature of point C will be



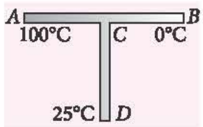
(a) $\frac{3T}{\sqrt{2}+1}$ (b) $\frac{T}{\sqrt{2}+1}$

(c) $\frac{T}{\sqrt{3}(\sqrt{2}-1)}$ (d) $\frac{T}{\sqrt{2}-1}$

18. An icebox made of 1.5 cm thick styrofoam has dimensions 60 cm × 60 cm × 30 cm. It contains ice at 0 °C and is kept in a room at 40 °C. Find the rate at which the ice is melting. Latent heat of fusion of ice = 3.36×10^5 J kg^{-1} and thermal conductivity of styrofoam = 0.04 W m^{-1}°C^{-1}.

(a) 0.5 g s^{-1} (b) 0.56 g s^{-1}
(c) 0.46 g s^{-1} (d) 0.48 g s^{-1}

19. A rod CD of thermal resistance 5.0 K W^{-1} is joined at the middle of an identical rod AB as shown in figure. The ends A, B and D are maintained at 100°C, 0°C and 25°C respectively. Find the heat current in CD.

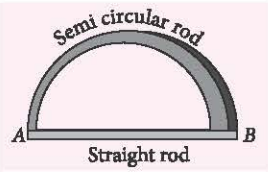


(a) 4 W (b) 5 W
(c) 6 W (d) 7 W

20. An electric heater emits 1000 W of thermal radiation. The coil has a surface area of 0.020 m^2. Assuming that the coil radiates like a blackbody, find its temperature.
(Stefan's constant, $\sigma = 6.00 \times 10^{-8}$ W m^{-2} K^{-4}.)

(a) 980 K (b) 970 K
(c) 950 K (d) 955 K

21. Two rods (one semi-circular and other straight) of same material and of same cross-sectional area are joined as shown in the figure. The points A and B are maintained at different temperatures. Find the ratio of the heat transferred through a cross-section of semi-circular rod to the heat transferred through a cross-section of the straight rod in a given time.



(a) $\frac{2}{\pi}$ (b) $\frac{1}{\pi}$ (c) π (d) $\frac{\pi}{2}$

22. An ice cube of mass 0.1 kg at 0°C is placed in an isolated container which is at 227°C. The specific heat s of the container varies with temperature T according to the empirical relation $s = A + BT$, where A = 100 cal kg^{-1} K^{-1} and $B = 2\times 10^{-2}$ cal kg^{-1} K^{-2}. If the final temperature of the container is 27°C, find the mass of the container.
(Latent heat of fusion of water = 8×10^4 cal kg^{-1} specific heat of water = 10^3 cal kg^{-1} K^{-1})
(a) 0.495 kg (b) 0.224 kg
(c) 0.336 kg (d) 0.621 kg

23. The two opposite faces of a cubical piece of iron (thermal conductivity = 0.2 cal cm^{-1} s^{-1} °C^{-1}) are at 100 °C and 0 °C in ice. If the area of surface is 4 cm^2, then the mass of ice melted in 10 minutes will be
(a) 30 g (b) 300 g (c) 5 g (d) 50 g

24. The sun's surface temperature is about 6000 K. The sun's radiation is maximum at a wavelength of 0.5 μm. A certain light bulb filament emits radiation with a maximum at 2 μm. If both the surface of the sun and of the filament have the same emissive characteristics, what is the temperature of the filament?
(a) 1500 K (b) 1600 K
(c) 1000 K (d) 1200 K

25. Three discs A, B and C having radii 2 m, 4 m, and 6 m, respectively are coated with carbon black on their outer surfaces. The wavelengths corresponding to maximum intensity are 3 m, 4 m and 5 m respectively. The power radiated by them are P_A, P_B and P_C respectively. Then
(a) P_A is minimum (b) P_B is minimum
(c) P_C is minimum (d) $P_A = P_B = P_C$

26. A hot liquid is filled in a container and kept in a room of temperature of 25 °C. The liquid emits heat at the rate of 200 J s^{-1} when its temperature is 75 °C. When the temperature of the liquid becomes 40 °C, the rate of heat loss in J s^{-1} is
(a) 160 (b) 140 (c) 80 (d) 60

27. If an anisotropic solid has coefficients of linear expansion α_x, α_y, and α_z for three mutually perpendicular directions in the solid, its coefficient of volume expansion will be
(a) $(\alpha_x\alpha_y\alpha_z)^{1/3}$ (b) $\alpha_x + \alpha_y + \alpha_z$
(c) $(\alpha_x^2+\alpha_y^2+\alpha_z^2)^{1/2}$ (d) $(\sqrt{\alpha_x}+\sqrt{\alpha_y}+\sqrt{\alpha_z})^2$

28. The surface temperature of the stars is determined using
(a) Planck's law
(b) Wien's displacement law
(c) Rayleigh Jeans law
(d) Kirchhoff's law

29. A sphere of 3 cm radius acts like a blackbody. It absorbs 30 kW of power radiated to it from the surroundings. What is the temperature of the sphere? (Stefan's constant, $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^4)
(a) 2600 K (b) 2500 K
(c) 2400 K (d) 2200 K

30. A refrigerator door is 150 cm high, 80 cm wide, and 6 cm thick. If the coefficient of conductivity is 0.0005 cal cm^{-1} s^{-1} °C^{-1}, and the inner and outer surfaces are at 0 °C and 30 °C, respectively, what is the heat loss per minute through the door, in cal?
(a) 2000 (b) 1500 (c) 1800 (d) 2200

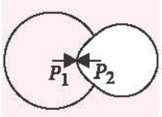
SOLUTIONS

1. **(b) :** Surface area of 8 droplets = $8 \times 4\pi r^2$
$= 32\pi \times 10^{-6}$ m^2
∴ Surface energy of 8 droplets = $32\pi \times 10^{-6} \times 0.55$ J
Let R be the radius of the big drop formed.
The volume of the big drop $= \frac{4\pi}{3}R^3$
Since volume remains constant in the process of combination
$$8\times\frac{4\pi}{3}\times(1\times10^{-3})^3 = \frac{4\pi}{3}\times R^3 \text{ or } R = 2\times 10^{-3}\text{ m}$$
Surface energy of big drop formed
$= 4\pi(2 \times 10^{-3})^2 \times 0.55 = 16\pi \times 10^{-6} \times 0.55$ J
∴ Energy evolved
$= 32\pi \times 10^{-6} \times 0.55 - 16\pi \times 10^{-6} \times 0.55$
$= 16\pi \times 0.55 \times 10^{-6} = 27.65 \times 0^{-6}$ J

2. **(a) :** P_1(excess pressure in the spherical bubble) $= \frac{4S}{r_1}$

P_2(excess pressure in the spherical soap bubble) $= \dfrac{4S}{r_2}$

$\therefore \quad P_2 - P_1 = 4S\left(\dfrac{1}{r_2} - \dfrac{1}{r_1}\right) = \dfrac{4S}{r}$



Where r = radius of curvature of the common surface

$\therefore \quad \dfrac{1}{r} = \dfrac{1}{r_2} - \dfrac{1}{r_1} = \dfrac{1}{2} - \dfrac{1}{4} = \dfrac{2-1}{4} = \dfrac{1}{4}$

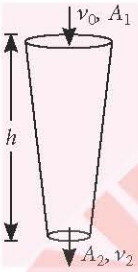
$\therefore \quad r = 4$ cm

3. (c) : Effective value of acceleration due to gravity becomes $(g + a_0)$.

Required velocity of efflux, $v = \sqrt{2(g + a_0)h}$

4. (b) : In the shown figure, $v_2^2 = v_0^2 + 2gh$

and $A_1 v_0 = A_2 v_2$



Solving, $\dfrac{A_2}{A_1} = \dfrac{v_0}{\sqrt{v_0^2 + 2gh}}$

$\dfrac{A_2}{A_1} = \dfrac{A_2}{2A_2} = \dfrac{v_0}{\sqrt{v_0^2 + 2gh}}$

$4v_0^2 = v_0^2 + 2gh \quad \Rightarrow \quad h = \dfrac{3v_0^2}{2g}$

5. (b) : Since the soap film has two surfaces,

$\therefore$ Increase in surface area,

$\Delta A = 2[10 \times 6 - 8 \times 3.75] \times 10^{-4}$ m^2

Surface tension, $S = \dfrac{W}{\Delta A} = \dfrac{2 \times 10^{-4}}{2 \times [60 - 30] \times 10^{-4}}$

$= 3.3 \times 10^{-2}$ N m^{-1}

6. (c) : Here : $v_1 = 1.0$ m s^{-1}, $A_1 = 10^{-4}$ m^2,

$h_1 - h_2 = 0.15$ m, $v_2 = ?$ $A_2 = ?$

According to Bernoulli's theorem,

$$P + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$[\because P_1 = P_2 = P]$

or $\dfrac{1}{2}v_1^2 + gh_1 = \dfrac{1}{2}v_2^2 + gh_2$

or $v_2^2 = v_1^2 + 2g(h_1 - h_2) = (1.0)^2 + 2 \times 10 \times 0.15 = 4$

or $v = 2$ m s^{-1}

By equation of continuity, $A_1 v_1 = A_2 v_2$

$\therefore \quad A_2 = \dfrac{A_1 v_1}{v_2} = \dfrac{10^{-4} \times 1}{2} = 5 \times 10^{-5}$ m^2

7. (a) : Equating the rate of flow, $v_1 A_1 = v_2 A_2$

But $v_1 = \sqrt{2gy}$, $A_1 = L^2$, $v_2 = \sqrt{2g \times 4y}$, $A_2 = \pi R^2$

$\therefore \quad \sqrt{2gy} \times L^2 = \sqrt{2g \times 4y} \times \pi R^2$

or $L^2 = 2\pi R^2$ or $R = \dfrac{L}{\sqrt{2\pi}}$

8. (a) : Here : $r_1 = \dfrac{3.0}{2} = 1.5 \text{ mm} = 1.5 \times 10^{-3}$ m,

$r_2 = \dfrac{6.0}{2} = 3.0 \text{ mm} = 3.0 \times 10^{-3}$ m

$S = 7.3 \times 10^{-2}$ N m^{-1}, $\theta = 0°$

$\rho = 1.0 \times 10^3$ kg m^{-3}, $g = 9.8$ m s^{-2}

Let h_1 and h_2 be the heights to which water rises in the two tubes. Then

$h_1 = \dfrac{2S\cos\theta}{r_1 \rho g}$ and $h_2 = \dfrac{2S\cos\theta}{r_2 \rho g}$

Therefore,

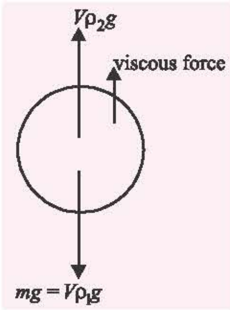
$$h_1 - h_2 = \frac{2S\cos\theta}{\rho g}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$= \frac{2 \times 7.3 \times 10^{-2} \cos 0°}{10^3 \times 9.8}\left[\frac{1}{1.5 \times 10^{-3}} - \frac{1}{3 \times 10^{-3}}\right]$$

$$= \frac{14.6 \times 10^{-2}}{10^3 \times 9.8 \times 10^{-3}}\left[\frac{1}{1.5} - \frac{1}{3}\right]$$

$$= \frac{14.6 \times 10^{-2}}{9.8} \times \frac{1}{3} = 0.5 \times 10^{-2} \text{ m}$$

9. (b): The forces acting on the solid ball when it is falling through a liquid are mg downwards, upthrust upwards and the viscous force also acting upwards. The viscous force rapidly increases with velocity, attaining a maximum when the ball reaches the terminal velocity.



Then the acceleration is zero. $mg - V\rho_2 g - kv^2 = ma$

where V is volume, v is the terminal velocity.

When the ball is moving with terminal velocity, $a = 0$

$\therefore \quad V\rho_1 g - V\rho_2 g - kv^2 = 0$

or $v^2 = \dfrac{Vg(\rho_1 - \rho_2)}{k}$ or $v = \sqrt{\dfrac{Vg(\rho_1 - \rho_2)}{k}}$

10. (b) : Upthrust in liquid A, $U_1 = \dfrac{1}{4}V\rho_1 g$

Upthrust in liquid B, $U_2 = \dfrac{3}{4}V\rho_2 g$

$\therefore \quad \dfrac{U_1}{U_2} = \dfrac{\rho_1}{3\rho_2}$

For floatation,

$\dfrac{1}{4}V\rho_1 g + \dfrac{3}{4}V\rho_2 g = V\rho g \quad \Rightarrow \quad \rho_1 + 3\rho_2 = 4\rho$

11. (d): $V = \frac{\pi P r^4}{8\eta l}$

As capillaries of same length are joined end to end, the volume rate V across each of the capillaries is same.

Here, $P_1 \cdot r_1^4 = P_2 \cdot r_2^4 = P_3 \cdot r_3^4$

or $P_1(2r)^4 = P_2(3r)^4 = P_3(4r)^4 \Rightarrow 16P_1 = 81P_2 = 256P_3$

$\Rightarrow P_2 = \frac{16}{81}P_1$ and $P_3 = \frac{P_1}{16}$

(Given) : $(P_1 + P_2 + P_3) = 20.2$ cm of Hg

$\Rightarrow P_1\left(1+\frac{16}{81}+\frac{1}{16}\right) = 20.2 \Rightarrow P_1 = 16$ cm of Hg.

12. (c) : Surface energy = surface tension × surface area

$E = S \times 2A$

($\because$ Soap film has two surfaces)

As $A' = \left[A - \frac{50A}{100}\right] = \frac{A}{2} \quad \therefore E' = S \times 2\left(\frac{A}{2}\right) = SA$

Percentage decrease in surface energy

$= \frac{E-E'}{E} \times 100\% = \frac{2SA - SA}{2SA} \times 100\%$

$= \frac{1}{2} \times 100\% = 50\%$

13. (b): The velocity of efflux from the hole, $v = \sqrt{2gh}$

Using equation of continuity, $A_1v_1 = A_2v_2$

$A\left(-\frac{dh}{dt}\right) = a(\sqrt{2gh}) \Rightarrow \frac{-dh}{\sqrt{h}} = \frac{a}{A}\sqrt{2g}dt$

$-\int_{H_1}^{H_2} h^{-\frac{1}{2}}dh = \frac{a}{A}\sqrt{2g}\int_0^t dt$ or $-\left[2h^{\frac{1}{2}}\right]_{H_1}^{H_2} = \frac{a}{A}\sqrt{2g}(t)$

or $2(\sqrt{H_1} - \sqrt{H_2}) = \frac{a}{A}\sqrt{2 \cdot g}\, t$

$\Rightarrow t = \frac{A}{a} \cdot \sqrt{\frac{2}{g}}(\sqrt{H_1} - \sqrt{H_2})$

14. (b): As the total mass of the air inside the bubble and the temperature remains constant.

$\therefore P_aV_a + P_bV_b = P_cV_c$

As pressure inside the soap bubble is $\frac{4S}{r}$ more than the external pressure, (here S is the surface tension)

$$\left(P+\frac{4S}{a}\right)\cdot\left(\frac{4}{3}\pi a^3\right) + \left(P+\frac{4S}{b}\right)\cdot\left(\frac{4}{3}\pi b^3\right) = \left(P+\frac{4S}{c}\right)\cdot\left(\frac{4}{3}\pi c^3\right)$$

$\therefore S = \frac{P(c^3 - a^3 - b^3)}{4(a^2 + b^2 - c^2)}$

15. (c) : According to ascent formula

$h = \frac{2S\cos\theta}{r\rho g}$

For a given liquid, $hr = \frac{2S\cos\theta}{\rho g}$ = constant

$\therefore h_1r_1 = h_2r_2$

or $h_2 = \frac{h_1r_1}{r_2} = \frac{(2.0)(r_1)}{(r_1/3)} = (2.0)(3) = 6.0$ cm

16. (a) : Rate of fall of temperature $= \frac{50-40}{5}$

$= 2°C$ per minute

Average excess of temperature $= \frac{40+50}{2} - 20$

$= 45 - 20 = 25$

According to law of cooling,

rate of cooling $\propto$ excess of temperature

$\therefore 2 = k \cdot 25$ or $k = \frac{2}{25}$

Let T = temperature after another 5 minutes

Then rate of fall of temperature $= \frac{40-T}{5}$

and average excess of temperature $= \frac{T+40}{2} - 20 = \frac{T}{2}$

$\therefore \frac{40-T}{5} = k\frac{T}{2}$

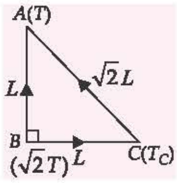
or $\frac{40-T}{5} = \frac{2}{25}\cdot\frac{T}{2}$ or $40 - T = \frac{T}{5}$ or $6T = 200$

or $T = 33.33°C$

17. (a) : As $T_B > T_A$, heat flows from B to A through both paths BA and BCA.

Rate of heat flow in BC = Rate of heat flow in CA

$\therefore \frac{KA(\sqrt{2}T - T_C)}{L} = \frac{KA(T_C - T)}{\sqrt{2}L}$



Solving this, we get

$T_C = \frac{3T}{\sqrt{2}+1}$

18. (c) : The total surface area of the walls

= 2(60 cm × 60 cm + 60 cm × 30 cm + 60 cm × 30 cm)

= 1.44 m^2.

The thickness of the walls = 1.5 cm = 0.015 m.

The rate of heat flow into the box is

$\frac{\Delta Q}{\Delta t} = \frac{KA(T_1 - T_2)}{x}$

$= \frac{(0.04\ W\ m^{-1}\,°C^{-1})(1.44\ m^2)(40°C)}{0.015\ m} = 154\ W$

The rate at which the ice melts is

$$= \frac{154\ \text{W}}{3.36\times10^5\ \text{J kg}^{-1}} = 0.46\ \text{g s}^{-1}$$

19. (a) : The thermal resistance of AC is equal to that of CB and is equal to 2.5 K W^{-1}. Suppose, the temperature at C is T. The heat currents through AC, CB and CD are

$$\frac{\Delta Q_1}{\Delta t} = \frac{100°C - T}{2.5\ \text{K W}^{-1}},\quad \frac{\Delta Q_2}{\Delta t} = \frac{T - 0°C}{2.5\ \text{K W}^{-1}}$$

and $\frac{\Delta Q_3}{\Delta t} = \frac{T - 25°C}{5.0\ \text{K W}^{-1}}$

We also have

$$\frac{\Delta Q_1}{\Delta t} = \frac{\Delta Q_2}{\Delta t} + \frac{\Delta Q_3}{\Delta t}$$

or $\frac{100°C - T}{2.5} = \frac{T - 0°C}{2.5} + \frac{T - 25°C}{5}$

or $225°C = 5\ T$

or $T = 45°C$

Thus, $\frac{\Delta Q_3}{\Delta t} = \frac{45°C - 25°C}{5.0\ \text{K W}^{-1}} = \frac{20\ \text{K}}{5.0\ \text{K W}^{-1}} = 4.0\ \text{W}.$

20. (d): Let the temperature of the coil be T. The coil will emit radiation at a rate $A\sigma T^4$. Thus,

$1000\ \text{W} = (0.020\ \text{m}^2) \times (6.0 \times 10^{-8}\ \text{W m}^{-2}\ \text{K}^{-4}) \times T^4$

or $T^4 = \frac{1000}{0.020 \times 6.00 \times 10^{-8}}\ \text{K}^4 = 8.33 \times 10^{11}\ \text{K}^4$

or $T = 955$ K

21. (a) : $\frac{dQ}{dt} = \frac{KA\Delta T}{l}$, for both rods K, A and ΔT are same

$\Rightarrow \frac{dQ}{dt} \propto \frac{1}{l}$

so $\frac{(dQ/dt)_{\text{semicircular}}}{(dQ/dt)_{\text{straight}}} = \frac{l_{\text{straight}}}{l_{\text{semicircular}}} = \frac{2r}{\pi r} = \frac{2}{\pi}$

22. (a) : Heat received by ice is

$Q_1 = mL + ms\Delta T$

$= (0.1)\ (8 \times 10^4) + (0.1) \times 10^3 \times 27 = 10700$ cal

Heat lost by the container

$$Q_2 = \int_{300}^{500} m\,(A + BT)dT = m\left[AT + \frac{BT^2}{2}\right]_{300}^{500}$$

$= 21600\ m$

By principle of calorimetry, $Q_1 = Q_2$

$\Rightarrow\ m = 0.495$ kg

23. (b): $Q = mL = KA\frac{(T_1 - T_2)}{l}t$

$$\Rightarrow m = \frac{1}{L} \times KA\frac{(T_1 - T_2)}{l} \times t$$

$$= \frac{1}{80} \times 0.2 \times 4 \times \frac{(100 - 0)}{\sqrt{4}} \times 10 \times 60$$

$(\because\ l^2 = 4\ \Rightarrow\ l = \sqrt{4})$

$$= \frac{0.2 \times 4 \times 100 \times 600}{80 \times 2} = 300\,g$$

24. (a) : Use Wien's displacement law,

$\lambda_m T$ = constant

$\therefore\ \lambda_1 T_1 = \lambda_2 T_2$

$\Rightarrow\ 0.5 \times 10^{-6} \times 6000 = 2 \times 10^{-6} \times T_2$

$\therefore\ T_2 = \frac{0.5 \times 6000}{2} = 1500$ K

25. (a) : Stefan's law, $P \propto AT^4$

Wien's law, $\lambda_m T$ = constant

$$\therefore\ P \propto \frac{A}{(\lambda_m)^4} \propto \frac{r^2}{\lambda_m^4}$$

$$P_A : P_B : P_C = \frac{2^2}{3^4} : \frac{4^2}{5^4} : \frac{6^2}{5^4} = \frac{4}{81} : \frac{1}{16} : \frac{36}{625}$$

$= 0.05 : 0.0625 : 0.0576$

Hence P_A is minimum.

26. (d): Applying Newton's law of cooling,

$$\frac{ms(75 - 25)}{dt} = 200\ \text{J s}^{-1}$$

and $\frac{ms(40 - 25)}{dt} = x\quad \therefore\ x = \frac{200}{50} \times 15 = 60\ \text{J s}^{-1}$

27. (b)

28. (b): The surface temperature of the stars is determined by Wien's displacement law.

29. (a) : The power absorbed by a blackbody is $P = \sigma AT^4$

or $(30 \times 10^3) = (5.67 \times 10^{-8})\ 4\pi(0.03)^2 T^4$.

$T^4 = 4.68 \times 10^{13}$

T = temperature of sphere = 2600 K

30. (c) : $Q = \frac{KA\,\Delta T(t)}{d}$

$$= \frac{0.0005(150 \times 80)(30° - 0°)(60)}{6} = 1800\ \text{cal}$$

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