PHYSICS
For Joint Entrance Examination (JEE)
Electrostatics and Current Electricity

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Preface

While the paper-setting pattern and assessment methodology have been revised many times over and newer criteria devised to help develop more aspirant-friendly engineering entrance tests, the need to standardize the selection processes and their outcomes at the national level has always been felt. A combined national-level engineering entrance examination has finally been proposed by the Ministry of Human Resource Development, Government of India. The Joint Entrance Examination (JEE) to India’s prestigious engineering institutions (IITs, IIITs, NITs, ISM, IISERs, and other engineering colleges) aims to serve as a common national-level engineering entrance test, thereby eliminating the need for aspiring engineers to sit through multiple entrance tests.

While the methodology and scope of an engineering entrance test are prone to change, there are two basic objectives that any test needs to serve:

1. The objective to test an aspirant's caliber, aptitude, and attitude for the engineering field and profession.
2. The need to test an aspirant's grasp and understanding of the concepts of the subjects of study and their applicability at the grassroots level.

Students appearing for various engineering entrance examinations cannot bank solely on conventional shortcut measures to crack the entrance examination. Conventional techniques alone are not enough as most of the questions asked in the examination are based on concepts rather than just formulae. Hence, it is necessary for students appearing for joint entrance examination to not only gain a thorough knowledge and understanding of the concepts but also develop problem-solving skills to be able to relate their understanding of the subject to real-life applications based on these concepts.

This series of books is designed to help students to get an all-round grasp of the subject so as to be able to make its useful application in all its contexts. It uses a right mix of fundamental principles and concepts, illustrations which highlight the application of these concepts, and exercises for practice. The objective of each book in this series is to help students develop their problem-solving skills/accuracy, the ability to reach the crux of the matter, and the speed to get answers in limited time. These books feature all types of problems asked in the examination—be it MCQs (one or more than one correct), assertion-reason type, matching column type, comprehension type, or integer type questions. These problems have skilfully been set to help students develop a sound problem-solving methodology.

Not discounting the need for skilled and guided practice, the material in the books has been enriched with a number of fully solved concept application exercises so that every step in learning is ensured for the understanding and application of the subject. This whole series of books adopts a multi-faceted approach to mastering concepts by including a variety of exercises asked in the examination. A mix of questions helps stimulate and strengthen multi-dimensional problem-solving skills in an aspirant.

It is imperative to note that this book would be as profound and useful as you want it to be. Therefore, in order to get maximum benefit from this book, we recommend the following study plan for each chapter.

Step 1: Go through the entire opening discussion about the fundamentals and concepts.

Step 2: After learning the theory/concept, follow the illustrative examples to get an understanding of the theory/concept.

Overall the whole content of the book is an amalgamation of the theme of physics with ahead-of-time problems, which equips the students with the knowledge of the field and paves a confident path for them to accomplish success in the JEE.

With best wishes!

B.M. Sharma
# Chapter 1

Coulomb's Laws and Electric Field

- Electric Charge
- Charging of a Body
- Work Function of a Body
- Properties of Electric Charge
- Coulomb's Law
- Coulomb's Law in Vector Form
- Electric Field
- Different Patterns of Electric Field Lines

- Field of Ring Charge
- Field of Uniformly Charged Disk
- Field of Two Oppositely Charged Sheets
- Electric Dipole
- Electric Field Due to a Dipole
- Electric Field Intensity Due to a Short Dipole at Some General Point
- Dipole in a Uniform Electric Field
ELECTRIC CHARGE

Electric charge, like mass, is one of the fundamental attributes of the particle of which the matter is made of. Charge is the physical property of certain fundamental particles (such as, electron, proton) by virtue of which they interact with the other similar fundamental particles.

- Charge is an intrinsic property of some fundamental particles which accompanies these particles wherever they exist.
- Charge is that property of a body/particle which is responsible for 'electrical force' between them.

To distinguish the nature of interaction, charges are divided into two parts: (i) positive (ii) negative

Figure 1.1 shows an experiment to demonstrate that there are two types of charges.

We know that matter consists of atoms. An atom consists of a central core (called nucleus) and electrons. Electrons orbit around the nucleus. Nucleus consists of neutrons and protons. Neutrons do not contain any net charge. Protons and electrons have equal charges, but of opposite nature. Protons are positively charged, whereas electrons are negatively charged. Protons, however, are very heavy when compared with electrons, about 1836 times. Protons are imprisoned in the nucleus along with neutrons due to the strongest binding force existing in nature called 'strong or nuclear force'. Thus, protons do not travel from atom to atom. The outermost electrons may travel from atom to atom. Hence, we say that electrons are the basis of electricity.

Charge on a proton or on an electron is of indivisible nature. We designate this charge as +e and −e, respectively. Hence, charge in or on any object is always an integral multiple of the electronic charge.

In a normal atom:

1. Number of protons are equal to number of electrons.
2. Protons have the basic +e charge and electrons have the basic −e charge.
3. Hence, a normal atom is electrically neutral.

Electrons can travel from one atom to another and from one body to another. If a body loses one electron, it becomes positively charged with +e charge and vice versa.

A body, however, cannot lose or gain any proton, which is heavy and remains imprisoned in the nucleus, by ordinary methods.

Note: Basic unit of charge = e, whose magnitude is equal to the magnitude of charge on an electron or proton, i.e., e = 1.6 × 10⁻¹⁹ C.

S.I. unit of charge: As mentioned, e = 1.6 × 10⁻¹⁹ C. Here, e stands for one electronic charge which is the basic unit of charge; C stands for coulomb (note the small c in coulomb). coulomb is the S.I. unit of charge.

CHARGING OF A BODY

Ordinarily, matter contains equal number of protons and electrons.

A body can be charged by the transfer of electrons or redistribution of electrons.

A body can be charged by the transfer of electrons and by the transfer of protons. Why?

It is because protons are inside the nucleus and it is very difficult to remove them from there. Electrons lie in the outer shells and it is easier to remove them.

![Image of experiments with glass rods and ebonite rods](https://via.placeholder.com/150)

**Fig. 1.1**

**Nature of charges acquired by both glass rods (or by both ebonite rods) should be the same.**

**Conclusion:** Like charges repel each other

**Nature of charges acquired by both glass rods (or by both ebonite rods) should be different from nature of charge acquired by glass rod. Hence, there are two kinds of charges.**

**Conclusion:** Unlike charges attract each other.

Final conclusion: Like charges repel each other and unlike charges attract each other. Later on, it was found that glass rod had positive charge and ebonite rod had negative charge. There is no third kind of charge.
To charge a body negatively: some electrons are given to it. To charge a body positively: some electrons are taken from it.

**WORK FUNCTION OF A BODY**

*It is the amount of work to be done on a body in order to remove an electron from its surface.* Obviously, it is easier to remove an electron from a body whose work function is lower.

Let us see how bodies get charged due to friction:

As shown in Fig. 1.2, let \( W_2 > W_1 \). Now, suppose \( A \) and \( B \) are rubbed together.

Net transfer of electrons will take place from \( A \) to \( B \).

It is because electrons in \( A \) are loosely bound as work function of \( A \) is less than \( B \).

It is to be noted that mass is also affected during charging.

(Mass of negatively charged body increases and that of positively charged body decreases.)

![Diagram of two bodies with electrons transfer](image)

**Fig. 1.2**

Basically charging can be done by three methods:

1. Friction, 2. Conduction, and 3. Induction

**Charging by Friction**

When two bodies are rubbed together, electrons are transferred from one body to the other making one body positively charged and the other negatively charged.

**Example:** When a glass rod is rubbed with silk, the rod becomes positively charged, whereas silk gets negatively charged. However, ebonite on rubbing with wool becomes negatively charged making the wool positively charged.

**Charging by Conduction**

The process of charging from an already charged body can happen by either conduction or induction. Conduction from a charged body involves transfer of like charges. A positively charged body can cause more bodies to get positively charged, but the sum of the total charge on all positively charged bodies will be the same as charge on initially considered charged body.

**Charging by Induction**

Induction is a process by which a charged body can be used to charge neutral bodies without touching them or losing its own charge. If a charged body is brought near a neutral body, the charged body attracts opposite charge and repels similar charge present on the neutral body. If the neutral body is now earthed, the like charge is neutralized by the flow of charge from earth, leaving unlike charge on the body. Now, the earthing and the charging body is removed leaving the initially neutral body charged. The whole process is as shown in Fig. 1.3.

**PROPERTIES OF ELECTRIC CHARGE**

**Quantization of Charge**

Charge exists in discrete packets rather than in continuous amount, i.e., charge on any body is the integral multiple of the charge on an electron or proton.

\[ Q = \pm n e, \text{ where } n = 0, 1, 2, \ldots \]

**Conservation of Charge**

Charge is conserved, i.e., total charge on an isolated system is constant. By isolated system, we here mean a system through the boundary of which no charge is allowed to escape or enter. This

![Diagram of charging process](image)

**Fig. 1.3**
Additivity of Charge

Total charge on a body is the algebraic sum of all the charges located anywhere on the body. While adding the charges, their sign must be taken into consideration.

For example, if a body has charges 2 C, -5 C, 4 C and 6 C (Fig. 1.4), then the total charge on the body is $2 - 5 + 4 + 6 = 7$ C.

Note that charges are added like real numbers. They have no direction. So, charge is a scalar quantity.

![Fig. 1.4](image)

Charge is Invariant

Charge does not depend on the speed of the body.

Points to Remember

- There are two types of forces which act between two charges. If the charges are stationary, there is only one type of force between them. It is called electric or electrostatic force. It is given by Coulomb's law for point charges. If the charges are moving, then two types of forces act between them. The first one is electric force. The other force which emerges due to motion is called magnetic force. We shall study magnetic force in the following chapter.

- Charge produces electric and magnetic fields and radiates energy: A stationary charged particle produces only electric field in the space surrounding it. A charged particle moving without acceleration produces electric as well as magnetic field. A charged particle in accelerated motion radiates energy as well, in the form of electromagnetic waves.

Illustration 1.1 A glass rod is rubbed with a silk cloth. The glass rod acquires a charge of $+19.2 \times 10^{-19}$C.

1. Find the number of electrons lost by glass rod.
2. Find the negative charge acquired by silk.
3. Is there transfer of mass from glass to silk?

Given, $m_r = 9 \times 10^{-31}$ kg.

Sol.

1. Number of electrons lost by glass rod is
   $$n = \frac{q}{e} = \frac{19.2 \times 10^{-19}}{1.6 \times 10^{-19}} = 12$$

2. Charge on silk = $-19.2 \times 10^{-19}$C

3. Since an electron has a finite mass ($m_e = 9 \times 10^{-31}$ kg), there will be transfer of mass from glass rod to silk cloth. Mass transferred = $12 \times (9 \times 10^{-31}) = 1.08 \times 10^{-29}$ kg

Note that mass transferred is negligibly small. This is expected because the mass of an electron is extremely small.

![Illustration 1.2](image)

Electric charges $A$ and $B$ attract each other.

Electric charges $B$ and $C$ repel each other. If $A$ and $C$ are held close together, they will

a. attract  
   b. repel.  
   c. not affect each other  
   d. more information is needed to answer.

Sol. a.

<table>
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<th>Case 2</th>
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<td>If $A$ and $B$ attract each other, then</td>
<td>If $A$ and $B$ repel each other, then</td>
</tr>
<tr>
<td>$+$ and $B$</td>
<td>$A$ and $+$</td>
</tr>
<tr>
<td>$-$ and $C$</td>
<td>$B$ and $+$</td>
</tr>
<tr>
<td>$B$ and $C$</td>
<td>$B$ and $+$</td>
</tr>
</tbody>
</table>

From both cases, we see that $A$ and $C$ will be having unlike charges. Hence, if the charges $A$ and $C$ are held together, they will attract each other.

![Illustration 1.3](image)

If an object made of substance $A$ is rubbed with an object made of substance $B$, then $A$ becomes positively charged and $B$ becomes negatively charged. If, however, an object made of substance $A$ is rubbed against an object made of substance $C$, then $A$ becomes negatively charged. What will happen if an object made of substance $B$ is rubbed against an object made of substance $C$?

a. $B$ becomes positively charged and $C$ becomes positively charged.

b. $B$ becomes positively charged and $C$ becomes negatively charged.

c. $B$ becomes negatively charged and $C$ becomes positively charged.

d. $B$ becomes negatively charged and $C$ becomes negatively charged.

Sol. c. When $A$ and $B$ are rubbed, $A$ becomes positively charged and $B$ becomes negatively charged. It means Electrons are loosely bound with $A$ in comparison to $B$. When $A$ and $C$ are rubbed together, $A$ becomes negatively charged and $C$ positively charged. It means Electrons are loosely bound with $C$ in comparison to $A$. Hence, in $C$, electrons are most loosely bound. So, if $B$ and $C$ are rubbed together, $C$ will lose electrons and $B$ will receive electrons. Hence, $C$ will become positively charged and $B$ will become negatively charged.

Illustration 1.4 Objects $A$, $B$ and $C$ are three identical, insulated, spherical conductors. Originally, $A$ and $B$ have
charges of +3 mC, whereas C has a charge of −6 mC. Objects A and C are allowed to touch, then they are moved apart. Objects B and C are allowed to touch before they are moved apart.

i. If objects A and B are now held near each other, they will
   a. attract  b. repel  c. have no effect on each other.

ii. If instead objects A and C are held near each other, they will
   a. attract  b. repel  c. have no effect on each other.

Sol.

Initially,

\[ A \] +3 mC \hspace{1cm} B \] +3 mC \hspace{1cm} C \] −6 mC

- When the objects A and C are allowed to touch and then moved apart:

\[ \begin{array}{c}
A \\
\leftrightarrow \\
C
\end{array} \]

\[ \left[+3 \text{ mC} + (-6 \text{ mC}) = -3 \text{ mC}\right] \]

\[ \frac{3}{2} \text{ mC} \hspace{1cm} -\frac{3}{2} \text{ mC} \]

- When the objects B and C are allowed to touch and then moved apart:

\[ \begin{array}{c}
B \\
\leftrightarrow \\
C
\end{array} \]

\[ \left[+3 \text{ mC} + \left(-\frac{3}{2} \text{ mC}\right) = +\frac{3}{2} \text{ mC}\right] \]

\[ +\frac{3}{4} \text{ mC} \hspace{1cm} +\frac{3}{4} \text{ mC} \]

i.a. Hence, if A and B are now held near each other, they will attract each other.

ii.a. If A and C are now held near each other, they will also attract each other.

Illustration 1.5 Figure 1.5 shows that a positively charged rod is brought near two uncharged metal spheres A and B clamped on insulated stands and placed in contact with each other.

i. What would happen if the rod is removed before the spheres are separated?

ii. Would the induced charges be equal in magnitude even if the spheres had different sizes or different conductors?

iii. What will happen if the spheres are separated first and then the rod is removed far away?

Sol.

i. When a positively charged rod is brought near A, the free electrons in the sphere A are attracted towards the rod and moved on the left side of A. This movement leaves an unbalanced positive charge on B. If the rod is removed before the spheres are separated, the excess electrons on sphere A would flow back to B. Both the spheres will become uncharged.

ii. Yes, net charge is conserved. Before the rod is brought near A, both A and B were neutral. They will remain so even if they have different sizes or materials.

iii. If the rod is removed after the spheres are separated, then sphere A will have net negative charge and sphere B will have net positive charge of same magnitude as shown in Fig. 1.6.

Concept Application Exercise 1.1

1. a. How many electrons are in 1 coulomb of negative charge?
   b. Which is the true test of electrification, attraction or repulsion?
   c. Can a body have charge of $0.8 \times 10^{-19}$ C?

2. If only one charge is available, can itself be used to obtain a charge many times greater than itself in magnitude?

3. a. Can two bodies having like charges attract each other? (Yes/No)
   b. Can a charged body attract an uncharged body? (Yes/No)
   c. Two identical metallic spheres of exactly equal masses are taken, one is given a positive charge $q$ and the other an equal negative charge. Their masses after charging are different. Comment on the statement.

4. A particle has charge of $+10^{-12}$ C.
   a. Does it contain more or less number of electrons as compared with the neutral state?
   b. Calculate the number of electrons transferred to provide this charge.

5. An ebonite rod is rubbed with fur. The ebonite rod is found to have a charge of $-3.2 \times 10^{-4}$ C on it.
   a. Calculate the number of electrons transferred.
   b. What is the charge on fur after rubbing?

6. The electric charge of macroscopic bodies is actually a surplus or deficiency of electrons. Why not protons?

7. A charged rod attracts bits of dry paper which after touching the rod often jump away from it violently. Explain.

8. A person standing on an insulating stool touches a charged insulated conductor. Will the conductor get completely discharged?

9. An electron moves along a metal tube with variable cross section. How will its velocity change when it approaches the neck of the tube (Fig. 1.7)?
10. Define the following statement "If there were only one electrically charged particle in the entire universe, the concept of electric charge would be meaningless".

**COULOMB’S LAW**

The force of interaction between two point charges is proportional to the product of magnitudes of the two charges and inversely proportional to the square of distance between them.

Let two point electric charges \( q_1 \) and \( q_2 \) be at rest, separated by a distance \( r \), then they exert a force on each other which is given by

\[
F = k \frac{q_1 q_2}{r^2}
\]

where \( k \) is a proportionality constant known as electrostatic force constant.

If there is free space (or vacuum) between the two charges, then \( k = \frac{1}{4\pi \varepsilon_0} = 9 \times 10^9 \text{Nm}^2 \text{C}^{-2} \) (in SI units).

where \( \varepsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12} \text{C}^2 \text{N}^{-1} \text{m}^{-2} \) is the absolute electric permittivity of the free space.

So, force between two charges is given as

\[
F = \frac{1}{4\pi \varepsilon_0} \frac{q_1 q_2}{r^2}
\]

Equation (i) is applicable only for point charges placed in vacuum. Now, what happens if the two charges are placed in some medium? In a medium, the force is given as

\[
F' = k' \frac{q_1 q_2}{r^2}
\]

where \( k' = \frac{1}{4\pi \varepsilon} \) and in this \( \varepsilon \) is known as absolute electrical permittivity of medium. Then,

\[
F' = \frac{1}{4\pi \varepsilon} \frac{q_1 q_2}{r^2}
\]

The ratio \( \varepsilon / \varepsilon_0 = \varepsilon_s \) is known as relative electrical permittivity of medium. It is also known as dielectric constant and denoted by \( K \).

So,

\[\varepsilon / \varepsilon_0 = \varepsilon_s = K\]

The value of \( K \) for different materials: vacuum = 1, air = 1.006, glass = 3 to 4, water = 81, conductor = \( \infty \).

In general, \( K \geq 1 \).

Now, from Eqs. (i) and (iii),

\[
\frac{F'}{F} = \frac{\varepsilon_s}{\varepsilon} = \frac{1}{K} \Rightarrow \frac{F'}{F} = \frac{1}{K}
\]

It means when the charges are placed in a medium, the force decreases \( K \) times. Also, \( F = F' / K \).

So, the dielectric constant of a medium may be defined as the ratio of force between two charges when they are placed in vacuum to that when they are placed in that medium at same separation.

Note:

- Coulomb’s law is not valid for distances < 10^{-11} m.
- Electrostatic forces are comparatively stronger than gravitational forces. Can you show this?

(As an example—when we hold a book in our hand, electric force between hand and the book is sufficient to balance the gravitational force of earth on the book due to entire earth.)

**Some Important Points**

- Coulomb’s law is applicable only for point charges.
- Coulomb’s law is similar to Newton’s gravitational law and both obey inverse square law.
- Coulomb’s law obeys Newton’s third law, i.e., the forces exerted by the two charges on each other are equal and opposite.
- This force acts along the line joining the two particles (called central force).
- Electrostatic force is a conservative force.

**COULOMB’S LAW IN VECTOR FORM**

Let \( q_1 \) and \( q_2 \) be two like charges placed at points \( A \) and \( B \), respectively, in vacuum.

\[
\vec{F}_{12} = \frac{\varepsilon_0}{4\pi r} \frac{q_1 q_2}{r^2} \hat{r}
\]

\( \vec{r} \) is the position vector of point \( A \) and \( \vec{r}_2 \) is the position vector of point \( B \).

Let \( \vec{r} \) be vector from \( A \) to \( B \), then

\[
\vec{r} = \vec{r}_2 - \vec{r}_1 \quad \text{and} \quad r = \| \vec{r}_2 - \vec{r}_1 \|.
\]

\[
\Rightarrow \quad \vec{F} = \frac{\varepsilon_0}{4\pi r} \left( \frac{q_1 q_2}{r^2} \right) \left( \frac{\vec{r}_2 - \vec{r}_1}{r} \right)
\]

Let \( \vec{F}_{21} \) be the force on charge \( q_2 \) due to \( q_1 \); and \( \vec{F}_{12} \) be the force on charge \( q_1 \) due to \( q_2 \).
From Fig. 1.9, it is clear that \( \vec{F}_{21} \) and \( \vec{r} \) are in the same direction, so

\[
\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2 \vec{r}}{r^2} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2 \vec{r}}{r^2} = \frac{q_1 q_2 \vec{r}}{4\pi\varepsilon_0 r^2}
\]

\[
\Rightarrow \quad \vec{F}_{21} = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{\vec{r} - \vec{r}_1}{|\vec{r} - \vec{r}_1|^3}
\]

The above equations give the Coulomb’s law in vector form.

As we know that charges equal and opposite forces on each other, so we have

\[
\vec{F}_{12} = -\vec{F}_{21} \Rightarrow \vec{F}_{12} = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{\vec{r} - \vec{r}_2}{|\vec{r} - \vec{r}_2|^3}
\]

**Superposition Principle**

It enables us to calculate the force acting on a charge due to more than one charge.

According to superposition principle, the total force on a given charge is vector sum of all the individual forces exerted by each of the other charges:

\[
\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots + \vec{F}_n
\]

Another important point is that the force between two charges remains unaffected due to the presence of a third charge.

![Fig. 1.10](image)

Note:
- Coulomb’s law and principle of superposition together can explain whole of the electrostatics.
- Both Coulomb’s law and Gravitational law describe inverse square law that involve a property of interacting particles—the charge in one case and mass in the other case.

**Illustration 1.7** Two identical He-filled spherical balloons each carrying a charge \( q \) are tied to a weight \( W \) with strings and float in equilibrium as shown in Fig. 1.12(a).

i. the magnitude of \( q \), assuming that the charge on each balloon acts as if it were concentrated at the centre.

ii. the volume of each balloon.

Take density of He as \( \rho_h \) and density of air as \( \rho_a \). Ignore the weight of the unfilled balloons.

![Fig. 1.12](image)

**Illustration 1.8** Two particles, each having a mass of 5 g and charge \( 10^{-7} \) C, stay in limiting equilibrium on a horizontal table with a separation of 10 cm between them. Find the coefficient of friction between each particle and the table, which is the same between each particle and table.

\[
\begin{align*}
\text{Sol.} & \quad 2T \cos \theta = W, T \sin \theta = F \\
\Rightarrow & \quad \frac{\tan \theta}{W} = \frac{F}{W} \Rightarrow F = W \tan \theta \\
\Rightarrow & \quad \frac{\tan \theta}{2} = \frac{W \tan \theta}{2} \Rightarrow q = \sqrt{8W \tan \theta \pi \varepsilon_0 x^2} \\
\text{ii.} & \quad T \cos \theta + mg = B \Rightarrow \frac{W}{2} + V \rho_{\text{air}} g = V \rho_a g \\
\Rightarrow & \quad V = \frac{W}{2(\rho_a - \rho_{\text{air}})g}
\end{align*}
\]
Sol. Friction force $f$ will balance the electrostatic repulsion, i.e.,

$$f = F = \mu F = \frac{q^2}{4\pi\varepsilon_0 r^2}$$

Fig. 1.13

$$\Rightarrow \mu \times \frac{5}{1000} \times 10 = \frac{9 \times 10^9 \times (10^{-7})^2}{(0.10)^2} \Rightarrow \mu = 0.18$$

Illustration 1.9. A particle of mass $m$ carrying a charge $-q_1$ starts moving around a fixed charge $+q_2$ along a circular path of radius $r$. Prove that period of revolution $T$ of charge $-q_1$ is given by $T = \sqrt{\frac{16\pi^2 \varepsilon_0 mr^3}{q_1 q_2}}$.

Sol. Electrostatic force on $-q_1$ due to $+q_2$ will provide the necessary centripetal force, hence

$$kq_1 q_2 = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{kq_1 q_2}{mr}}$$

Now,

$$T = \frac{2\pi r}{v} = \sqrt{\frac{16\pi^2 \varepsilon_0 mr^3}{q_1 q_2}}$$

Fig. 1.14

Illustration 1.10. Consider three charges $q_1$, $q_2$, and $q_3$, each equal to $q$, at the vertices of an equilateral triangle of side $a$. What is the force on a charge $Q$ placed at the centroid of the triangle?

Sol. Method 1. The resultant of three equal coplanar vectors acting at a point will be zero if these vectors form a closed polygon (Fig. 1.15). Hence, the vector sum of the forces $\vec{F}_1$, $\vec{F}_2$, and $\vec{F}_3$ is zero.

Method 2. The forces acting on the charge $Q$ are

$$\vec{F}_1 = \text{force on } Q \text{ due to } q_1 = \frac{1}{4\pi\varepsilon_0} \frac{Qq}{AO}$$

$$\vec{F}_2 = \text{force on } Q \text{ due to } q_2 = \frac{1}{4\pi\varepsilon_0} \frac{Qq}{BO}$$

Fig. 1.15

$$\vec{F}_3 = \text{force on } Q \text{ due to } q_3 = \frac{1}{4\pi\varepsilon_0} \frac{Qq}{CO}$$

The resultant force is $\vec{F}_r = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$

$$= \frac{Qq}{4\pi\varepsilon_0} \frac{1}{AO} (\vec{AO} + \vec{BO} + \vec{CO}) = 0$$

(As $|q_1| = |q_2| = |q_3|$ and $|AO| = |BO| = |CO|$)

Also, $\vec{AO} + \vec{BO} + \vec{CO} = 0$ because these are three equal vectors in a plane making angles of $120^\circ$ with each other.

Method 3. The resultant force $\sum \vec{F}$ is the vector sum of individual forces, i.e.,

$$\sum \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$\sum F_x = F_{1x} + F_{2x} + F_{3x} = 0 + F_1 \cos 30^\circ + F_2 \cos 30^\circ$$

(i)

Fig. 1.16

And $\sum F_x = F_{1x} + F_{2x} + F_{3x} = F_1 \cos 30^\circ + F_2 \sin 30^\circ + F_3 \sin 30^\circ$

(ii)

As $|F_1| = |F_2| = |F_3| = |F|$ (say), Eqs. (i) and (ii) become

$$\sum F_x = 0 \text{ and } \sum F_y = 0.$$ 

Hence, resultant force $\sum \vec{F} = 0$. 

Fig. 1.17
Illustration 1.11

Point charges are placed at the vertices of a square of side $a$ as shown in Fig. 1.18. What should be the sign of the charge $q$ and magnitude of the ratio $|q/Q|$ so that

i. net force on each $Q$ is zero?
ii. net force on each $q$ is zero?
iii. Is it possible that the entire system could be in electrostatic equilibrium?

![Fig. 1.18](image)

Sol.

i. Consider the forces acting on charge $Q$ placed at $A$ (shown in Fig. 1.19(a) and (b))

Case I. Let the charges $q$ and $Q$ are of same sign.

![Fig. 1.19](image)

Here, $F_1 = \frac{1}{4\pi\varepsilon_0} \frac{qQ}{a^2}$

Here, $F_2 = \frac{1}{4\pi\varepsilon_0} \frac{qQ}{2a^2}$

In Fig. 1.19(a), resultant of forces $\vec{F}_1$ and $\vec{F}_2$ will lie along $\vec{F}_1$, so that net force on $Q$ cannot be zero. Hence, $q$ and $Q$ have to be of opposite signs.

Case II. Let the charges $q$ and $Q$ are of opposite sign.

In this case, as shown in Fig. 1.19(b), resultant of $\vec{F}_1$ and $\vec{F}_2$ will be opposite to $\vec{F}_1$, so that it becomes possible to obtain a condition of zero net force.

Let us write $\vec{F}_q = \vec{F}_1 + \vec{F}_2$

\[ \sqrt{F_1^2 + F_2^2} = \frac{1}{4\pi\varepsilon_0} \frac{qQ}{a^2} \sqrt{2} \]

Direction of $\vec{F}_q$ will be along $AC$ ($\vec{F}_q$, being resultant of forces of equal magnitude, bisects the angle between the two) $\vec{F}_1$ and $\vec{F}_2$ are in opposite directions. Net force on $Q$ can be zero if their magnitudes are also equal, i.e.,

\[ \frac{1}{4\pi\varepsilon_0} \frac{qQ}{a^2} \sqrt{2} = \frac{1}{4\pi\varepsilon_0} \frac{qQ}{2a^2} \]

\[ \Rightarrow q = \frac{Q}{2\sqrt{2}} \Rightarrow \left| \frac{q}{Q} \right| = \frac{1}{2\sqrt{2}} \quad (Q \neq 0) \]

Therefore, the sign of $q$ should be negative of $Q$.

ii. Consider now the forces acting on charge $q$ placed at $B$ (see Fig. 1.20(a) and (b)).

In a similar manner, as discussed in (i), for net force on $q$ to be zero, $q$ and $Q$ have to be of opposite signs. This is also shown in the given figures.

Now, $F_1 = \frac{1}{4\pi\varepsilon_0} \frac{Qq}{a^2}$

$F_2 = \frac{1}{4\pi\varepsilon_0} \frac{Qq}{a^2}$

$F_3 = \frac{1}{4\pi\varepsilon_0} \frac{q^2}{2a^2}$

Referring to Fig. 1.20(b), let us write $\vec{F}_q = \vec{F}_3 + \vec{F}_2$

\[ \Rightarrow F_q = \sqrt{F_3^2 + F_2^2} = \frac{1}{4\pi\varepsilon_0} \frac{Qq}{a^2} \sqrt{2} \]

Resultant of $\vec{F}_1$ and $\vec{F}_2$, i.e., $\vec{F}_q$, is opposite to $\vec{F}_1$. Net force can become zero if their magnitudes are also equal, i.e.,

\[ \frac{1}{4\pi\varepsilon_0} \frac{Qq}{\sqrt{2}} = \frac{1}{4\pi\varepsilon_0} \frac{q^2}{2a^2} \Rightarrow q = \frac{Q}{2\sqrt{2}} \Rightarrow \left| \frac{q}{Q} \right| = 2\sqrt{2} \quad (Q \neq 0) \]

Therefore, the sign of $q$ should be negative of $Q$.

In this case, we need not to repeat the calculation as the present situation is same as the previous one; we can directly write $|q/Q| = 2\sqrt{2}$. 

Coulomb's Laws and Electric Field 1.9
iii. The entire system cannot be in equilibrium since both conditions, i.e., \( q = -\frac{Q}{2\sqrt{2}} \) and \( Q = -\frac{q}{2\sqrt{2}} \) cannot be satisfied together.

Illustration 1.2 Two identical small charged spheres, each having a mass \( m \), hang in equilibrium as shown in Fig. 1.21(a). The length of each string is \( l \) and the angle made by any string, with vertical is \( \theta \). Find the magnitude of the charge on each sphere.

Sol. The forces acting on the sphere are tension in the string \( T \); force of gravity, \( mg \); repulsive electric force, \( F_e \) as shown in the free-body diagram of the sphere (Fig. 1.21(b)). The sphere is in equilibrium. The forces in the horizontal and vertical directions must separately add up to zero.

\[
\sum F_x = T \sin \theta - F_e = 0 \tag{i}
\]
\[
\sum F_y = T \cos \theta - mg = 0 \tag{ii}
\]

Fig. 1.21

From Eq. (ii), \( T = \frac{mg}{\cos \theta} \). Thus, we can eliminate \( T \) from Eq. (i) to obtain

\[
F_e = mg \tan \theta \text{ or } \frac{kq^2}{r^2} = mg \tan \theta \tag{iii}
\]

where \( k = \frac{1}{4\pi\epsilon_0} \) and \( r = 2l \sin \theta \).

Equation (iii) now reduces to

\[
\frac{1}{4\pi\epsilon_0} \frac{q^2}{2l \sin^2 \theta} = mg \tan \theta
\]

or

\[
q = \sqrt{16\pi\epsilon_0 l^2 mg \tan \theta \sin^2 \theta}
\]

Illustration 1.3 Two identical balls each having a density \( \rho \) are suspended from a common point by two insulating strings of equal length. Both the balls have equal mass and charge. In equilibrium, each string makes an angle \( \theta \) with the vertical. Now, both the balls are immersed in a liquid. As a result, the angle \( \theta \) does not change. The density of the liquid is \( \sigma \). Find the dielectric constant of the liquid.

Sol. Let \( V \) be the volume of each ball, then mass of each ball is

\[
m = \rho V
\]

When the balls are in air, from the previous problem,

\[
F = mg \tan \theta = \rho V g \tan \theta
\]

Fig. 1.22

When the balls are suspended in liquid, the Coulombic force is reduced to \( F' = F/K \) and apparent weight \( = \text{weight} - \text{upthrust} \), i.e.,

\[
W' = (\rho V g - \sigma V g)
\]

According to the problem, angle \( \theta \) is unchanged. Therefore,

\[
F' = W' \tan \theta = (\rho V g - \sigma V g) \tan \theta \tag{ii}
\]

From Eqs. (i) and (ii), we get

\[
F = \frac{F'}{F'} = K = \frac{\rho V g}{\rho V g - \sigma V g} = \frac{\rho}{\rho - \sigma}
\]

Illustration 1.4 Three particles, each of mass \( \rho m \) and carrying a charge \( q \) each, are suspended from a common point by insulating massless strings, each of length \( L \). If the particles are in equilibrium and are located at the corners of an equilateral triangle of side \( 'a' \), calculate the charge \( q \) on each particle. Assume \( L >> a \).

Sol. From Fig. 1.23(a), for equilibrium of a particle along a vertical line, we get

\[
T \cos \theta = mg \tag{i}
\]

While for equilibrium in the plane of equilateral triangle, we get

\[
T \sin \theta = 2F \cos 30^0 \tag{ii}
\]

So, from Eqs. (i) and (ii), we have

\[
\tan \theta = \frac{\sqrt{3}F}{mg} \tag{iii}
\]

Fig. 1.23

Here, \( F = \frac{1}{4\pi\epsilon_0} \frac{q^2}{d^2} \) and \( \tan \theta = \frac{OA}{OP} = \frac{OA}{\sqrt{L^2 - OA^2}} \)

Also, from Fig. 1.23(c), we get

\[
OA = \frac{2}{3} AD = \frac{2}{3} a \sin 60^0 = \frac{a}{\sqrt{3}}
\]
So, \[ \tan \theta = \frac{(a/\sqrt{3})}{L (a^2/3)} = \frac{a}{L} \quad \text{(as } L \gg a) \]

On substituting the above values of \( F \) and \( \tan \theta \) in Eq. (iii), we get:

\[ \frac{a}{(a^2)} = \frac{3q^2}{mg 4\pi \varepsilon_0 a^3}, \text{i.e., } q = \left[ \frac{4\pi \varepsilon_0 a' mg}{3L} \right]^{1/2} \]

Illustration 1.15 A thin fixed ring of radius ‘a’ has a positive charge ‘q’ uniformly distributed over it. A particle of mass ‘m’, having a negative charge ‘Q’, is placed on the axis at a distance of \( x (x \ll a) \) from the centre of the ring. Show that the motion of the negatively charged particle is approximately simple harmonic. Calculate the time period of oscillation.

![Fig. 1.24](image)

Sol. The force on the point charge \( Q \) due to the element \( dq \) of the ring is

\[ dF = \frac{1}{4\pi \varepsilon_0} \frac{dqQ}{r^2} \text{ along } AB \]

For every element of the ring, there is symmetrically situated diametrically opposite element, the components of forces along the axis will add up while those perpendicular to it will cancel each other. Hence, net force on the charge \(-Q\) is zero which shows that this force will be towards the centre of ring.

\[ F = \int dF \cos \theta = \cos \theta \int dF \]

\[ = \frac{x}{r} \int \frac{Q dq}{r^2} \]

So,

\[ F = \frac{Qx}{4\pi \varepsilon_0 r^3} \int dq = \frac{1}{4\pi \varepsilon_0} \frac{Qq}{(a^2 + x^2)^{3/2}} \quad \text{(i)} \]

(as \( r = (a^2 + x^2)^{1/2} \) and \( \int dq = q \))

As the restoring force is not linear, the motion will be oscillatory. However, if \( x \ll a \) so that \( x^2 \ll a^2 \), then

\[ F = -\frac{1}{4\pi \varepsilon_0} \frac{Qq}{a^2} x = -kx \quad \text{with } k = \frac{Qq}{4\pi \varepsilon_0 a^2} \]

i.e., the restoring force will become linear and so the motion is simple harmonic with time period

\[ T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{4\pi \varepsilon_0 ma^2}{qQ}} \]

Concept Application Exercise 1.2

1. A negatively charged particle is placed exactly midway between two fixed particles having equal positive charges. What will happen to the charge

   i. if it is displaced at right angle to the line joining the positive charges?
   
   ii. if it is displaced along the line joining the positive charges?

2. Does an electric charge experience a force due to the field produced by itself? (Yes/No)

3. Two negative charges of a unit magnitude and a positive charge ‘\( q \)’ are placed along a straight line. At what position and value of \( q \) will the system be in equilibrium? (Negative charges are fixed.)

4. Figure 1.25 shows three arrangements of an electron \( e \) and two protons \( p \) (where \( O > d \)).

   a. Rank the arrangements according to the magnitude of the net electrostatic force on the electron due to the protons, largest first.

   ![Fig. 1.25](image)

   b. In situation (c), is the angle between the net force on the electron and the line labeled horizontal less than or more than 45°?

5. Figure 1.26 shows two charged particles on an axis. The charges are free to move. At one point, however, a third charged particle can be placed such that all three particles are in equilibrium.

   ![Fig. 1.26](image)

   a. Is that point to the left of the first two particles, to their right, or between them?
   
   b. Should the third particle be positively or negatively charged?
   
   c. Is the equilibrium stable or unstable?

6. In Fig. 1.27, a central particle of charge \(-q\) is surrounded by two circular rings of charged particles of radii \( r \) and \( R \), with \( R > r \). What is the magnitude and direction of the net electrostatic force on the central particle due to the other particles?
7. Figure 1.28 shows four situations in which particles of charge +q or −q are fixed in place. In each, the particles on the x-axis are equidistant from the y-axis. The particle on the y-axis experiences an electrostatics force F from each of these two particles.
   a. Are the magnitudes F of those forces the same or different?
   b. Is the magnitude of the net force on the particle on the y-axis equal to, greater than or less than 2F?
   c. Do the x components of the two forces add or cancel?
   d. Do the y components of the forces add or cancel?
   e. Is the direction of the net force on the middle particle that of the canceling components or the adding components?
   f. What is the direction of the net force on the middle particle?

8. Force between two point electric charges kept at a distance \(d\) apart in air is \(F\). If these charges are kept at the same distance in water, the force between the charges is \(F'\). The ratio \(F'/F\) is equal to _______.

9. Two small balls each having charge \(q\) are suspended by two insulating threads of equal length \(L\) from a hook in an elevator. The elevator is freely falling. Calculate the angle between the two threads and tension in each thread.

10. Suppose we have a large number of identical particles, very small in size. Any two of them at 10 cm separation repel with a force of \(3 \times 10^{-18}\) N.
   a. If one of them is at 10 cm from a group (of very small size) of \(n\) others, how strongly do you expect it to be repelled?
   b. Suppose you measure the repulsion and find it \(6 \times 10^{-18}\) N. How many particles were there in the group?

---

**ELECTRIC FIELD**

If we place a single charge \(q\) at some point in space, it will experience no force. But if some other charge (say \(Q\)) is placed near it, \(q\) will start experiencing a force given by

\[
F = \frac{kQq}{r^2}
\]

Now, question arises, how does \(Q\) apply a force on \(q\) or how does \(q\) know the presence of \(Q\) when there is no direct contact between them.

Basically, the force between two charges can be seen as a two-step process:

1. Firstly, charge \(Q\) creates something around itself known as electric field.
2. Secondly, any other charge particle like \(q\) if placed at some point in that field experiences a force, or we can say that charges interact with each other through electric field.

So, we can define electric field as the space around a charge in which its influence can be felt by any other charged particle.

**How to Measure Electric Field**

Strength of electric field at a point in space can be measured in terms of two measurable quantities:

1. Electric field intensity is denoted by \(E\). It is a vector quantity.
2. Electric field potential is denoted by \(V\). It is a scalar quantity.

First, we will discuss them separately and then we will see what is the relation between them and how to obtain one from the other.

**Electric Field Intensity \(E\)**

How to find electric field intensity \(E\) at a point?

**General method:** Electric field intensity, \(E\), is a vector quantity. At a point in a given space, it has both magnitude and direction. Let us calculate \(E\) at some point \(P\) created due to some charges around \(P\). Bring a small charge \(q\) [test charge, generally positive] at point \(P\). Let this charge experiences a force \(F\) due to charges placed in the vicinity of \(P\). Then we define electric field intensity at \(P\) as force experienced per unit test charge (Fig. 1.30).

\[
E = \lim_{q_0 \to q_0} \frac{F}{q_0}.
\]

The direction \(E\) will be same as that of \(F\).

**Note:** \(Q\). Why the magnitude of test charge is kept small?

*Ans.* Otherwise, large magnitude may disturb the original charge distribution and then we will get electric field due to disturbed configuration and not original.

**What is the minimum possible value of \(q_0\)?**

*Ans.* \(1.6 \times 10^{-19}\) C
Unit of \(E\): \(\text{N/C (newton per coulomb)}\)

Dimensional formula of \(E\):
\[
E = \frac{\text{Force}}{\text{Charge} \times \text{ampere} \times \text{time}} = \frac{MLT^{-3}}{AT} = [MLT^{-3} A^{-1}]
\]

Note: If a test charge experiences no force at a point, the electric field at that point must be zero.

Electric field due to a point charge is illustrated in Fig. 1.31.

(i) Positive point charge
\[
E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \text{ away from the charge}
\]

(ii) Negative point charge
\[
E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \text{ towards the charge}
\]

Electric field due to a point charge is spherically symmetric.

**Point Charge in an Electric Field**

What happens if a point charge \(q\) is placed at any point in an electric field which is produced by some other stationary charges. Let this electric field is \(\vec{E}\). Charge \(q\) will experience a force at this point, let this force is \(\vec{F}\). Then, value of electric field at that point must be
\[
\vec{E} = \frac{\vec{F}}{q} \Rightarrow \vec{F} = q\vec{E}. \text{ This is the force on } q \text{ by } \vec{E}.
\]

**Direction of \(\vec{F}\):** The direction of \(\vec{F}\) will be same as that of \(\vec{E}\) if \(q\) is +ve and opposite if \(q\) is -ve (Fig. 1.32).

![Fig. 1.32](image)

(a) Positive charge \(q_0\) placed in an electric field: force on \(q_0\) is in the same direction as \(\vec{E}\).

(b) Negative charge \(q_0\) placed in an electric field: force on \(q_0\) is in the opposite direction as \(\vec{E}\).

**Electric Field Intensity due to a Point Charge in Position Vector Form**

Electric field at \(P\) due to charge \(Q\) is
\[
\vec{E} = \frac{Q(\vec{r} - \vec{r}_0)}{4\pi\varepsilon_0 |\vec{r} - \vec{r}_0|^3}
\]

**Electric Field Intensity due to a Group of Charges**

Using the principle of superposition, net field at point \(P\) (see Fig. 1.34) is
\[
\vec{E} = \vec{E}_1 + \vec{E}_2 + \ldots + \vec{E}_n
\]
\[
\Rightarrow \vec{E} = \frac{q_1}{4\pi\varepsilon_0 r_1^3} \vec{r}_1 + \frac{q_2}{4\pi\varepsilon_0 r_2^3} \vec{r}_2 + \ldots + \frac{q_n}{4\pi\varepsilon_0 r_n^3} \vec{r}_n
\]
\[
\Rightarrow \vec{E} = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i^3} \vec{r}_i
\]

In terms of position vectors:
\[
\vec{E} = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i(\vec{r} - \vec{r}_i)}{|\vec{r} - \vec{r}_i|^3}
\]

**Illustration:** Two point-like charges \(a\) and \(b\) whose strengths are equal in absolute value are positioned at a certain distance from each other. Assuming the field strength is positive in the direction coinciding with the positive direction of the \(r\) axis, determine the signs of the charges for each distribution of the field strength between charges shown in Figs. 1.35(a), (b), (c) and (d).

**Sol.**

a. As electric field tends away at \(a\) and towards at \(b\), hence there should be + charge at \(a\) and negative charge at \(b\), i.e., \(q_a\) is '+' and \(q_b\) is '-'.

![Fig. 1.34](image)
b. The neutral point exists between \( a \) and \( b \) only when both \( q_a \) and \( q_b \) are of same sign. As direction of electric field is away from both, both charges are positive, i.e., \( q_a \) is '+' and \( q_b \) is '+'.

Similarly, for (c) and (d) in Fig. 1.36:

c. \( q_a \) is '−' and \( q_b \) is '+'
d. \( q_a \) is '−' and \( q_b \) is '−'.

**Illustration 1.17** Two point charges \( ±q \) are placed on the axis at \( x = −a \) and \( x = +a \), as shown in Fig. 1.37.

**Fig. 1.37**

i. Plot the variation of \( E \) along the \( x \)-axis.

ii. Plot the variation of \( E \) along the \( y \)-axis

**Sol.**

i. Variation of \( E \) along the \( x \)-axis as shown in Fig. 1.38(a).

ii. Variation of \( E \) along the \( y \)-axis as shown in Fig. 1.38(b). In this case, field will be maximum at origin because at origin, field due to both charges is directly added.

**Fig. 1.38**

**Illustration 1.18** In Fig. 1.39, determine the point (other than infinity) at which the electric field is zero.

**Fig. 1.39**

**Sol.** Electric field will be zero at a point closer to the charge smaller in magnitude. Let electric field is zero at \( P \) (see Fig. 1.40).

**Fig. 1.40**

Then, we have

\[
\frac{k(2.5 \times 10^9)}{x^2} = \frac{k(6 \times 10^8)}{(1+x)^2}
\]

\( x = 1.82 \) m

**Illustration 1.19** Four charges are arranged as shown in Fig. 1.41. A point \( P \) is located at distance \( r \) from the centre of the configuration. Assuming \( r \gg l \), find

i. the magnitude of the field at point \( P \).

ii. the angle of its vector with the \( x \)-axis.

**Sol.**

i. Electric field due to charges placed on the \( y \)-axis [Fig. 1.41(a)].

\[
E_y = 2E_0 \sin \theta = 2 \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2 + \left(\frac{1}{2}\right)^2} \left(\frac{1}{r^2 + \left(\frac{1}{2}\right)^2}\right)^{1/2}
\]

**Fig. 1.41**
\[ E_y = \frac{1}{4\pi\varepsilon_0} \frac{q l}{(r^2 + \frac{l^2}{2})^{3/2}} = \frac{1}{4\pi\varepsilon_0} \frac{q l}{r^3} \text{ (as } r >> l) \]

Electric field due to charges placed on x-axis [Fig. 1.42(b)]

\[ E_y = E_1 - E_2 = \frac{1}{4\pi\varepsilon_0} \frac{q}{r} - \frac{1}{4\pi\varepsilon_0} \frac{q}{r + l/2} = \frac{1}{2\pi\varepsilon_0} \frac{ql}{r^3} \]

![Diagram](image)

\[ E_{net} = \sqrt{E_1^2 + E_2^2} = \sqrt{\frac{q l}{4\pi\varepsilon_0 r^3}} \]

ii. The angle that \( E_{net} \) makes with x-axis [Fig. 1.42(c)] is

\[ \alpha = \tan^{-1}\left( \frac{E_y}{E_x} \right) = \tan^{-1}\left( \frac{1}{2} \right) \text{ below x-axis} \]

Illustration 1.20: A uniform electric field \( E \) exists between two metal plates one -ve and other +ve. The plate length is \( l \) and the separation of the plates is \( d \).

i. An electron and a proton start from the negative plate and positive plate, respectively, and go to opposite plates. Which one of them wins this race?

ii. An electron and a proton start moving parallel to the plates. They are at the midpoint from the separation of plates at one end of the plates. Which of the two will have greater deviation when they come out of the plates if they start with the

a. same initial velocity,

b. same initial kinetic energy and
c. same initial momentum.

Sol.

i. \( a_e = \frac{eE}{m_e}, a_p = \frac{eE}{m_p}, \) \( d = \frac{1}{2} ar^2 \)

\[ t = \sqrt{\frac{2d}{a}} \text{ or } t = \sqrt{\frac{2md}{eE}} \]

As \( m_e < m_p \), therefore \( t_e < t_p \). Hence, electron will take less time, i.e., the electron wins the race.

![Diagram](image)

Fig. 1.43

ii. Time to cross the plates \( t = \frac{d}{a} \)

Deviation: \( \delta = \frac{1}{2} ar^2 = \frac{1}{2} \frac{eE}{m} \left( \frac{l}{2} \right)^2 \)

\[ \frac{\delta_e}{\delta_p} = \frac{m_p \left( \frac{u_p}{u_e} \right)^2}{m_e \left( \frac{u_e}{u_p} \right)^2} \]

a. If \( u_p = u_e \), then \( \frac{\delta_e}{\delta_p} = \frac{m_p}{m_e} \)

As \( m_e > m_p \), therefore \( \delta_e > \delta_p \)

Hence, deviation of electron will be more.

![Diagram](image)

Fig. 1.44

b. From Eq. (i), \( \frac{\delta_e}{\delta_p} = \left( \frac{m_p u_e}{m_e u_p} \right)^2 \) = 1 (as given)

Hence, deviation of both electron and proton will be the same.

c. From Eq. (i), \( \frac{\delta_e}{\delta_p} = \left( \frac{m_p u_e}{m_e u_p} \right)^2 \) = \( \frac{m_e}{m_p} \)

As \( m_e < m_p \), \( \delta_e < \delta_p \)

Hence, the deviation of proton will be more.

Illustration 1.21: A charge \( 10^{-8} \) coulomb is located at origin in free space and another charge \( Q \) at \((2, 0, 0)\). If the x-component of the electric field at \((3, 1, 1)\) is zero, calculate the value of \( Q \). Is the y-component zero at \((3, 1, 1)\)?

Sol. The electric field due to a point charge \( q \) at position \( \vec{r} \) in vector form is given by

\[ \vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \vec{r} \]
Here \( \vec{r}_1 = (3-0)\hat{i} + (1-0)\hat{j} + (1-0)\hat{k} = 3\hat{i} + \hat{j} + \hat{k} \)

\[
= \sqrt{(3^2 + 1^2 + 1^2)} = \sqrt{11} \text{ m}
\]

\( \vec{r}_2 = (3-2)\hat{i} + (1-0)\hat{j} + (1-0)\hat{k} = \hat{i} + \hat{j} + \hat{k} \)

\[
= \sqrt{(1^2 + 1^2 + 1^2)} = \sqrt{3} \text{ m}
\]

So,

\[
\vec{E}_1 = \frac{1}{4\pi\epsilon_0} \frac{10^{-9}}{(11)^{3/2}} [3\hat{i} + \hat{j} + \hat{k}]
\]

and

\[
\vec{E}_2 = \frac{1}{4\pi\epsilon_0} \frac{Q}{(3)^{3/2}} [\hat{i} + \hat{j} + \hat{k}]
\]

Hence, net field is

\[
\vec{E} = \vec{E}_1 + \vec{E}_2
\]

\[
= \frac{1}{4\pi\epsilon_0} \left\{ \left[ \frac{3 \times 10^{-9}}{11\sqrt{11}} + \frac{Q}{3\sqrt{3}} \right] \hat{i} + \left[ \frac{10^{-9}}{11\sqrt{11}} + \frac{Q}{3\sqrt{3}} \right] \hat{j} + \left[ \frac{10^{-9}}{11\sqrt{11}} + \frac{Q}{3\sqrt{3}} \right] \hat{k} \right\}
\]

According to the given problem,

\[
E_x = 0, \text{ i.e., } \frac{1}{4\pi\epsilon_0} \left( \frac{3 \times 10^{-9}}{11\sqrt{11}} + \frac{Q}{3\sqrt{3}} \right) = 0
\]

So,

\[
Q = \left[ \frac{3}{11} \right] \times 3 \times 10^{-3} \text{ coulomb}
\]

And for this value of \( Q \),

\[
E_x = \frac{1}{4\pi\epsilon_0} \left\{ \frac{10^{-9}}{11\sqrt{11}} \times \frac{(3/11)^{3/2} \times 3 \times 10^{-9}}{3\sqrt{3}} \right\}
\]

\[
= \frac{1}{4\pi\epsilon_0} \frac{2 \times 10^{-9}}{11\sqrt{11}} \neq 0 \text{ i.e., } E_x \text{ is not zero.}
\]

**Lines of Force**

This idea was given by Michael Faraday. The lines of force provide a nice idea to visualize the pattern of electric field in a given space. We assume that space around a charged body is filled with some lines known as electric lines of force. These lines of force are drawn in space in such a way that tangent to the line at any point gives the direction of electric field at that point. It has been found quite convenient to visualize the electric field in terms of lines of force.

**Properties of Electric Lines of Force**

- Electric lines of force start (or diverge out) from a positive charge and end (or converge) on a negative charge.
- The tangent drawn at any point on the line of force gives the direction of force acting on a positive charge at that point (see Fig. 1.47).
- In S.I. system of units, the number of electric lines of force originating or terminating on a charge of \( q \) coulomb is equal to \( q/\varepsilon_0 \).

![Fig. 1.46]

- Two electric lines of force never cross each other, because if they do so then at the point of intersection, intensity will have two directions which is absurd.
- Electric lines of force can never be closed loops, as a line can never start and end on the same charge.
- The electric lines of force do not pass through a conductor as electric field inside a conductor is always zero.
- Lines of force have a tendency to contract longitudinally like a stretched elastic string producing attraction between opposite charges and repel each other laterally resulting in repulsion between similar charges and edge effect (curving of lines of force near the edges of a charged conductor).
- Electric lines of force end or start normally on the surface of a conductor.
- Tangent to the line of force at a point in an electric field gives the direction of intensity or force or acceleration which a positive charge will experience there but not the direction of motion always, therefore a positive point charge free to move in an electric field may or may not follow the line of force. It will follow the line of force if it is a straight line (as direction of velocity and acceleration will be same) and will not follow the line if it is curved as the direction of motion will be different from that of acceleration and the particle will move in the direction of neither motion nor acceleration (line of force).

The use of the electric lines of force is that we can compare the intensities at two points just by looking at the distribution of lines of force. Where the field lines are close together, \( E \) is large and where they are far apart, \( E \) is small.
Coulomb's Laws and Electric Field

Fig. 1.48

In Fig. 1.48, electric lines of forces are shown. The electric field intensity at point 2 will be greater in comparison to that at point 1.

**DIFFERENT PATTERNS OF ELECTRIC FIELD LINES**

- **Magnitude is not constant**
- **Direction is not constant**
- **Both magnitude and direction are not constant**
- **Both magnitude and direction are constant**

(a) A metal plate and a point charge

(b) A single positive charge

(c) A positive charge and a negative charge of equal magnitude (an electric dipole)

(d) Two equal positive charges. N is the neutral point lying at the middle of the charges.

(e) A is a positive charge and B a negative charge of different magnitudes \(|q_A| < |q_B|\)

(f) Two positive charges of different magnitudes \(q_1 < q_2\)

FIG. 1.49

Note: Neutral point \(N\) is the location where the net electric field due to charges is zero. It lies near the charge of smaller magnitude.

FIG. 1.50

Illustration 1.22—Figure 1.51 shows the sketch of field lines for two point charges \(2Q\) and \(-Q\). The pattern of field lines can be deduced by considering the following points:

Fig. 1.51

Sol.

i. Symmetry: For every point above the line joining the two charges, there is an equivalent point below it. Therefore, the pattern must be symmetrical about the line joining the two charges.

ii. Near field: Very close to a charge, its field predominates. Therefore, the lines are radial and spherically symmetric.

iii. Far field: Far from the system of charges, the pattern should look like that of a single point charge of value \((2Q - Q) = +Q\), i.e., the lines should be radially outward.

iv. Null point or neutral point: There is one point at which \(E = 0\). No lines should pass through this point. Neutral point lies near the position of charge of smaller magnitude.

v. Number of lines: Twice as many lines leave \(+2Q\) as enter \(-Q\).

Note: Excess lines from \(2Q\) charge will meet at infinity.

FIG. 1.52

Charges \(+q\) and \(-2q\) are fixed at distance \(d\) apart as shown in the figure.
i. Sketch roughly the pattern of electric field lines, showing position of neutral point.

ii. Where should a charge particle \( q \) be placed so that it experiences no force?

Sol. Let net force on \( q \) at \( P \) is zero, then

\[
\frac{kq^2}{x^2} = \frac{kq 2q}{(d+x)^2} \Rightarrow x = \frac{d}{\sqrt{2} - 1}
\]

Fig. 1.53

\( P \) is the neutral potential where electric field will be zero.

\[
\frac{1}{4\pi\varepsilon_0 \sqrt{(x+1)^2}} = \frac{q_A}{4\pi\varepsilon_0 x^2} = \frac{q_B}{4\pi\varepsilon_0 x^2}
\]

Fig. 1.56

vi. At infinity.

vii. No. As lines of force are curved, the direction of velocity and acceleration will be different. Hence, a charge cannot follow strictly the line of force. Also to move on some curved path, centripetal force is required, whereas lines of force will provide only tangential force.

**FIELD OF RING CHARGE**

A ring-shaped conductor with radius \( a \) carries a total charge \( Q \) uniformly distributed around it. Let us calculate the electric field at a point \( P \) that lies on the axis of the ring at a distance \( x \) from its centre.

\[
A = B \quad E_A \quad E_B
\]

Fig. 1.57

As shown in the figure, the ring is divided into infinitesimal segments each of length \( ds \). Each segment has charge \( dQ \) and acts as a point charge source of electric field. Let \( dE \) be the electric field from one such segment; the net electric field at \( P \) is then the sum of all contributions \( dE \) from all the segments that make up the ring. (The same technique works for any situation where charges are distributed along a line or a curve.) The calculation of \( E \) is greatly simplified because the field point \( P \) is on the symmetry axis of the ring. If we consider two ring segments at the top and bottom of the ring, we see that the contributions \( dE \) to the field at \( P \) from these segments have the same \( x \)-component but opposite \( y \)-components. Hence, the total \( y \)-component of field due to this pair of segments is zero. When we add up the contributions from all such pairs of segments, the total field \( E \) will have only a component along the ring’s symmetry axis (the \( x \)-axis), with no component perpendicular to that axis (that is, no \( y \)-component or \( z \)-component). So, the field at \( P \) is described completely by its \( x \)-component \( E_x \).

To calculate \( E_x \) note that the square of the distance \( r \) from a ring segment to the point \( P \) is \( r^2 = x^2 + a^2 \). Hence, the magnitude of this segment’s contribution to the electric field at \( P \) is

\[
dE = \frac{1}{4\pi\varepsilon_0} \frac{dQ}{(x^2 + a^2)^{3/2}}
\]

Using \( \cos \alpha = \frac{x}{r} = \frac{x}{(x^2 + a^2)^{1/2}} \) the component \( dE_x \) of this field along the \( x \)-axis is
\[ dE_x = \frac{dQ}{4\pi \varepsilon_0 (x^2 + a^2)^{3/2}} \]

To find the total x-component \( E_x \) of the field at \( P \), we integrate this expression over all segments of the ring, i.e.,

\[ E_x = \frac{1}{4\pi \varepsilon_0} \int \frac{xdQ}{(x^2 + a^2)^{3/2}} \]

Since \( x \) does not vary as we move from point to point around the ring, all the factors on the right side except \( dQ \) are constant and can be taken outside the integral. The integral of \( dQ \) is just the total charge \( Q \) and we finally get

\[ E_x = \frac{1}{4\pi \varepsilon_0} \frac{Q}{(x^2 + a^2)^{3/2}} \]  \[ \text{(i)} \]

- Electric field is directed away from positively charged ring.
- For \( x = 0 \), \( E = 0 \). This conclusion may be arrived at by the symmetry consideration.
- At a large distance from the ring, the electric field will be zero. Hence, it should have certain maximum value between \( x = 0 \) and \( x = \infty \) (or \( x = -\infty \)).
- If we maximize Eq. (i), we can get the value of \( x_0 \) as well as \( E_{\text{max}} \).

For the maximum value of \( E_x \), we get

\[ \frac{dE_x}{dx} \left\{ \frac{1}{4\pi \varepsilon_0} \frac{x}{(x^2 + a^2)^{3/2}} \right\} = 0 \]

\[ \frac{(x^2 + a^2)^{3/2}(1-x^2)^{3/2}}{(x^2 + a^2)^{3/2}} = 0 \]

\[ (x^2 + a^2) - 3x^2 = 0 \Rightarrow x = \pm \frac{a}{\sqrt{2}} \]

and the maximum value of the electric field is

\[ E_{\text{max}} = \frac{1}{4\pi \varepsilon_0} \frac{2Q}{3\sqrt{3}a^2} \]

**Fig. 1.58** Two identical point charges having magnitude \( q \) each are placed as shown in the figure.

**Fig. 1.59**

i. Plot the variation of electric field on x-axis.
ii. Where will the magnitude of electric field be maximum on x-axis? Find the maximum value of electric field at x-axis.
iii. If we place a negative charge (of magnitude \( -q \) and mass \( m \)) at the mid point of charges and displaced along x-axis examine whether it will perform simple harmonic motion. If yes, then find the time period of oscillation of the particle.

**Sol. i.**

**Fig. 1.60**

\[ E = \frac{q}{2\pi \varepsilon_0 (a^2 + x^2)^{3/2}} \cos \theta \]

For \( E \) to be maximum, \( \frac{dE}{dx} = 0 \)

Solving, we get \( x = \pm \frac{a}{\sqrt{2}} \Rightarrow E_{\text{max}} = -\frac{q}{3\sqrt{3}\pi \varepsilon_0 a^2} \)

iii. Force on particle: \( F = -qE = -\frac{q^2}{2\pi \varepsilon_0 (a^2 + x^2)^{3/2}} \)

For \( x << a \), particle will execute S.H.M. with time period

\[ T = 2\pi \sqrt{\frac{2\pi \varepsilon_0 ma^3}{q^2}} \]

**Electric Field due to an Infinite Line Charge**

Positive electric charge \( Q \) is distributed uniformly along a line, lying along the y-axis. Let us find the electric field at point \( D \) on the x-axis at a distance \( r_0 \) from the origin.

We divide the line charge into infinitesimal segments, each of which acts as a point charge; let the length of a typical segment at height \( l \) be \( dl \). If the charge is distributed uniformly with the linear charge density \( \lambda \), then the charge \( dQ \) in a segment of length \( dl \) is \( dQ = \lambda dl \). At point \( D \), the differential electric field \( dE \) created by this element is

**Fig. 1.61**
\[ dE = \frac{dQ}{4\pi \varepsilon_0 r^2} = \frac{\lambda dl}{4\pi \varepsilon_0 r^2} = \frac{\lambda dl}{4\pi \varepsilon_0 \theta \sec^2 \theta} \]  

(1)

In triangle AOD; \( OA = OD \tan \theta \), i.e., \( l = r_0 \tan \theta \).

Differentiating this equation with respect to \( \theta \), we get

\[ dl = r_0 \sec^2 \theta d\theta. \]

Substituting the value of \( dl \) in Eq. (i), we get

\[ dE = \frac{\lambda d\theta}{4\pi \varepsilon_0 r_0}. \]

Field \( dE \) has components \( dE_x, dE_y \) given by

\[ dE_x = \frac{\lambda \cos \theta d\theta}{4\pi \varepsilon_0 r_0} \quad \text{and} \quad dE_y = \frac{-\lambda \sin \theta d\theta}{4\pi \varepsilon_0 r_0}. \]

On integrating expression for \( dE_x \) and \( dE_y \) in limits \( \theta = -\pi/2 \) to \( \theta = +\pi/2 \), we obtain \( E_x \) and \( E_y \). Note that as the length of wire increases, the angle \( \theta \) also increases. For a very long wire (infinitely long wire), \( \theta \) approaches \( \pi/2 \).

\[ E_x = \int_{-\pi/2}^{\pi/2} \frac{\lambda \cos \theta d\theta}{4\pi \varepsilon_0 r_0} = \frac{\lambda}{2\pi \varepsilon_0 r_0} \]

and

\[ E_y = \int_{-\pi/2}^{\pi/2} \frac{\lambda \sin \theta d\theta}{4\pi \varepsilon_0 r_0} = 0. \]

Thus, \( E = E_x = \frac{\lambda}{4\pi \varepsilon_0 r_0} \).

Note: Using a symmetry argument, we could have guessed that \( E_y \) would be zero; if we place a positive test charge at \( D \), the upper half of the line of charge pushes downward on it, and the lower half pushes upward with equal magnitude.

- If the wire has finite length and the angles subtended by ends of wire at a point are \( \theta_1 \) and \( \theta_2 \), the limits of integration would change.

\[ E_x = \int_{\theta_1}^{\theta_2} \frac{\lambda \cos \theta d\theta}{4\pi \varepsilon_0 r_0} = \frac{\lambda}{4\pi \varepsilon_0 r_0} (\sin \theta_2 - \sin \theta_1). \]

\[ E_y = \int_{\theta_1}^{\theta_2} \frac{\lambda \sin \theta d\theta}{4\pi \varepsilon_0 r_0} = \frac{\lambda}{4\pi \varepsilon_0 r_0} (\cos \theta_1 - \cos \theta_2). \]

If we wish to determine field at the end of a long wire, we may substitute \( \theta_1 = 0 \) and \( \theta_2 = \pi/2 \) in the expressions for \( E_x \) and \( E_y \), i.e.,

\[ E_x = \frac{\lambda}{4\pi \varepsilon_0 r_0} \left[ \sin(0) + \sin \left( \frac{\pi}{2} \right) \right] = \frac{\lambda}{4\pi \varepsilon_0 r_0} \]

and

\[ E_y = \frac{\lambda}{4\pi \varepsilon_0 r_0} \left[ \cos(0) - \cos \left( \frac{\pi}{2} \right) \right] = \frac{\lambda}{4\pi \varepsilon_0 r_0}. \]

**FIELD OF UNIFORMLY CHARGED DISK**

Let us find the electric field caused by a disk of radius \( R \) with a uniform positive surface charge density (charge per unit area) \( \sigma \) at a point on the axis of the disk at a distance \( x \) from its centre.

The situation is shown in Fig.1.64. We can represent this charge distribution as a collection of concentric rings of charge. We already know how to find the field of a single ring on its axis of symmetry, therefore we will add the contribution of all the rings. As shown in the figure, a typical ring has charge \( dQ \), inner radius \( r \) and outer radius \( r + dr \). Its area \( dA \) is approximately equal to its width \( dr \) times its circumference \( 2\pi r \), or \( dA = 2\pi r \, dr \). The charge per unit area is \( \sigma = dQ/dA \), so the charge of ring is \( dQ = \sigma (2\pi r dr) \), or \( dQ = 2\pi \sigma r dr \). The field component \( dE \) at point \( P \) due to charge \( dQ \) of a ring of radius \( r \) is

\[ dE = \frac{1}{4\pi \varepsilon_0} \frac{dQ}{(x^2 + r^2)^{3/2}} \]

To find the total field due to all the rings, we integrate \( dE \) over \( r \). To include the whole disk, we must integrate from 0 to \( R \) (not from \(-R \) to \( R \)), i.e.,
\[ E_x = \int dE_x = \int_0^d dE_x = \int_0^d \frac{1}{4\pi \varepsilon_0} \left( \frac{2\pi \alpha r dr}{(x^2 + r^2)^{3/2}} \right) \]

Remember that \( x \) is a constant during the integration and that the integration variable is \( r \). The integral can be evaluated by the use of the substitution \( z = x^2 + r^2 \). We will let you work out the details; the result is

\[ E_x = \frac{\sigma x}{2\varepsilon_0} \left[ \frac{1}{\sqrt{x^2 + R^2}} + \frac{1}{x} \right] = \frac{\sigma}{2\varepsilon_0} \left[ \frac{x}{1 - \frac{x}{\sqrt{x^2 + R^2}}} \right] \]

(i)

In this figure, the charge is assumed to be positive. At a point on the symmetry axis of a uniformly charged ring, the electric field due to the ring has no components perpendicular to the axis. Hence, at point \( P \) in the figure, \( dE_x = dE_z = 0 \) for each ring, and thus the total field has \( E_x = E_z = 0 \).

Again, we can ask what happens if the charge distribution gets very large. Suppose we keep increasing the radius \( R \) of the disk, simultaneously adding charge so that the surface charge density \( \sigma \) (charge per unit area) is constant. In the limit that \( R \) is much larger than the distance \( x \) of the field point from the disk \( (R \gg x) \), i.e., the situation becomes the electric field near infinite plane sheet of charge.

From Eq. (i), we get

\[ E_x = \frac{\sigma}{2\varepsilon_0} \left[ \frac{x}{\sqrt{x^2 + R^2}} \right] \]

As \( R \gg x \), then the term \( \frac{1}{\sqrt{1 + \frac{R^2}{x^2}}} \) → 0

And we get \( E_x = \frac{\sigma}{2\varepsilon_0} \)

Our final result does not contain the distance \( x \) from the plane. This is a correct but rather surprising result.

We observe the following.

- The electric field produced by an infinite plane sheet of charge is independent of the distance from the sheet.
- The electric field is uniform; everywhere its direction is perpendicular to the sheet and away from it.
- Infinite plane sheet of charge is a hypothetical case. In real practice, there is no such infinite plane sheet of charge.
- Again, there is no such thing as an infinite sheet of charge, but if the dimensions of the sheet are much larger than the distance \( x \) of the observation point \( P \) from the sheet, the field is very nearly the same as for an infinite sheet.

**FIELD OF TWO OPPositely CHARGED SHEETS**

Two infinite plane sheets are placed parallel to each other, separated by a distance \( d \) (as shown in Fig. 1.65). The lower sheet has a uniform positive surface charge density \( \sigma \), and the upper sheet has a uniform negative surface charge density \( -\sigma \) with the same magnitude. Let us find the electric field between the two sheets, above the upper sheet and below the lower sheet.

![Fig. 1.65](image)

The situation described in this example is an idealization of two finite, oppositely charged sheets, like the plates shown in the figures. If the dimensions of the sheets are large in comparison to the separation \( d \), then good approximation we can consider the sheets to be infinite in extent. We know the field due to a single infinite plane sheet of charge. We can then find the total field by using the principle of superposition of electric fields. Let sheet 1 be the lower sheet of positive charge, and let sheet 2 be the upper sheet of negative charge; the fields due to each sheet are \( E_1 \) and \( E_2 \), respectively, and both have the same magnitude at all points, no matter how far from either sheet, i.e., \( E_1 = E_2 = \frac{\sigma}{2\varepsilon_0} \).

At all points, the direction of \( \vec{E}_1 \) is away from the positive charge of sheet 1, and the direction of \( \vec{E}_2 \) is towards the negative charge of sheet 2. These fields, as well as the \( x \)- and \( y \)-axes, are shown in the figure. At points between the sheets, the fields add each other and at points above the upper sheet or below the lower sheet cancel each other. Thus, the total field is

\[ E = E_1 + E_2 = \frac{\sigma}{\varepsilon_0} \]

above the upper sheet

\[ E = \frac{\sigma}{\varepsilon_0} \]

between the sheets

\[ 0 \]

above the upper sheet

Because we considered the sheets to be infinite, our result does not depend on the separation \( d \).

Symmetry plays very important role in problem solving. Electric field is in the direction along the line which divides the charge distribution symmetrically.

![Diagram](image)

\[ E_{\text{lin}} = \int dE \cos \theta \]
Some Useful Results

A charged rod of fixed length having charge density $\lambda$.

Semi-infinite rod having charge density $\lambda$.

**Semicircular ring having charge density $\lambda$.**

**Quarter circular ring having charge density $\lambda$.**

**Infinite line charge.**

**Charged ring.**

**Charged disk.**

**Infinite sheet of charge.**

**Two point charge**

Three point charge at the corner of an equilateral triangle.

Four point charges at the corner of a square.

Charged disk.

Net field at a point on the axis is along the axis of disk.

The electric field at point $P$ due to charges (1), (2), (3) and (4) is $|E_1| = |E_2| = |E_3| = |E_4|$

Hence, net electric field at $P$ is $|E_{net}| = 4|E_1|\cos\theta$

**Charged ring.**

**Charged disk.**

**Infinite sheet of charge.**

**Infinite line charge.**

**Charged ring.**

**Charged disk.**

**Infinite sheet of charge.**
20. A droplet of ink in an industrial ink-jet printer carries a charge of $1.6 \times 10^{-10}$ C and is deflected onto the paper by a force of $3.2 \times 10^4$ N. Find the strength of the electric field to produce this force.

**ELECTRIC DIPOLE**

- An electric dipole is a system of two equal and opposite point charges separated by a very small and finite distance.
- Figure 1.77 shows an electric dipole consisting of two equal and opposite point charges $-q$ and $+q$ separated by a small distance $2l$. The strength of an electric dipole is measured by a vector quantity known as electric dipole moment. Its magnitude is equal to the product of the magnitude of either charge and the distance between the two charges, i.e.,

$$p = 2ql$$

![Fig. 1.77](image)

The direction of $p$ is from negative charge to positive charge.
- In S.I. system of units, $p$ is measured in coulomb-metre.

**ELECTRIC FIELD DUE TO A DIPOLE**

**Electric Field Intensity due to an Electric Dipole at a Point on the Axial Line**

- A line passing through the negative and positive charges of the electric dipole is called the axial line of the electric dipole.

![Fig. 1.78](image)

- Suppose an electric dipole $AB$ is located in a medium of dielectric constant $K$ (as shown in Fig.1.78). Let the dipole consists of two point charges $-q$ and $+q$ separated by a short distance $2l$ metre. Let $P$ be an observation point on the axial line such that its distance from the midpoint $O$ of the electric dipole is $r$. We are interested to calculate the intensity of electric field at $P$.

$$E_1 = \frac{1}{4\pi\varepsilon_0 K} \frac{q}{(r-l)^2} \text { due to } q \text { at } P$$  

(along the direction $OX$)

and $E_2 = \frac{1}{4\pi\varepsilon_0 K} \frac{q}{(r+l)^2} \text { due to } -q \text { at } P$  

(along the direction $OB$)

The intensities $E_1$ and $E_2$ are along the same line but in opposite directions. Since $E_1 > E_2$, the resultant intensity $E$ at the point $P$ will be equal to their differences and in the direction $\overrightarrow{AP}$. Thus,

$$E = E_1 - E_2 = \frac{1}{4\pi\varepsilon_0 K} \frac{q}{(r-l)^2} - \frac{1}{4\pi\varepsilon_0 K} \frac{q}{(r+l)^2}$$

$$= \frac{q}{4\pi\varepsilon_0 K} \left[ \frac{4rl}{(r^2-l^2)^2} \right] = \frac{1}{4\pi\varepsilon_0 K} \left[ \frac{2(2ql)r}{(r^2-l^2)^2} \right]$$

But $2ql = p$ = electric dipole moment;

$$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0 K} \frac{2pr}{(r^2-l^2)^2}$$

- If $l$ is very small as compared to $r$ ($l \ll r$), then $l^2$ can be neglected in comparison to $r^2$. Then, the electric field intensity at the point $P$ due to a short dipole is given by

$$E = \frac{1}{4\pi\varepsilon_0 K} \frac{2pr}{r^4} = \frac{1}{4\pi\varepsilon_0 K} \frac{2p}{r^2}$$

$$= \frac{1}{4\pi\varepsilon_0 K} \frac{2p}{r^2}$$

- If dipole is placed in air or vacuum, then $K = 1$ and

$$E = \frac{1}{4\pi\varepsilon_0} \frac{2p}{r^2}$$

**Note:** The direction of electric field $E$ is in the direction of $\overrightarrow{p}$, i.e., parallel to the axis of dipole from the negative charge towards the positive charge.

In vector form, we can write

$$E = \frac{1}{4\pi\varepsilon_0} \frac{2\overrightarrow{p}}{r^2}$$

**Electric Field Intensity due to an Electric Dipole at a Point on the Equatorial Line**

An equatorial line of the electric dipole is a line perpendicular to the axial line and it passes through a point midway between charges.

- Let us now suppose that the observation point $P$ is situated on the equatorial line of dipole such that its distance from midpoint $O$ of the electric dipole is $r$ (as shown in Fig.1.79). Let us assume again that the medium between the electric dipole and the observation point has dielectric constant $K$.

$$E_1 = \frac{1}{4\pi\varepsilon_0 K} \frac{q}{(r^2+l^2)} \quad \text{(along the direction } PD)$$

and $$E_2 = \frac{1}{4\pi\varepsilon_0 K} \frac{q}{(r^2+l^2)} \quad \text{(along the direction } PC)$$

The magnitude of $E_1$ and $E_2$ is equal but directions are different.

Net intensity: $E = E_1 \cos \theta + E_2 \cos \theta$

$$= \frac{1}{4\pi\varepsilon_0 K} \frac{q}{(r^2+l^2)} \cos \theta + \frac{1}{4\pi\varepsilon_0 K} \frac{q}{(r^2+l^2)} \cos \theta$$

(since components cancel out)
\[
E = \frac{q}{4\pi\varepsilon_0(r^2 + l^2)} \times 2\cos\theta \text{ along } PR
\]

But from the figure,
\[
\cos\theta = \frac{OA}{PA} = \frac{OA}{(OP^2 + OA^2)^{1/2}} = \frac{1}{(r^2 + l^2)^{1/2}}
\]
\[
\therefore E = \frac{1}{4\pi\varepsilon_0K}(r^2 + l^2) \times \frac{2l}{(r^2 + l^2)^{3/2}} = \frac{1}{4\pi\varepsilon_0K} \times \frac{2ql}{(r^2 + l^2)^{3/2}}
\]

But \(2ql = p\) = electric dipole moment
\[
\therefore E = \frac{1}{4\pi\varepsilon_0K} \times \frac{p}{(r^2 + l^2)^{3/2}}.
\]

- If \(l\) is very small as compared to \(r(l \ll r)\), then \(l^2\) can be neglected in comparison to \(r^2\).

![Diagram of dipole and electric field intensity](image)

**Note:** The direction of electric field \(E\) is opposite to the direction of \(\vec{p}\), i.e., antiparallel to the axis of dipole from the positive charge towards the negative charge.

### ELECTRIC FIELD INTENSITY DUE TO A SHORT DIPOLE AT SOME GENERAL POINT

- Let \(AB\) be a short electric dipole of dipole moment \(\vec{p}\) (directed from \(B\) to \(A\)). We are interested to find the electric field at some general point \(P\). The distance of observation point \(P\) w.r.t. midpoint \(O\) of the dipole is \(r\) and the angle made by the line \(OP\) w.r.t. axis of dipole is \(\theta\).
- We know that dipole moment of a dipole is a vector quantity. It can be resolved into two rectangular components \(\vec{p}_1\) and \(\vec{p}_2\) as shown in Fig. 1.80, so that \(\vec{p} = \vec{p}_1 + \vec{p}_2\). The magnitudes of \(\vec{p}_1\) and \(\vec{p}_2\) are \(p_1 = p\cos\theta\) and \(p_2 = p\sin\theta\).
- It is clear from figure that point \(P\) lies on the axial line of dipole with moment \(\vec{p}\). Hence, the magnitude of the electric field intensity \(E\) at \(P\) due to \(\vec{p}_1\) is

![Diagram of dipole and electric field intensity components](image)

\[
E_1 = \frac{1}{4\pi\varepsilon_0} \times \frac{2p\cos\theta}{r^3} \text{(along } OC) \tag{i}
\]

Similarly, \(P\) lies on the equatorial line of dipole with moment \(\vec{p}_2\). Hence, the magnitude of electric field intensity \(E_2\) at \(P\) due to \(\vec{p}_2\) is

\[
E_2 = \frac{1}{4\pi\varepsilon_0} \times \frac{p\sin\theta}{r^3} \text{(opposite to } p_2) \tag{ii}
\]

Hence, the resultant intensity at \(P\) is \(\vec{E} = \vec{E}_1 + \vec{E}_2\).

The magnitude of \(\vec{E}\) is \(E = \sqrt{(E_1^2 + E_2^2)}\) (as \(\vec{E}_1\) and \(\vec{E}_2\) are mutually perpendicular).

or

\[
E = \frac{\sqrt{2p^2\cos^2\theta}}{4\pi\varepsilon_0 r^3} + \left(\frac{p^2\sin^2\theta}{4\pi\varepsilon_0 r^3}\right) = \frac{p}{4\pi\varepsilon_0 r^3} \sqrt{\cos^2\theta + \sin^2\theta} = \frac{p}{4\pi\varepsilon_0 r^3} \sqrt{1 + 3\cos^2\theta}
\]

- If the resultant field intensity vector \(\vec{E}\) makes an angle \(\phi\) with the direction of \(\vec{E}_1\), then
\[
\tan \phi = \frac{E_z}{E_i} = \frac{(\rho \sin \theta / 4\pi \varepsilon_0 r^3)}{(2 \rho \cos \theta / 4\pi \varepsilon_0 r^3)} = \frac{\tan \theta}{2}
\]

**Illustration 1.26** Three charges \(-q, +2q, -q\) are arranged on a line as shown in Fig. 1.81. Calculate the field at a distance \(r >> a\) on the line.

**Sol.** The field at point \(P\) is superposition of fields \(E_1, E_2, E_3\) due to each charge.

\[
\begin{align*}
E_1 &= -\frac{q}{4\pi \varepsilon_0 (r-a)^2} \hat{i}; \\
E_2 &= \frac{2q}{4\pi \varepsilon_0 r^2} \hat{i}; \\
E_3 &= -\frac{q}{4\pi \varepsilon_0 (r+a)^2} \hat{i}
\end{align*}
\]

Fig. 1.81

Now,
\[
E = E_1 + E_2 + E_3 = \frac{q}{4\pi \varepsilon_0} \left[ \frac{1}{(r-a)^2} + \frac{2}{r^2} - \frac{1}{(r+a)^2} \right] \hat{i}
\]

If \(r >> a\), we can use binomial approximation:
\[
(1 + \alpha)^n = 1 + n\alpha + \frac{n(n-1)}{2!} \alpha^2 + \cdots \text{ for } \alpha << 1
\]

Therefore,
\[
E = \frac{q}{4\pi \varepsilon_0 r^2} \left[ \left(1 - \frac{a}{r} \right)^2 + \frac{1}{(r+a)^2} \right] \hat{i}
\]

The charge in this problem may be considered as two dipoles placed close together. Such an arrangement of charge is called an electric quadrupole.

**Illustration 1.27** What is the force on a dipole of dipole moment \(p\) placed as shown in Fig. 1.82.

**Fig. 1.82**

**Sol.** Force on any \(q\) by dipole is
\[
F = qE_{dipole} = \frac{q}{4\pi \varepsilon_0} \frac{p}{a^3} \text{ downwards}
\]

So from the third law, force on dipole due to both charges is
\[
2F = \frac{2qp}{2\pi \varepsilon_0 a^3} \text{ upwards}
\]

**Net Force on a Dipole in a Non-Uniform Field**

Suppose an electric dipole with dipole moment \(p\) is placed in a non-uniform electric field \(E = E_x\) that points along the x-axis (Fig. 1.83). Let \(E\) depends only on \(x\). The electric field at the position of negative charge is \(E\) and at the position of positive charge \((E + \Delta E)\). Then, the net force acting on the dipole is

\[
F = q(E + \Delta E) - qE = q\Delta E = q \left[ \frac{\Delta E}{\Delta x} \right] \\
F = q\Delta x \frac{dE}{dx} = \frac{p}{\varepsilon_0} \frac{dE}{dx}
\]

where \(\frac{dE}{dx}\) is the gradient of the field in the x-direction.

**Illustration 1.28** Find the force on a small electric dipole of dipole moment \(p\) due to a point charge \(Q\) placed at a distance \(r\).

**Fig. 1.84**

**Sol.** Electric field of a point charge is a non-uniform electric field. Electric field at a distance \(x\) from the point charge is
\[
E = \frac{1}{4\pi \varepsilon_0} \frac{Q}{x^3} \Rightarrow \frac{dE}{dx} = -\frac{1}{4\pi \varepsilon_0} \frac{2Q}{x^4}
\]

The magnitude of force on the dipole is
\[
F = \left| p \frac{dE}{dx} \right| = -\frac{1}{4\pi \varepsilon_0} \frac{2Q}{r^3}, \text{ negative sign indicates that force is towards } Q \text{ or it is attractive.}
\]

**Alternatively:** The same result can be calculated as force on the point charge due to dipole which is the same as the force on dipole due to the point charge (Newton’s third law). The electric field of small dipole at a distance \(r\) is
\[
E = \frac{1}{4\pi \varepsilon_0} \frac{2p}{r^3}
\]

Hence, force on the point charge \(Q\) is
\[
F = EQ = \frac{1}{4\pi \varepsilon_0} \frac{2pQ}{r^3}
\]
**DIPOLE IN A UNIFORM ELECTRIC FIELD**

**Torque:** When a dipole is placed in a uniform field as shown in Fig. 1.85, the net force on it is \( \mathbf{F}_\perp = q\mathbf{E} + (-q)\mathbf{E} = 0 \)

![Fig. 1.85](image)

Hence, net force on a dipole is zero in a uniform electric field.

While the torque \( \tau = q\mathbf{E} \times d \sin \phi \)

i.e., \( \tau = p\mathbf{E} \sin \phi \) [as \( p =qd \)]

or \( \tau = \mathbf{p} \times \mathbf{E} \) (by electric field)

and \( \tau = \mathbf{E} \times \mathbf{p} \) (by us if the dipole is in equilibrium)

From the expression, it is clear that couple acting on a dipole is maximum (= \( p\mathbf{E} \)) when dipole is perpendicular \((\phi = 90^\circ)\) to the field and minimum (= 0) when dipole is parallel \((\phi = 0^\circ)\) or antiparallel \((\phi = 180^\circ)\) to the field.

By applying a torque, electric field tends to align a dipole in its own direction.

**Illustration 1.29** An electric dipole consists of two charges of 0.1 \( \mu \text{C} \) separated by a distance of 2.0 cm. The dipole is placed in an external field of 10\(^{6} \) \( \text{NC}^{-1} \). What maximum torque does the field exert on the dipole?

**Sol.** \( \tau = p\mathbf{E} \sin \theta = q\times 2a \times \mathbf{E} \sin \theta \). The value of \( \tau \) will be maximum when \( \sin \theta = 1 \)

\[ \tau_{\text{max}} = 10^{-7} \times 2 \times 10^{-2} \times 10^6 \times 1 = 2 \times 10^{-4} \text{ Nm} \]

**Concept Application Exercise 1.4**

1. State the following statements as true/false:
   a. An electric dipole is kept in a uniform electric field at some angle with it. It experiences a force but no torque.
   b. An electric dipole may experience a net force when it is placed in a non-uniform electric field.
   c. An electric dipole is kept in a non-uniform electric field.

2. Electric intensity due to an electric dipole varies with distance as \( \mathbf{E} \propto r^n \), where \( n \) is ______.

3. An electric dipole of moment \( \mathbf{p} \) is placed at the origin along the x-axis. The electric field \( \mathbf{E} \) at a point \( P \), whose position vector makes an angle \( \theta \) with the x-axis, will make an angle with x-axis which is ______.

4. Two point charges of +1 \( \mu \text{C} \) and -1 \( \mu \text{C} \) are separated by a distance of 100 \( \AA \). A point \( P \) is at a distance of 10 cm from the midpoint and on the perpendicular bisector of the line joining the two charges. Find the electric field at \( P \).

5. An electric dipole consists of two opposite charges of magnitude 2 \( \times 10^{-4} \) \( \text{C} \) each and separated by a distance of 3 cm. It is placed in an electric field of 2 \( \times 10^{6} \) \( \text{NC}^{-1} \). Determine the maximum torque on the dipole.

6. Three charges are arranged on the vertices of an equilateral triangle as shown in Fig. 1.87. Find the dipole moment of the combination.

7. The electric field at \( A \) due to dipole \( \mathbf{p} \) is perpendicular to \( \mathbf{p} \). the angle \( \theta \) is ______.

8. An electric dipole is formed by two particles fixed at the end of a light rod of length \( l \). The mass of each particle is \( m \) and the charges are -\( q \) and +\( q \). The system is placed in such a way that the dipole axis is parallel to a uniform electric field \( \mathbf{E} \) that exists in region. The dipole is slightly rotated about its centre and released. Show that for small angular displacement, the motion is S.H.M. Evaluate its time period.

9. A dipole consists of two particles carrying charges +2 \( \mu \text{C} \) and -2 \( \mu \text{C} \) and masses 1 and 2 kg, respectively, separated by a distance of 6 m. It is placed in a uniform electric field of 8 \( \times 10^{4} \) \( \text{Vm}^{-1} \). For small oscillations about its equilibrium position, find the angular frequency.

10. A small electric dipole of dipole moment \( P \) is placed near a point charge +\( Q \) as shown. Then, the net force on the dipole is towards ______.
Solved Examples

**Example 1.1**  A uniformly charged wire with linear charge density \( \lambda \) is laid in the form of a semicircle of radius \( R \). Find the electric field generated by the semicircle at the centre.

**Solution** We consider a differential element \( dl \) on the ring that subtends an angle \( d\theta \) at the centre of the ring, i.e., \( dl = Rd\theta \).

Charge on this element = \( dQ = \lambda Rd\theta \).

This element creates a field \( dE \) which makes an angle \( \theta \) at the centre as shown in Fig. 1.91.

![Fig. 1.91](image)

For each differential element in the upper half of the ring, there corresponds a symmetrically placed charge element in the lower half. The \( y \)-components of the field due to these symmetric elements cancel out and the \( x \)-components remain. We get

\[
dE_x = \frac{dQ}{4\pi \varepsilon_0 R^2} \cos \theta = \frac{\lambda (Rd\theta) \cos \theta}{4\pi \varepsilon_0 R^2}
\]

On integrating the expression for \( dE_x \) w.r.t. angle \( \theta \) in limits \( \theta = -\pi/2 \) to \( \theta = +\pi/2 \), we obtain

\[
E_x = \int_{-\pi/2}^{\pi/2} \frac{\lambda R}{4\pi \varepsilon_0 R^2} \cos \theta \, d\theta = \frac{\lambda}{2\pi \varepsilon_0 R^2} \]

In terms of the total charge, say \( Q \), on the ring, \( \lambda = Q/\pi R \) and we get \( E = Q/(2\pi \varepsilon_0 R) \).

If we consider the wire in the form of an arc as shown in Fig. 1.92, the symmetry consideration is not useful in cancelling out \( x \)- and \( y \)-components of the fields, if \( \theta_1 \) and \( \theta_2 \) are different. We will integrate \( dE_x \) as well as \( dE_y \) in limits \( \theta = -\theta_1 \) to \( \theta = +\theta_2 \).

![Fig. 1.92](image)

\[
E_x = \int_{-\theta_1}^{\theta_1} \frac{\lambda R}{4\pi \varepsilon_0 R^2} \cos \theta \, d\theta = \frac{\lambda}{4\pi \varepsilon_0 R} (\sin \theta_1 + \sin \theta_2)
\]

\[
E_y = \int_{-\theta_1}^{\theta_1} \frac{\lambda R}{4\pi \varepsilon_0 R^2} \sin \theta \, d\theta = \frac{\lambda}{4\pi \varepsilon_0 R} (\cos \theta_1 - \cos \theta_2)
\]

For a symmetrical arc, \( \theta_1 = \theta_2 \). Thus, \( E_y \) vanishes and

**Example 1.2**  A long wire with a uniform charge density \( \lambda \) is bent in two configurations shown in Fig. 1.93 (a) and (b). Determine the electric field intensity at point \( O \).

**Solution** In Fig. 1.93(a),

a. Field due to segment 1:

\[
\vec{E}_1 = \left( \frac{\lambda}{4\pi \varepsilon_0 R} \right) \hat{i} + \left( -\frac{\lambda}{4\pi \varepsilon_0 R} \right) \hat{j}
\]

![Fig. 1.93](image)

Field due to segment 2:

\[
\vec{E}_2 = \left( -\frac{\lambda}{4\pi \varepsilon_0 R} \right) \hat{i} + \left( \frac{\lambda}{4\pi \varepsilon_0 R} \right) \hat{j}
\]

![Fig. 1.94](image)

Field due to quarter shape wire segment 3 (3):

\[
\vec{E}_3 = \left( \frac{\lambda}{4\pi \varepsilon_0 R} \right) \hat{i} + \left( \frac{\lambda}{4\pi \varepsilon_0 R} \right) \hat{j}
\]

(C : \( \theta_1 = 90^\circ \); \( \theta_2 = 0^\circ \))

Resultant field is superposition of the fields due to each part, i.e.,

\[
\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3
\]

Substituting the values of \( \vec{E}_1, \vec{E}_2 \), and \( \vec{E}_3 \) in Eq. (i), we get

\[
\vec{E} = \left( \frac{\lambda}{4\pi \varepsilon_0 R} \right) \hat{i} + \left( \frac{\lambda}{4\pi \varepsilon_0 R} \right) \hat{j}
\]
Electrostatics and Current Electricity

Fig. 1.95

\[ |\vec{E}| = \left( \frac{\lambda}{4\pi\varepsilon_0 R} \right)^2 + \left( \frac{\lambda}{4\pi\varepsilon_0 R} \right)^2 = \frac{\sqrt{2}\lambda}{4\pi\varepsilon_0 R} \]

Here, \( E_x = E_y = -\frac{\lambda}{4\pi\varepsilon_0 R} \). Hence, the resultant field will make an angle of 45° with the axis.

b. Field due to segment 1

\[ \vec{E}_1 = \frac{\lambda}{4\pi\varepsilon_0 R} [\hat{j} - \hat{j}] \]

Field due to segment 2:

\[ \vec{E}_2 = -\frac{\lambda}{4\pi\varepsilon_0 R} \hat{i} \]

\[ \vec{E}_2 = -\frac{\lambda}{4\pi\varepsilon_0 R} [\hat{i} + \hat{j}] \]

\[ \vec{E}_3 = \frac{\lambda}{2\pi\varepsilon_0 R} \hat{j} \]

From the principle of superposition of electric fields,

\[ \vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = \frac{\lambda}{4\pi\varepsilon_0 R} [\hat{j} - \hat{j}] - \frac{\lambda}{4\pi\varepsilon_0 R} [\hat{i} + \hat{j}] \]

\[ + \frac{\lambda}{2\pi\varepsilon_0} \hat{j} = 0 \]

Hence, the net field is zero.

Example 1.3

A particle having charge that of an electron and mass \( 1.6 \times 10^{-30} \text{ kg} \) is projected with an initial speed \( u \) at an angle 45° to the horizontal from the lower plate of a parallel-plate capacitor as shown in the figure. The plates are sufficiently long and have separation of 2 cm. Find the maximum value of the velocity of the particle so that it does not hit the upper plate. Take electric field between the plates = \( 10^5 \text{ V/m} \) directed upwards.

Sol. Resolving the velocity of the particle parallel and perpendicular to the plate, we get

\[ u_x = u \cos 45° = \frac{u}{\sqrt{2}} \quad \text{and} \quad u_y = u \sin 45° = \frac{u}{\sqrt{2}} \]

Force on the charged particle in downward direction normal to the plate = \( eE \).

\[ \text{Fig. 1.97} \]

Therefore, acceleration \( a = eE/m \), where \( m \) is the mass of the charged particle.

The particle will not hit the upper plate, if the velocity component normal to the plate becomes zero before reaching it, i.e., \( 0 = -u_x - 2ay \) with \( y \leq d \), where \( d \) is the distance between the plates.

Therefore, the maximum velocity for the particle not to hit the upper plate (for this \( y = d = 2 \text{ cm} \)) is

\[ u_x = \sqrt{2ay} = \sqrt{\frac{2 \times 1.6 \times 10^{-10} \times 10^3 \times 2 \times 10^{-2}}{1.6 \times 10^{-30}}} \]

\[ = 2 \times 10^6 \text{ ms}^{-1} \]

\[ u_{\max} = u_x / \cos 45° = 2\sqrt{2} \times 10^6 \text{ ms}^{-1} \]

Example 1.4

A particle of mass \( m \) and charge \( q \) is released from rest in a uniform field of magnitude \( E \). The uniform field is created between two parallel-plates of charge densities \( +\sigma \) and \( -\sigma \), respectively. The particle accelerates towards the other plate a distance \( d \) apart. Determine the speed at which it strikes the opposite plate.

\[ \text{Fig. 1.98} \]

Sol. The applied electric field is \( \vec{E} = -E_0 \hat{j} \).

The force experienced by the charge \( q \) is, \( \vec{F} = q\vec{E} = -qE_0 \hat{j} \).

The force is constant, and so the acceleration is constant as well, i.e.,

\[ \vec{a} = \frac{\vec{F}}{m} = -\frac{qE_0}{m} \hat{j} \]

Because of constant acceleration, the particle moves in the –ve \( y \)-direction; the problem is analogous to motion of a mass released from rest in a gravitational field.
From equations of motion, we get
\[ v_y = v_{y0} + a_y t = 0 - \frac{q E_0}{m} t \]  
(i)

And
\[ y = y_0 + v_y t + \frac{1}{2} a_y t^2; 0 = y_0 + \frac{1}{2} \frac{q E_0}{m} t^2 \]  
(ii)

Particle starts at \( y_0 = d \) and impact occurs at \( y = 0 \)

From Eq. (ii), we get
\[ t = \left( \frac{2d}{q E_0} \right)^{1/2} \]

From Eq. (i), we get
\[ v_y = -\frac{q E_0}{m} \left( \frac{2d}{q E_0} \right)^{1/2} = -\sqrt{\frac{2q E_0 d}{m}} \]

**Example 1.4**

Two balls of charges \( q_1 \) and \( q_2 \) initially have a velocity of the same magnitude and direction. After a uniform electric field has been applied for a certain time interval, the direction of first ball changes by 60° and the velocity magnitude is reduced by half. The direction of velocity of the second ball changes by 90°. In what ratio will the velocity of the second ball change? Determine the magnitude of the charge-to-mass ratio of the second ball if it is equal to \( \alpha \) for the first ball. Ignore the electrostatic interaction between the balls.

---

**Fig. 1.99**

**Sol.** Let the electric field on each ball be given by
\[ E = E_0 i + E_0 j \]

From impulse–momentum equation, we have
\[ \text{Impulse} = \text{Change in momentum} \]

Let the final velocities of the balls be \( v_1 \) and \( v_2 \). Noting that \( v_1 = \frac{v}{2} \), we have
\[ q_1 (E_0 i + E_0 j) \Delta t = m_1 \left( \frac{v}{2} \cos 60^\circ + \frac{v}{2} \sin 60^\circ \right) - m_1 v_1 \]
(i)

\[ q_2 (E_0 i + E_0 j) \Delta t = m_2 \left( v \cos 90^\circ + v \sin 90^\circ \right) - m_2 v_2 \]
(ii)

On comparing the \( x \)– and \( y \)-components on both sides of Eq. (i), we get
\[ \frac{q_1}{m_1} E_0 \Delta t = -\frac{3}{4} v \]
and
\[ \frac{q_1}{m_1} E_0 \Delta t = \frac{\sqrt{3}}{4} v \]
(iii)

Similarly, for Eq. (ii), we get
\[ \frac{q_2}{m_2} E_0 \Delta t = -v \]
and
\[ \frac{q_2}{m_2} E_0 \Delta t = v_2 \]
(iv)

From Eq. (iii) and (iv), by dividing the equations expression for \( x \)-components, we get
\[ \frac{q_1}{m_1} = \frac{3}{4} \]
(v)

or
\[ \frac{q_2}{m_2} = \frac{4 q_1}{3 m_1} = \frac{4 \alpha}{3} \]

Also
\[ \frac{q_1/m_1}{q_2/m_2} = \frac{\sqrt{3} v}{4 v_2} = \frac{3}{4} \Rightarrow \frac{v_2}{v} = \frac{\sqrt{3}}{3} \]

**Example 1.5**

A rigid insulated wire frame in the form of a right-angled triangle \( ABC \) is set in a vertical plane as shown in the figure. Two beads of equal masses \( m \) each and carrying charges \( q_1 \) and \( q_2 \) are connected by a cord of length \( l \) and slide without friction on the wires.

(IIT-JEE, 1978)

**Fig. 1.100**

Considering the case when the beads are stationary
(i) the angle \( \alpha \)
(ii) the tension in the cord
(iii) the normal reaction on the beads

If the cord is now cut what are the value of the charges for which the beads continue to remain stationary?

Sol. Tension and electrostatic force are in opposite direction and along the string. Now each bead is in equilibrium under the concurrent forces: Normal reaction (N), weight (mg) and the resultant of tension and electrostatic force, i.e., \( T - F_e \)

where
\[ F_e = \frac{q_1 q_2}{4 \pi \epsilon_0} \frac{1}{r^2} \]

Applying Lami's theorem for both the beads, we get
\[ \frac{N_1}{\sin (120^\circ - \alpha)} = \frac{mg}{\cos \alpha} = \frac{T - F_e}{\cos 60^\circ} \]
(i)

\[ \frac{N_2}{\sin (60^\circ + \alpha)} = \frac{mg}{\sin \alpha} = \frac{T - F_e}{\cos 30^\circ} \]
(ii)

Dividing Eq. (i) by Eq. (ii), we have
\[ \frac{\tan \alpha}{\cos 30^\circ} = \frac{\cos 60^\circ}{\cos 30^\circ} = \frac{\sqrt{3}}{3}, \text{ therefore } \alpha = 60^\circ \]
(i)

\[ T = F_e + mg = \frac{1}{4 \pi \epsilon_0} \frac{q_1 q_2}{l^2} + mg \]
(ii)

\[ N_1 = \sqrt{3} \text{mg} \quad \text{and} \quad N_2 = mg \]

From Eq. (iii), \( T = 0 \) when string is cut
or
\[ q_1 q_2 = -\left(4 \pi \epsilon_0 \right) mg l^2 \]
EXERCISES

1. Calculate the number of electrons in a small, electrically neutral silver pin that has a mass of 10.0 g. Silver has 47 electrons per atom, and its molar mass is 107.87 g mol⁻¹.

2. A charged particle of radius $5 \times 10^{-7}$ m is located in a horizontal electric field of intensity $6.28 \times 10^2$ V m⁻¹. The surrounding medium has coefficient of viscosity $\eta = 1.6 \times 10^3$ N s m⁻². The particle starts moving under the effect of electric field and finally attains a uniform horizontal speed of 0.02 ms⁻¹. Find the number of electrons on it. Assume gravity free space.

3. Suppose that 1.00 g of hydrogen is separated into electrons and protons. Suppose also that the protons are placed at the Earth’s north pole and the electrons are placed at the south pole. What is the resulting compression force on the Earth? (Given that radius of earth is 6400 km).

4. Two identical conducting small spheres are placed with their centres 0.300 m apart. One is given a charge of 12.0 nC and the other a charge of -18.0 nC.
   a. Find the electric force exerted by one sphere on the other.
   b. If the spheres are connected by a conducting wire, find the electric force between the two after they attain equilibrium.

5. Four equal point charges, each of magnitude +Q, are to be placed in equilibrium at the corners of a square. What should be the magnitude and sign of the point charge that should be placed at the centre of square to do this job?

6. Two point electric charges of values $q$ and $2q$ are kept at a distance $d$ apart from each other in air. A third charge $Q$ is to be kept along the same line in such a way that the net force acting on $q$ and $2q$ is zero. Find the location of the third charge from charge ‘q’.

7. Two fixed point charges +4e and -e unit are separated by a distance ‘d’. Where the third point charge should be placed from +4e charge for it to be in equilibrium?

8. Two identical particles are charged and held at a distance of 1 m from each other. They are found to be attracting each other with a force of 0.027 N. Now, they are connected by a conducting wire, so that charge flows between them. When the charge flow stops, they are found to be repelling each other with a force of 0.009 N. Find the initial charge on each particle.

9. Two similarly and equally charged identical metal spheres $A$ and $B$ repel each other with a force of $2 \times 10^{-5}$ N. A third identical uncharged sphere $C$ is touched with $A$ and then placed at the mid-point between $A$ and $B$. Find the net electric force on $C$.

10. Three point charges of +2 $\mu$C, -3 $\mu$C and -3 $\mu$C are kept at the vertices $A$, $B$ and $C$, respectively, of an equilateral triangle of side 20 cm as shown in the figure. What should be the sign and magnitude of the charge $(q)$ to be placed at the midpoint $(M)$ of side $BC$ so that the charge at $A$ remains in equilibrium?

Fig. 1.102

11. Two small beads having positive charges $3q$ and $q$ are fixed at the opposite ends of a horizontal, insulating rod, extending from the origin to the point $x = d$. As shown in the figure, a third small charged bead is free to slide on the rod. At which position is the third bead in equilibrium? Can it be in stable equilibrium?

Fig. 1.103

12. A copper atom consists of copper nucleus surrounded by 29 electrons. The atomic weight of copper is 63.5 g mol⁻¹. Let us now take two pieces of copper each weighing 10 g. Let us consider one electron from one piece is transferred to another for every 1000 atoms in a piece.
   a. Find the magnitude of charge appearing on each piece.
   b. What will be the Coulomb force between the two pieces after the transfer of electrons if they are 10 cm apart?

   [Avogadro's number = $6 \times 10^{23}$ mol⁻¹]

13. A flat square sheet of charge of side 50 cm carries a uniform surface charge density. An electron 0.5 cm from a point near the centre of the sheet experiences a force of $1.8 \times 10^{-12}$ N directed away from the sheet. Determine the total charge on the sheet.

14. A particle of mass $9 \times 10^{-31}$ kg having a negative charge of $1.6 \times 10^{-19}$ C is projected horizontally with a velocity of $10^6$ m s⁻¹ into a region between two infinite horizontal parallel plates of metal. The distance between the plates is $d = 0.3$ cm and the particle enters 0.1 cm below the top plate. The top and bottom plates are connected, respectively, to the positive and negative terminals of a 30 V battery. Find the components of the velocity of the particle just before it hits one of the plates.

Fig. 1.104
15. Point charges $q$ and $-q$ are located at the vertices of a square with diagonals $2l$ as shown in the figure. Evaluate the magnitude of the electric field strength at a point located symmetrically with respect to the vertices of the square at a distance $x$ from the centre.

![Fig. 1.105](image)

16. Two mutually perpendicular long straight conductors carrying uniformly distributed charges of linear charge densities $\lambda_1$ and $\lambda_2$ are positioned at a distance $a$ from each other. How does the interaction between the rods depend on $a$?

![Fig. 1.106](image)

17. A ring of radius 0.1 m is made out of a thin metallic wire of area of cross section $10^{-6}$ m$^2$. The ring has a uniform charge of $\pi$ coulombs. Find the charge in the radius of the ring when a charge of $10^{-4}$ C is placed at the centre of the ring. Young’s modulus of the metal is $2 \times 10^{11}$ Nm$^{-2}$.

18. A charged cork ball of mass $m$ is suspended on a light string in the presence of a uniform electric field as shown in the figure. When $E = (A\hat{i} + B\hat{j}) NC^{-1}$, where $A$ and $B$ are positive numbers, the ball is in equilibrium at the $\theta$. Find (a) the charge on the ball and (b) the tension in the string.

![Fig. 1.107](image)

19. A ring of radius $R$ has charge $-Q$ distributed uniformly over it. Calculate the charge that should be placed at the centre of the ring such that the electric field becomes zero at a point on the axis of the ring distant ‘$R$’ from the centre of the ring.

20. Two identical small equally charged conducting balls are suspended from long threads secured at one point. The charges and masses of the balls are such that they are in equilibrium when the distance between them is $a$ (the length of thread $L \gg a$). Then one of the balls is discharged. What will be the distance $b$ ($b \ll L$) between the balls when equilibrium is restored?

21. Two point charges $Q_a$ and $Q_b$ are positioned at points $A$ and $B$. The field strength to the right of charge $Q_a$ on the line that passes through the two charges varies according to a law that is represented schematically in the figure accompanying the problem (without employing a definite scale). The field strength is assumed to be positive if its direction coincides with the positive direction of the $x$-axis. The distance between the charges is $l = 21$ cm (Fig. 1.108). Find

![Fig. 1.108](image)

a. the signs of the charges
b. the ratio between the absolute values of charges $Q_a$ and $Q_b$
c. the coordinate $x$ of the point where the field strength is maximum

22. Two semicircular wires $ABC$ and $ADC$, each of radius ‘$R$’, are lying on $x-y$ and $x-z$ plane, respectively, as shown in Fig. 1.109. If the linear charge density of the semicircular parts and straight parts is $\lambda$, find the electric field intensity $E$ at the origin.

![Fig. 1.109](image)

23. An infinite wire having linear charge density $\lambda$ is arranged as shown in Fig 1.110. A charge particle of mass $m$ and charge $q$ is released from point $P$. Find the initial acceleration of the particle (at $t = 0$) just after the particle is released.

![Fig. 1.110](image)
24. Two similar balls, each of mass $m$ and charge $q$, are hung from a common point by two silk threads, each of length $l$ (Fig. 1.111). Prove that separation between the balls is

$$x = \left(\frac{q^2 l}{2\pi \varepsilon_0 mg}\right)^{\frac{1}{3}}, \text{ if } \theta \text{ is small.}$$

![Fig. 1.111](image_url)

25. Three equal negative charges, $-q$, each, form the vertices of an equilateral triangle. A particle of mass $m$ and a positive charge $q$, is constrained to move along a line perpendicular to the plane of triangle and through its centre which is at a distance $r$ from each of the negative charges as shown in the figure. The whole system is kept in gravity free space. Find the period of vibration of the particle for small displacement from equilibrium position.

![Fig. 1.112](image_url)

26. A ball of radius $R$ carries a positive charge whose volume density at a point is given as $\rho = \rho_0 (1 - r/R)$, where $\rho_0$ is a constant and $r$ is the distance of the point from the centre. Assuming the permittivity of the ball and the environment to be equal to unity, find
a. the magnitude of the electric field strength as a function of the distance $r$ both inside and outside the ball
b. the maximum intensity $E_{\text{max}}$ and the corresponding distance $r_{\text{max}}$

4. The dimensional formula of electric intensity is
a. $MLT^{-2}A^{-1}$  
   b. $MLT^{-2}A^{-1}$
   c. $MLT^{-2}A^{-1}$  
   d. $MLT^{-2}A^{-2}$

5. The dielectric constant $K$ of an insulator can be
a. $-1$  
   b. $0$  
   c. $0.5$  
   d. $5$

6. Choose the correct statement:
a. The total charge of the universe is constant.
b. The total number of the charged particles is constant.
c. The total positive charge of the universe remains constant.
d. The total negative charge of the universe remains constant.

7. Two neutrons are placed at some distance apart from each other. They will
a. attract each other  
   b. repel each other  
   c. neither attract nor repel each other  
   d. cannot say

8. When a soap bubble is charged, its size
a. increases  
   b. decreases  
   c. remains the same  
   d. increases if it is given positive charge and decreases if it is given negative charge

9. Two point charges at certain distance apart in air repel each other with a force $F$. A glass plate is introduced between the charges. The force becomes $F'$, where
a. $F' < F$  
   b. $F' = F$  
   c. $F' > F$  
   d. data is insufficient

10. There are two charges $+1$ $\mu$C and $+5$ $\mu$C. The ratio of the forces (force on one due to other) acting on them will be
a. $1:1$  
   b. $1:2$  
   c. $1:3$  
   d. $1:4$

11. Two point charges $Q_1$ and $Q_2$ are $3$ m apart, and their sum of charges is $10$ $\mu$C. If force of attraction between them is $0.075$ N, then the values of $Q_1$ and $Q_2$, respectively, are
a. $5$ $\mu$C, $5$ $\mu$C  
   b. $15$ $\mu$C, $-5$ $\mu$C  
   c. $5$ $\mu$C, $15$ $\mu$C  
   d. $-15$ $\mu$C, $5$ $\mu$C

12. A certain charge ‘$Q$’ is to be divided into two parts $q$ and $Q - q$. What is the relationship ‘$Q$’ and ‘$q$’ if the two parts, placed at a given distance ‘$r$’ apart, are to have maximum Coulomb repulsion?
   a. $q = Q/2$  
   b. $q = Q/3$  
   c. $q = 2Q/2$  
   d. $q = Q/4$

13. Three charged particles are placed on a straight line as shown in the figure. $q_1$ and $q_2$ are fixed but $q_3$ can be moved. Under the action of the forces from $q_1$ and $q_2$, $q_3$ is in equilibrium. What is the relation between $q_1$ and $q_2$?

![Fig. 1.113](image_url)

14. Two particles $A$ and $B$ (B is right of A) having charges $8 \times 10^{-6}$ C and $-2 \times 10^{-6}$ C, respectively, are held fixed with separation of $20$ cm. Where should a third charged particle be placed so that it does not experience a net electric force?
   a. $5$ cm right of $B$  
   b. $5$ cm left of $A$  
   c. $20$ cm left of $A$  
   d. $20$ cm right of $B$
15. Five balls numbered 1, 2, 3, 4, 5 are suspended using separate threads. The balls (1, 2), (2, 4) and (4, 1) show electrostatic attraction, while balls (2, 3) and (4, 5) show repulsion. Therefore, ball 1 must be 
   a. negatively charged  
   b. positively charged  
   c. neutral  
   d. made of metal
16. Electric charges A and B repel each other. Electric charges B and C also repel each other. If A and C are held close together, they will 
   a. attract  
   b. repel  
   c. not affect each other  
   d. none of these
17. Two point charges repel each other with a force of 100 N. One of the charges is increased by 10% and the other is reduced by 10%. The new force of repulsion at the same distance would be 
   a. 100 N  
   b. 121 N  
   c. 99 N  
   d. none of these
18. Three charges +\( \Omega \), +\( \Omega \), and +\( q \) are placed on a straight line such that \( q \) is somewhere in between +\( \Omega \) and +\( \Omega \). If this system of charges is in equilibrium, what should be the magnitude and sign of charge \( q \)? 
   a. \( \frac{\Omega \Omega}{\sqrt{\Omega + \sqrt{\Omega \Omega}}} \), +ve  
   b. \( \frac{\Omega + \Omega}{2} \), +ve  
   c. \( \frac{\Omega \Omega}{\sqrt{\Omega + \sqrt{\Omega \Omega}}} \), -ve  
   d. \( \frac{\Omega + \Omega}{2} \), -ve
19. Two positive and equal charges are fixed at a certain distance. A third small charge is placed in between the two charges and it experiences zero net force due to the other two. 
   a. The equilibrium is stable if small charge is positive  
   b. The equilibrium is stable if small charge is negative  
   c. The equilibrium is always stable  
   d. The equilibrium is not stable
20. An isolated charge \( q \) of mass \( m \) is suspended freely by a thread of length \( l \). Another charge \( q_2 \) is brought near it (\( r > l \)). When \( q_1 \) is in equilibrium, tension in thread will be 
   a. \( mg \)  
   b. >\( mg \)  
   c. <\( mg \)  
   d. none of these
21. Three equal charges, each +\( q \), are placed on the corners of an equilateral triangle of side \( a \). Then, the coulomb force experienced by one charge due to the rest of the two is 
   a. \( \frac{kq^2}{a^2} \)  
   b. \( 2kq^2/a^2 \)  
   c. \( \sqrt{3} \frac{kq^2}{a^2} \)  
   d. zero
22. A positively charged ball hangs from a long silk thread. Electric field at a certain point (at the same horizontal level of ball) due to this charge is \( E \). Let us put a positive test charge \( q_o \) at this point and measure \( F/q_o \) on this charge. Then, \( E \) 
   a. >\( F/q_o \)  
   b. <\( F/q_o \)
23. Electric field near a straight wire carrying a steady current is 
   a. proportional to the distance from the wire  
   b. proportional to inverse square of the distance from the wire  
   c. inversely proportional to the distance from the wire  
   d. zero
24. A force of 2.25 N acts on a charge of \( 15 \times 10^{-4} \) C. Calculate the intensity of electric field at the point. 
   a. 1500 NC\(^{-1} \)  
   b. 150 NC\(^{-1} \)  
   c. 15000 NC\(^{-1} \)  
   d. none of these
25. An \( \alpha \) particle is situated in an electric field of strength \( 15 \times 10^4 \) NC\(^{-1} \). Force acting on it is 
   a. \( 4.8 \times 10^{-12} \) N  
   b. \( 4.8 \times 10^{-14} \) N  
   c. \( 48 \times 10^{-14} \) N  
   d. none of these
26. Two particles of masses in the ratio 1 : 2, with charges in the ratio 1 : 1, are placed at rest in a uniform electric field. They are released and allowed to move for the same time. The ratio of their kinetic energies will be finally 
   a. 2 : 1  
   b. 8 : 1  
   c. 4 : 1  
   d. 1 : 4
27. Three equal charges, each +\( q \), are placed on the corners of an equilateral triangle. The electric field intensity at the centroid of the triangle is 
   a. \( kq/q^2 \)  
   b. \( 3kq/q^2 \)  
   c. \( \sqrt{3} kq/q^2 \)  
   d. zero
28. A point charge of \( 100 \) \( \mu \)C is placed at \( 3\ell + 4\ell \) m. Find the electric field intensity due to this charge at a point located at \( 9\ell /2 \) m. 
   a. \( 8000 \) Vm\(^{-1} \)  
   b. \( 9000 \) Vm\(^{-1} \)  
   c. 2250 Vm\(^{-1} \)  
   d. 4500 Vm\(^{-1} \)
29. Electric lines of force 
   a. exist everywhere  
   b. exist only in the immediate vicinity of electric charges  
   c. exist only when both positive and negative charges are near one another  
   d. are imaginary
30. Two charges \( Q_1 = 18 \) \( \mu \)C and \( Q_2 = -2 \) \( \mu \)C are separated by a distance \( R \) and \( Q_1 \) is to the left of \( Q_2 \). The distance of the point where the net electric field is zero is 
   a. between \( Q_1 \) and \( Q_2 \)  
   b. left of \( Q_1 \) at \( R/2 \)  
   c. right of \( Q_1 \) at \( R \)  
   d. right of \( Q_2 \) at \( R/2 \)
31. An oil drop, carrying six electronic charges and having a mass of \( 1.6 \times 10^{-12} \) g, falls with some terminal velocity in a medium. What magnitude of vertical electric field is required to make the drop move upwards with the same speed as it was formerly moving downwards with? Ignore buoyancy. 
   a. \( 10^6 \) NC\(^{-1} \)  
   b. \( 10^4 \) NC\(^{-1} \)  
   c. \( 3.3 \times 10^4 \) NC\(^{-1} \)  
   d. \( 3.3 \times 10^6 \) NC\(^{-1} \)
32. What is the largest charge a metal ball of 1 mm radius can hold? Dielectric strength of air is \( 3 \times 10^6 \) Vm\(^{-1} \). 
   a. 3 nC  
   b. 1/3 nC  
   c. 2 nC  
   d. 1/2 nC
33. Five point charges, +\( q \) each, are placed at the five vertices of a regular hexagon. The distance of centre of the hexagon from any of the vertices is \( a \). The electric field at the centre of the hexagon is
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34. A ring of charge with radius 0.5 m has 0.002 \( \pi \) m gap. If the ring carries a charge of +1 C, the electric field at the centre is

\[ \frac{q}{4\pi\varepsilon_0 a^2} \]
\[ \frac{q}{8\pi\varepsilon_0 a^2} \]
\[ \frac{q}{16\pi\varepsilon_0 a^2} \]
\[ \text{zero} \]

35. A block of mass \( m \) containing a net negative charge \(-q\) is placed on a frictionless horizontal table and is connected to a wall through an unstretched spring of spring constant \( k \) as shown. If the horizontal electric field \( E \) parallel to the spring is switched on, then the maximum compression of the spring is

\[ \sqrt{\frac{qE}{k}} \]
\[ 2qE/k \]
\[ qE/k \]
\[ \text{zero} \]

36. Figure 1.117 shows the electric lines of force emerging from a charged body. If the electric fields at \( A \) and \( B \) are \( E_a \) and \( E_b \), respectively, and if the distance between \( A \) and \( B \) is \( r \), then

\[ E_a > E_b \]
\[ E_a < E_b \]
\[ E_a = E_b/r \]
\[ E_a = E_b/r^2 \]

37. If an electron has an initial velocity in a direction different from that of a uniform electric field, the path of the electron is

a. a straight line
b. a circle
c. an ellipse
d. a parabola

38. A point charge \( q \), is moved along a circular path of radius \( r \) in the electric field of another point charge \( q_2 \) at the centre of the path. The work done by the electric field on the charge \( q \), in half revolution is

a. zero
b. positive
c. negative
d. none of these

39. A spherically conducting ball is suspended by a grounded conducting thread. A positive point charge is moved near the ball. The ball will

a. be attracted to the point charge and swing toward it.
b. be repelled from the point charge and swing away from it.
c. not be affected by the point charge

d. none of these

40. Three +ve charges of equal magnitude \( q \) are placed at the vertices of an equilateral triangle of side \( l \). How can the system of charges be placed in equilibrium?

a. By placing a charge \( Q = (-q\sqrt{3}) \) at the centroid of the triangle
b. By placing a charge \( Q = (q\sqrt{3}) \) at the centroid of the triangle
c. By placing a charge \( Q = q \) at a distance \( l \) from all the three charges
d. By placing a charge \( Q = -q \) above the plane of the triangle at a distance \( l \) from all the three charges

41. In the figure, two equal positive point charges \( q_1 = q_2 = 2.0 \mu\text{C} \) interact with a third point charge \( Q = 4.0 \mu\text{C} \). The magnitude as well as direction of the net force on \( Q \) is

\[ q_1 = 2.0 \mu\text{C} \]
\[ q_2 = 2.0 \mu\text{C} \]
\[ Q = 4.0 \mu\text{C} \]

42. Three identical spheres, each having a charge \( q \) and radius \( R \), are kept in such a way that each touches the other two. The magnitude of the electric force on any sphere due to other two is

\[ \frac{1}{4\pi\varepsilon_0} \left( \frac{q}{R} \right)^2 \]
\[ \frac{\sqrt{3}}{4\pi\varepsilon_0} \left( \frac{q}{R} \right)^2 \]
\[ \frac{\sqrt{3}}{16\pi\varepsilon_0} \left( \frac{q}{R} \right)^2 \]
\[ \frac{\sqrt{3}}{16\pi\varepsilon_0} \left( \frac{q}{R} \right)^2 \]

43. Five point charges, each of value \(+q\), are placed on five vertices of a regular hexagon of side \( L \). The magnitude of the force on a point charge of value \(-q\) coulomb placed at the centre of the hexagon is

\[ \frac{1}{\pi\varepsilon_0} \left( \frac{q}{L} \right)^3 \]
\[ \frac{2}{\pi\varepsilon_0} \left( \frac{q}{L} \right)^3 \]
\[ \frac{1}{2\pi\varepsilon_0} \left( \frac{q}{L} \right)^2 \]
\[ \frac{1}{4\pi\varepsilon_0} \left( \frac{q}{L} \right)^2 \]

44. It is required to hold equal charges \( q \) in equilibrium at the corners of a square. What charge when placed at the center of the square will do this?

\[ \frac{-q}{2(1+2\sqrt{2})} \]
\[ \frac{q}{2(1+2\sqrt{2})} \]
45. A point charge \( q = -8.0 \text{ nC} \) is located at the origin. The electric field (in NC\(^{-1}\)) vector at the point \( x = 1.2 \text{ m}, y = -1.6 \text{ m} \) as shown in Fig.1.119 is:

a. \(-14.4\hat{i} + 10.8\hat{j}\)  

b. \(-14.4\hat{i} - 10.8\hat{j}\)  

c. \(-10.8\hat{i} + 14.4\hat{j}\)  

d. \(-10.8\hat{i} - 14.4\hat{j}\)

46. A positive point charge 50 \( \mu\text{C} \) is located in the plane xy at a point with radius vector \( \mathbf{r} = 2\hat{i} + 3\hat{j} \). The electric field vector \( \mathbf{E} \) at a point with radius vector \( \mathbf{r} = 8\hat{i} - 5\hat{j} \), where \( r_0 \) and \( r \) are expressed in metre, is:

a. \((1.4\hat{i} - 2.6\hat{j}) \text{ kNC}^{-1}\)  

b. \((1.4\hat{i} + 2.6\hat{j}) \text{ kNC}^{-1}\)  

c. \((2.7\hat{i} - 3.6\hat{j}) \text{ kNC}^{-1}\)  

d. \((2.7\hat{i} + 3.6\hat{j}) \text{ kNC}^{-1}\)

47. Four identical charges \( Q \) are fixed at the four corners of a square of side \( a \). The electric field at a point \( P \) located symmetrically at a distance \( a/\sqrt{2} \) from the centre of the square is:

a. \( Q/2\sqrt{2}\pi\varepsilon_0 a^3 \)  

b. \( Q/2\sqrt{2}\pi\varepsilon_0 a^3 \)  

c. \( 2\sqrt{2}Q/\pi\varepsilon_0 a^2 \)  

d. \( 2\sqrt{2}Q/\pi\varepsilon_0 a^3 \)

48. A thin glass rod is bent into a semicircle of radius \( r \). A charge \( +Q \) is uniformly distributed along the upper half and a charge \( -Q \) is uniformly distributed along the lower half, as shown in Fig.1.120. The electric field \( E \) at \( P \), the centre of the semicircle, is:

a. \( Q/\pi\varepsilon_0 r^2 \)  

b. \( 2Q/\pi\varepsilon_0 r^2 \)  

c. \( 4Q/\pi\varepsilon_0 r^2 \)  

d. \( Q/\pi\varepsilon_0 r^2 \)

49. A system consists of a thin charged wire ring of radius \( r \) and a very long uniformly charged wire oriented along the axis of the ring, with one of its ends coinciding with the centre of the ring. The total charge on the ring is \( q \) and the linear charge density on the straight wire is \( \lambda \). The interaction force between the ring and the wire is:

a. \( \frac{q\lambda}{4\pi\varepsilon_0 r^2} \)  

b. \( \frac{\lambda q}{2\pi\varepsilon_0 r} \)  

c. \( \frac{\sqrt{2}\lambda q}{\pi\varepsilon_0 r} \)  

d. \( \frac{4\lambda q}{\pi\varepsilon_0 r} \)

50. Find the electric field vector at \( P (a, a, a) \) due to three infinitely long lines of charges along the x-, y- and z-axes respectively. The charge density, i.e., charge per unit length of each wire is \( \lambda \).

51. A particle of mass \( m \) and charge \( -q \) moves diametrically through a uniformly charged sphere of radius \( R \) with total charge \( Q \). The angular frequency of the particle's simple harmonic motion, if its amplitude \( < R \), is given by:

a. \( \frac{1}{4\pi\varepsilon_0 mR} \)  

b. \( \frac{1}{4\pi\varepsilon_0 mR^2} \)  

c. \( \frac{1}{A\pi\varepsilon_0 mR^2} \)  

d. \( \frac{1}{4\pi\varepsilon_0 qR} \)

52. A particle of mass \( m \) carrying a positive charge \( q \) moves simple harmonically along x-axis under the action of a varying electric field \( E \) directed along x-axis. The motion of the particle is confined between \( x = 0 \) and \( x = 2l \). The angular frequency of the motion is \( \omega \). Then, which of the following is correct?

a. \( qE = -ma^2(x - l) \)  

b. \( qE = ma^2(x - l) \)  

c. Electric field to the right of origin is directed along +ve x-axis for all values of \( x \).  

d. Electric field to the right of origin is directed along -ve x-axis for all values of \( x \).

53. A circular ring carries a uniformly distributed positive charge and lies in X–Y plane with centre at origin of coordinate system. If \( q \) at a point \( (0, 0, z) \) the electric field is \( E \), then which of the following graphs is correct?
1. For the arrangement shown in Fig. 1.122, the two positive charges, +Q each, are fixed. Mark out the correct statement(s) regarding a third charged particle q placed at midpoint P that can be displaced along or perpendicular to the line connecting the charges.

- The particle will perform S.H.M. for x << a.
- The particle will oscillate about P but not harmonically for any x.
- The particle will perform S.H.M. for y << a.
- The particle will oscillate about P but not harmonically for y comparable to a.

2. A particle of mass m and charge −q has been projected from ground as shown in Fig. 1.123. Mark out the correct statement(s). XY plane is vertical.

- The path of motion of the particle may be parabolic.
- The path of motion of the particle may be a straight line.
- Time of flight of the particle is \(2a\sin \theta / g\).
- Range of motion of the particle can be less than, greater than or equal to \((a^2 \sin 2\theta) / g\).

3. For the arrangement shown in Fig. 1.124, the two point charges are in equilibrium. The infinite wire is fixed in the horizontal plane and the two point charges are placed one above and the other below the wire.

Considering the gravitational effect of the earth, the nature of \(q_1\) and \(q_2\) can be

- \(q_1 \rightarrow +ve, q_2 \rightarrow +ve\)  
- \(q_1 \rightarrow +ve, q_2 \rightarrow -ve\)  
- \(q_1 \rightarrow -ve, q_2 \rightarrow -ve\)  
- \(q_1 \rightarrow -ve, q_2 \rightarrow +ve\)

**Assertion-Reasoning Type**

In the following questions, each question contains Statement I (Assertion) and Statement II (Reason). Each question has four choices (a), (b), (c), and (d), out of which ONLY ONE is correct.

- Statement I is True, Statement II is True; Statement II is a correct explanation for Statement I.
- Statement I is True, Statement II is True; Statement II is NOT a correct explanation for Statement I.
- Statement I is True, Statement II is False.
- Statement I is False, Statement II is True.

1. Statement I: If a point charge is rotated in a circle around another charge at the centre of circle, the work done by electric field will be zero.

Statement II: Work done is equal to dot product of force and displacement.

2. Statement I: A positive point charge initially at rest in a uniform electric field starts moving along electric lines of forces. (Neglect all other forces except electric forces.)

Statement II: A point charge released from rest in an electric field always moves along the lines of force.


Statement II: A body acquires +ve charge when it loses electrons.

4. Statement I: Two similarly charged bodies may attract each other.

Statement II: When charge on one body (Q) is much greater than that on another (q) and they are close enough to each other, then force of attraction between Q and induced charges exceeds the force of repulsion between Q and q.

5. Statement I: Charge is quantized because only integral number of electrons can be transferred.

Statement II: There is no possibility of transfer of some fraction of electron.
For Problems 1–2
Two small identical conducting balls A and B of charges of +10 μC and +30 μC, respectively, are kept at a separation of 50 cm. These balls have been connected by a wire for a short time.
1. The final charge on each of the balls A and B will be
   a. 10 μC and 30 μC, respectively
   b. 20 μC on each ball
   c. 30 μC and 10 μC, respectively
   d. −40 μC and 80 μC, respectively
2. The force of interaction between the balls is
   a. 28.8 N  b. 32.6 N  c. 14.4 N  d. 72 N

For Problems 3–4
Two free point charges A and B having charges +q and +4q, respectively, are a distance l apart. A third charge is so placed that the entire system is in equilibrium.
3. The third charge should be placed
   a. left of A at a distance l/3 from A
   b. right of A at a distance l/3 from B
   c. between A and B at a distance 2l/3 from A
   d. between A and B at a distance l/3 from A
4. The third charge has magnitude and sign
   a. \( Q = \left( \frac{4}{9} \right) q \)
   b. \( Q = \left( \frac{4}{9} \right) q \)
   c. \( Q = \left( \frac{3}{5} \right) q \)
   d. \( Q = \left( \frac{3}{5} \right) q \)

For Problems 5–7
Three charges are placed as shown in Fig. 1.125. The magnitude of q1 is 2.00 μC, but its sign and the value of the charge q2 are not known. Charge q3 is +4.00 μC, and the net force on q3 is entirely in the negative x-direction.
5. As per the condition given in the problem, the sign of q1 and q2 will be
   a. +, +  b. +, −  c. −, +  d. −, −
6. The magnitude of q3 is
   a. \( \frac{27}{64} \) μC  b. \( \frac{27}{32} \) μC  c. \( \frac{13}{32} \) μC  d. \( \frac{13}{64} \) μC
7. The magnitude of force acting on q3 is
   a. 25.25 N  b. 32.5 N  c. 56.25 N  d. 13.5 N

For Problems 8–10
Two point like charges \( Q_1 \) and \( Q_2 \) are positioned at points 1 and 2. The field intensity to the right of the charge \( Q_2 \) on the line that passes through the two charges varies according to a law that is represented schematically in Fig. 1.126. The field intensity is assumed to be positive if its direction coincides with the positive direction on the x-axis. The distance between the charges is l.
8. The sign of each charge \( Q_1 \) and \( Q_2 \) is
   a. +, −  b. −, +  c. +, +  d. −, −
9. The ratio of the absolute values of the charges \( |Q_1/Q_2| \) is
   a. \( \left( \frac{a+l}{a} \right)^2 \)  b. \( \left( \frac{l}{a} \right)^2 \)  c. \( \left( \frac{a}{a+l} \right)^2 \)  d. \( \left( \frac{a}{l} \right)^2 \)
10. The value of b, where the field intensity is maximum, is
    a. \( \frac{l}{Q_1/Q_2} + 1 \)  b. \( \frac{l}{Q_1/Q_2} - 1 \)  c. \( \frac{l}{Q_1/Q_2} + 1 \)  d. \( \frac{l}{Q_1/Q_2} - 1 \)

For Problems 11–12
Four equal positive charges, each of value Q, are arranged at the four corners of a square of diagonal 2a. A small body of mass m carrying a unit positive charge is placed at a height h above the centre of the square.
11. What should be the value of Q in order that this body may be in equilibrium?
    a. \( \frac{\pi e_0 mg}{2h} \) \( (h^2 + 2a^2)^{3/2} \)  b. \( \frac{\pi e_0 mg}{h} \) \( (h^2 + a^2)^{3/2} \)
    c. \( \frac{2m^2}{a} \) \( (h^2 + 2a^2)^{-1/2} \)  d. \( \frac{\pi e_0 mg}{2h} \) \( (h^2 - a^2)^{1/2} \)
12. The type of equilibrium of the point mass is (consider only vertical displacement)
    a. stable equilibrium  b. unstable equilibrium  c. neutral equilibrium  d. cannot be determined

For Problems 13–16
An electron is projected with an initial speed \( v_0 = 1.60 \times 10^6 \) m/s into the uniform field between the parallel plates as shown in Fig. 1.127. Assume that the field between the plates is uniform and directed vertically downwards, and that the field outside the
plates is zero. The electron enters the field at a point midway between the plates. Mass of electron = $9.1 \times 10^{-31}$ kg.

13. If the electron just misses the upper plate, the time of flight of electron up to this instant is
   a. $1.25 \times 10^{-5}$ s  
   b. $32.5 \times 10^{-6}$ s
   c. $1.25 \times 10^{-5}$ s  
   d. $32.5 \times 10^{-8}$ s

14. For condition of previous situation, the magnitude of electric field is
   a. 124 NC$^{-1}$  
   b. 364 NC$^{-1}$
   c. 224 NC$^{-1}$  
   d. 520 NC$^{-1}$

15. If instead of electron, a proton were projected with the same speed, then compare the paths travelled by the electron and the proton.
   a. The proton will hit the upper plate.
   b. The proton will hit the lower plate.
   c. The proton will not hit either plate.
   d. None of these.

16. The vertical displacement travelled by the proton as it exits the region between the plates is. Mass of proton = $1.67 \times 10^{-27}$ kg.
   a. $1.6 \times 10^{-4}$ m  
   b. $3.25 \times 10^{-8}$ m
   c. $5.25 \times 10^{-4}$ m  
   d. $2.73 \times 10^{-4}$ m

For Problems 17–18
An electron is projected as shown in Fig. 1.128, with kinetic energy $K$, at an angle $\theta = 45^\circ$ between two charged plates. Ignore the gravity.

17. The magnitude of the electric field, so that the electron just fails to strike the upper plate is
   a. $K/qd$  
   b. $2K/qd$  
   c. $K/2qd$  
   d. infinite

![Fig. 1.128](image)

18. At what distance from the starting point will the electron strike the lower plate?
   a. $d$  
   b. $2d$  
   c. $3d$  
   d. $4d$

For Problems 19–20
In 1909, Robert Millikan was the first to find the charge of an electron in his now-famous oil-drop experiment. In that experiment, tiny oil drops were sprayed into a uniform electric field between a horizontal pair of oppositely charged plates. The drops were observed with a magnifying eyepiece, and the electric field was adjusted so that the upward force on some negatively charged oil drops was just sufficient to balance the downward force of gravity. That is, when suspended, upward force $qE$ just equaled $mg$. Millikan accurately measured the charges on many oil drops and found the values to be whole number multiples of $1.6 \times 10^{-19}$ C—the charge of the electron. For this, he won the Nobel prize.

19. If a drop of mass $1.08 \times 10^{-14}$ kg remains stationary in an electric field of $1.68 \times 10^{3}$ NC$^{-1}$, then the charge of this drop is
   a. $6.40 \times 10^{-19}$ C  
   b. $3.2 \times 10^{-19}$ C
   c. $1.6 \times 10^{-19}$ C  
   d. $4.8 \times 10^{-19}$ C

20. Extra electrons on this particular oil drop (given the presently known charge of the electron) are
   a. 4  
   b. 3  
   c. 5  
   d. 8

Matching Column Type

<table>
<thead>
<tr>
<th>Column I</th>
<th>Column II</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Let us place a charge $+q$ at $O$, displace it slightly along $X$-axis and release. Assume that it is allowed to move only along $X$-axis. At position $O$,</td>
<td></td>
</tr>
<tr>
<td>a. force on the charge is zero</td>
<td></td>
</tr>
<tr>
<td>ii. Place a charge $-q$ at $O$. Displace it slightly along $X$-axis and release. Assume that it is allowed to move only along $X$-axis. At position $O$,</td>
<td></td>
</tr>
<tr>
<td>b. potential energy of the system is maximum</td>
<td></td>
</tr>
<tr>
<td>iii. Place a charge $+q$ at $O$. Displace it slightly along $Y$-axis and release. Assume that it is allowed to move only along $Y$-axis. At position $O$,</td>
<td></td>
</tr>
<tr>
<td>c. potential energy of the system is minimum</td>
<td></td>
</tr>
<tr>
<td>iv. Place a charge $-q$ at $O$. Displace it slightly along $Y$-axis and release. Assume that it is allowed to move only along $Y$-axis. At position $O$,</td>
<td></td>
</tr>
<tr>
<td>d. the charge is in equilibrium</td>
<td></td>
</tr>
</tbody>
</table>

2. Match the forces given in Column I with the properties given in Column II:
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Chapter 2 Kinetic Theory of Gases and First Law of Thermodynamics
Chapter 3 Archives on Chapters 1 and 2

UNIT II: OSCILLATION AND WAVES

Chapter 4 Linear and Angular Simple Harmonic Motion
Chapter 5 Travelling Waves
Chapter 6 Sound Waves and Doppler Effect
Chapter 7 Superposition and Standing Waves
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Appendix: Solutions to Concept Application Exercises A.1
Preface

While the paper-setting pattern and assessment methodology have been revised many times over and newer criteria devised to develop more aspirant-friendly engineering entrance tests, the need to standardize the selection processes and their outcomes at the national level has always been felt. A combined national-level engineering entrance examination has finally been proposed by the Ministry of Human Resource Development, Government of India. The Joint Entrance Examination (JEE) to India's prestigious engineering institutions (IITs, IIITs, NITs, ISM, IISERs, and other engineering colleges) aims to serve as a common national-level engineering entrance test, thereby eliminating the need for aspiring engineers to sit through multiple entrance tests.

While the methodology and scope of an engineering entrance test are prone to change, there are two basic objectives that any test needs to serve:

1. The objective to test an aspirant’s caliber, aptitude, and attitude for the engineering field and profession.
2. The need to test an aspirant’s grasp and understanding of the concepts of the subjects of study and their applicability at the grassroots level.

Students appearing for various engineering entrance examinations cannot bank solely on conventional shortcut measures to crack the entrance examination. Conventional techniques alone are not enough as most of the questions asked in the examination are based on concepts rather than just formulae. Hence, it is necessary for students appearing for joint entrance examination to not only gain a thorough knowledge and understanding of the concepts but also develop problem-solving skills to be able to relate their understanding of the subject to real-life applications based on these concepts.

This series of books is designed to help students to get an all-round grasp of the subject so as to be able to make its useful application in all its contexts. It uses a right mix of fundamental principles and concepts, illustrations which highlight the application of these concepts, and exercises for practice. The objective of each book in this series is to help students develop their problem-solving skills/accuracy, the ability to reach the crux of the matter, and the speed to get answers in limited time. These books feature all types of problems asked in the examination—be it MCQs (one or more than one correct), assertion-reason type, matching column type, comprehension type, or integer type questions. These problems have skillfully been set to help students develop a sound problem-solving methodology.

Not discounting the need for skilled and guided practice, the material in the books has been enriched with a number of fully solved concept application exercises so that every step in learning is ensured for the understanding and application of the subject. This whole series of books adopts a multi-faceted approach to mastering concepts by including a variety of exercises asked in the examination. A mix of questions helps stimulate and strengthen multi-dimensional problem-solving skills in an aspirant.

It is imperative to note that this book would be as profound and useful as you want it to be. Therefore, in order to get maximum benefit from this book, we recommend the following study plan for each chapter.

Step 1: Go through the entire opening discussion about the fundamentals and concepts.
Step 2: After learning the theory/concept, follow the illustrative examples to get an understanding of the theory/concept.

Overall the whole content of the book is an amalgamation of the theme of physics with ahead-of-time problems, which equips the students with the knowledge of the field and paves a confident path for them to accomplish success in the JEE.

With best wishes!

B.M. Sharma
UNIT I
THERMAL PHYSICS

CHAPTER 1: THERMAL PROPERTIES OF MATTER
CHAPTER 2: KINETIC THEORY OF GASES AND FIRST LAW OF THERMODYNAMICS
CHAPTER 3: ARCHIVES ON CHAPTERS 1 AND 2
CHAPTER 1

Thermal Properties of Matter

- Heat
- Calorimetry
- Thermal Expansion
- Transmission of Heat
- Combination of Conductors
- Determination of Thermal Conductivity
- Growth of Ice on Lake
- Convection
- Radiation
- Prevost’s Theory of Heat Exchange
- Kirchhoff’s Law
- Stefan’s Law
- Newton’s Law of Cooling
- Distribution of Energy in the Spectrum of Black Body
- Wien’s Displacement Law
- Temperature of the Sun and Solar Constant
HEAT

Heat is a form of energy which appears when two bodies at different temperatures come into contact and flows from the body at higher temperature to that at lower temperature. It is energy in motion or energy in transit. Heat is not a property of a system. A system can give out or absorb heat, but does not contain heat. It is the form of energy which determines the change in thermal state of a body and is defined as the flow of energy from one body to the other due to difference in the degree of hotness of two bodies (temperature). It flows from the body which is at a high temperature to the other at low temperature.

The energy associated with configuration and random motion of the atoms and molecules within a body is called internal energy and the part of this internal energy which is transferred from one body to the other due to temperature difference is called heat.

One calorie is defined as the amount of heat energy required to raise the temperature of 1 g of water through 1°C (more specifically from 14.5°C to 15.5°C).

As heat is a form of energy it can be transformed into others and vice versa. For example, thermocouple converts heat energy into electrical energy, resistor converts electrical energy into heat energy. Friction converts mechanical energy into heat energy. Heat engine converts heat energy into mechanical energy.

Here it is important that whole of mechanical energy, i.e., work can be converted into heat but whole of heat can never be converted into work.

Temperature

Temperature is defined by zeroth law of thermodynamics, which states that when two bodies A and B are separately in thermal equilibrium with a third body C, then A and B are also in thermal equilibrium with each other (thermal equilibrium implies equality of temperature). Temperature is a scalar quantity which is a property of all thermodynamic systems such that the equality of temperature is necessary and sufficient for thermal equilibrium.

1. Temperature is one of the seven fundamental quantities with dimension [Θ].
2. It is a scalar physical quantity with SI unit kelvin.
3. When heat is given to a body and its state does not change, the temperature of the body rises and if heat is taken from a body its temperature falls, i.e., temperature can be regarded as the effect of cause ‘heat’.
4. According to the kinetic theory of gases, temperature (macroscopic physical quantity) is a measure of average translational kinetic energy of a molecule (microscopic physical quantity).

\[
\text{Temperature } \propto \text{kinetic energy } \quad \text{As } E = \frac{3}{2} kT
\]

5. Although the temperature of a body can be raised without limit, it cannot be lowered without limit and theoretically limiting low temperature is taken to be zero of the kelvin scale.
6. Highest possible temperature achieved in laboratory is about 10^6 K, while the lowest possible temperature attained is 10^-4 K.
7. Branch of physics dealing with production and measurement of temperatures close to 0 K is known as cryogenics, while that dealing with the measurement of very high temperature is called pyrometry.
8. Temperature of the core of the sun is 10^7 K while that of its surface is 6000 K.
9. Normal temperature of human body is 310.15 K = 37°C = 98.6°F.
10. NTP or STP implies 273.15 K (0°C = 32°F).

As the temperature is measured by the value of the thermodynamic property of a substance, i.e., a property which varies linearly with the temperature, two fixed points are needed to define a temperature scale.

These two fixed points in modern thermometry are taken as
1. Triple point of water, i.e., the state of water where the liquid, solid and vapour phases of water coexist in equilibrium. It is characterized by unique values of temperature and pressure.
2. On this scale, the other fixed point may be taken as the absolute zero.

We then need to assign some numbers to these two fixed points. The lowest temperature may be taken as zero. The triple point of water on Celsius scale is 0.01°C. Thus the absolute temperature \( T \) for triple point of water will be given by

\[
T = t + 273.15 = 0.01 + 273.15 = 273.16 K
\]

Thermometry

An instrument used to measure the temperature of a body is called a thermometer.

The linear variation in some physical property of a substance with change of temperature is the basic principle of thermometry and these properties are defined as thermometric property (x) of the substance.

\[ x \text{ may be } \quad (i) \text{ length of liquid in capillary;} \]
\[ (ii) \text{ pressure of gas at constant volume;} \]
\[ (iii) \text{ volume of gas at constant pressure and} \]
\[ (iv) \text{ resistance of a given platinum wire.} \]

In old thermometry, two arbitrarily fixed points ice and steam point (freezing point and boiling point at 1 atm) are taken to define the temperature scale. In Celsius scale, freezing point of water is assumed to be 0°C while boiling point 100°C and the temperature interval between these is divided into 100 equal parts.

So, if the thermometric property at temperatures 0°C, 100°C and 7°C is \( x_0, x_{100} \) and \( x \), respectively, then by linear variation \( y = mx + c \) we can say that
\[ O = ax_o + b \]  
\[ 100 = ax_{100} + b \]  
\[ T_c = ax + b \]  

From these equations \( \frac{T_c - 0}{100 - 0} = \frac{x - x_o}{x_{100} - x_o} \)  
\[ T_c = \frac{x - x_o}{x_{100} - x_o} \times 100^\circ C \]

In modern thermometry instead of two fixed points only one reference point is chosen (triple point of water 273.16 K at which ice, water and water vapours coexist).

So, if the values of thermometric property at 0 K, 273.16 K and \( T_c \) K are \( x_o \), \( x_c \) and \( x \), respectively, then by linear variation \( (y = mx + c) \) we can say that  
\[ O = a \times 0 + b \]  
\[ 273.16 = a \times x_c + b \]  
\[ T_c = a \times x + b \]

From these equations \( \frac{T_c}{273.16} = \frac{x}{x_c} \)  
\[ T_c = 273.16 \left[ \frac{x}{x_c} \right] K \]

**Measurement of Temperature**

There are different systems of measurement of temperature. The lower fixed point (LFP) and the upper fixed point (UFP) in any system of units are corresponding to freezing point and boiling point of water at 1 atm.

For different system of units the LFP and UFP are given as

<table>
<thead>
<tr>
<th>System of units</th>
<th>Units</th>
<th>Lower fixed point (LFP)</th>
<th>Upper fixed point (UFP)</th>
<th>Different - LFP &amp; UFP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Degree celsius (contagrade)</td>
<td>°C</td>
<td>0°C</td>
<td>100°C</td>
<td>100</td>
</tr>
<tr>
<td>Kelvin scale (SI unit)</td>
<td>K</td>
<td>273.15 K</td>
<td>373.15 K</td>
<td>100</td>
</tr>
<tr>
<td>Fahrenheit</td>
<td>°F</td>
<td>32°F</td>
<td>212°F</td>
<td>180</td>
</tr>
</tbody>
</table>

Temperature on one scale can be converted into other scale by using the following identity.

\[
\text{Reading on any scale} - \text{lower fixed point (LFP)} = \frac{\text{Constant for Upper fixed point (UFP) - lower fixed point(LFP)}}{\text{all scales}}
\]

The relation between Celsius (C), Kelvin (K), Fahrenheit (F) and any other new scale \( \theta \) is  
\[ \frac{C - 0}{100} = \frac{F - 32}{180} = \frac{K - 273}{100} = \frac{\theta - \theta_s}{n} \]  
where \( n \) is the number of divisions between ice point and steam point on the new scale and \( \theta_s \) is the ice point on it.

**Illustration 1.2** Liquid nitrogen has a boiling point of \(-195.81^\circ C\) at atmospheric pressure. Calculate this temperature (a) in degrees Fahrenheit and (b) in kelvin.

**Sol.** We can use Eq. (i) to convert degree celsius into Fahrenheit and kelvin.

a. Temperature in Fahrenheit is given by \[ T_f = \frac{9}{5} T_c + 32^\circ F = \frac{9}{5} (-195.81) + 32 = -320.46^\circ F \]

b. Temperature in Kelvin \( T_c = 273.15 \text{ K} - 195.81 \text{ K} = 77.3 \text{ K} \)

A convenient way to change one scale to another is to remember the freezing and boiling points of water in each form:

\[ T_{freeze} = 32^\circ F = 0^\circ C = 273.15 \text{ K} \]
\[ T_{boil} = 212^\circ F = 100^\circ C \]

To convert from Fahrenheit to Celsius, subtract 32 (the freezing point) and then adjust the scale by the liquid range of the water.

\[ \text{Scale factor} = \frac{(100 - 0)^\circ C}{(212 - 32)^\circ F} = \frac{5^\circ C}{9^\circ F} \]

A Kelvin is the same size change as a degree celsius, but the Kelvin scale takes its zero point at absolute zero, instead of the freezing point of water. Therefore, to convert from Kelvin to Celsius, subtract 273.15 K from given Kelvin temperature.

**Illustration 1.2** Two ideal gas thermometers A and B use oxygen and hydrogen, respectively. The following observations are made:

<table>
<thead>
<tr>
<th>Temperature</th>
<th>Pressure thermometer A</th>
<th>Pressure thermometer B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triple point of water</td>
<td>1.250 x 10^6 Pa</td>
<td>0.200 x 10^4 Pa</td>
</tr>
<tr>
<td>Normal freezing point of sulphur</td>
<td>1.797 x 10^6 Pa</td>
<td>0.287 x 10^4 Pa</td>
</tr>
</tbody>
</table>

a. What is the absolute temperature of normal melting point of sulphur as read by thermometer A and B?  
b. What do you think is the reason for slightly different answers from A and B?

**Sol.**

a. For thermometer A, \[ T_u = 273 \text{ K}, P_u = 1.250 \times 10^6 \text{ Pa} \]

We have \[ T = \frac{P}{P_u} \times T_u \]
\[ = \frac{1.797 \times 10^6}{1.250 \times 10^6} \times 273 = 392.46 \text{ K} \]

For thermometer B,
\[ T_u = 273 \text{ K}, P_u = 0.200 \times 10^4 \text{ Pa} \]

We have \[ T = \frac{P}{P_u} \times T_u \]
\[ = \frac{1.250 \times 10^6}{0.200 \times 10^4} \times 273 = 392.46 \text{ K} \]
1.4 Waves & Thermodynamics

\[
\frac{0.287 \times 10^3 \times 273}{0.200 \times 10^3} = 391.75 K
\]

b. The slight difference in the temperatures as read by two thermometers is due to the fact that oxygen and hydrogen do not behave like an ideal gas.

Illustration 1.3. What will be the following temperatures on the Kelvin scale: a. 37°C, b. 80°F, c. -196°C?

Sol.
a. Temperature on Kelvin scale \( T_K \) is related to temperature \( T_C \) on Celsius scale as

\[
T_K = T_C + 273
\]

or

\[
T_K = 37 + 273 = 310 K
\]

b. Temperatures \( T_K \) on Kelvin scale and \( T_F \) on Fahrenheit scale are related as

\[
\frac{T_F - 273}{373 - 273} = \frac{T_K - 32}{212 - 32}
\]

or

\[
T_K = \frac{5}{9} (T_F - 32) + 273
\]

or

\[
T_F = \frac{9}{5} (T_K - 32) + 273
\]

here \( T_F = 80°F \); thus

\[
T_K = \frac{5}{9} (80 - 32) + 273 = 299.66 K
\]

c. Again from relation used in part (a)

\[
T_K = T_C + 273 = -196 + 273 = 77 K
\]

CAlORIMETRY

This is the branch of heat transfer that deals with the measurement of heat. The heat is usually measured in calories or kilocalories.

One Calorie

One calorie is the quantity of heat required to raise the temperature of 1 g of water by 1°C.

Mechanical Equivalent of Heat (J)

According to Joule, work may be converted into heat and vice versa. The ratio of work done \( W \) to heat produced \( Q \) by that work without any wastage is always constant.

\[
\frac{W}{Q} = \text{constant}
\]

This constant is called mechanical equivalent of heat (J). The value of this constant is taken as 4.18 J/cal.

Illustration 1.4. In the Joule experiment, a mass of 20 kg falls through 1.5 m at a constant velocity to stir the water in a calorimeter. If the calorimeter has a water equivalent of 2 g and contains 12 g of water, what is \( f \), the mechanical equivalent of heat, for a temperature rise of 5.0°C?

Sol. Expressing \( \Delta PE \) in Joules and \( Q \) in Calories, we have

\[
f = \frac{\Delta PE}{Q} = \frac{mgv}{m_c c \Delta T} = \frac{20 \times 9.8 \times 1.5}{(12 + 2)(1)(5.0)} = 4.2 \text{ J/cal}
\]

Thermal Capacity and Water Equivalent

1. Thermal capacity: It is defined as the amount of heat required to raise the temperature of the whole body (mass \( m \)) through 1°C or 1 K.

\[
\text{Thermal capacity} = \frac{m c \Delta T}{\Delta T} = m c
\]

The value of thermal capacity of a body depends upon the nature of the body and its mass. Dimension: [ML²T⁻¹⁸] or unit cal/C (practical J/K or SI).

2. Water equivalent: Water equivalent of a body is defined as the mass of water which would absorb or evolve the same amount of heat as is done by the body in rising or falling through the same range of temperature. It is represented by \( W \).

If \( m = \) mass of the body, \( c = \) specific heat of body, \( \Delta T = \) rise in temperature.

Then heat given to body

\[
\Delta Q = m c \Delta T
\]

(i)

If same amount of heat is given to \( W \) grams of water and its temperature also rises by \( \Delta T \).

Then heat given to water

\[
\Delta Q = W \times 1 \times \Delta T \quad \text{[As } c \text{ water } = 1]\]

(ii)

From Eqs. (i) and (ii), \( \Delta Q = m c \Delta T = W \times 1 \times \Delta T \)

∴ Water equivalent \( (W) = m c \) grams

Unit: kg (SI); dimension: [ML⁰T⁻¹⁸].

Note:

- Unit of thermal capacity is J/kg while unit of water equivalent is kg.
- Thermal capacity of the body and its water equivalent are numerically equal.
- If thermal capacity of a body is expressed in terms of mass of water it is called water equivalent of the body.

Specific Heat

1. Gram specific heat: When heat is given to a body and its temperature increases, the heat required to raise the temperature of unit mass of a body through 1°C (or K) is called specific heat of the material of the body.
1. Heat changes the temperature of mass \( m \) by \( \Delta T \)

Specific heat \( c = \frac{Q}{m\Delta T} \)

Units: \( \text{cal/g} \times ^\circ \text{C} \) (practical), \( \text{J/kg} \times ^\circ \text{K} \) (SI); dimension: \( [L^2T^{-2}\theta^{-1}] \)

2. Molar specific heat: Molar specific heat of a substance is defined as the amount of heat required to raise the temperature of 1 g mole of the substance through a unit degree; it is represented by \( C \).

By definition, 1 mole of any substance is a quantity of the substance, whose mass \( M \) grams is numerically equal to the molar mass of the substance.

\[
C = M \cdot c
\]

or

\[
C = M \cdot \frac{Q}{m\Delta T} = \frac{1}{\mu} \cdot \frac{Q}{\Delta T}
\]

As \( c = \frac{Q}{m\Delta T} \) and \( \mu = \frac{m}{M} \),

\[
C = \frac{Q}{\mu\Delta T}
\]

Units: \( \text{cal/mol} \times ^\circ \text{C} \) (practical), \( \text{J/mol} \times \text{kelvin} \) (SI); dimension: \( [ML^2T^{-2}\theta^{-1}\mu^{-1}] \)

**Important Points**

1. Specific heat for hydrogen is maximum (3.5 cal/g \( \times ^\circ \text{C} \)) and for water, it is 1 cal/g \( \times ^\circ \text{C} \).

   For all other substances, the specific heat is less than 1 cal/g \( \times ^\circ \text{C} \) and it is minimum for radon and actinium (0.022 cal/g \( \times ^\circ \text{C} \)).

2. Specific heat of a substance also depends on the state of the substance, i.e., solid, liquid or gas.

   For example, \( C_p = 0.5 \text{ cal/g} \times ^\circ \text{C} \) (solid), \( C_{\text{water}} = 1 \text{ cal/g} \times ^\circ \text{C} \) (liquid) and \( C_{\text{steam}} = 0.47 \text{ cal/g} \times ^\circ \text{C} \) (gas).

3. The specific heat of a substance when it melts or boils at constant temperature is infinite.

   As \( \frac{Q}{m\Delta T} \) \( \times 0 \) \( (\text{As } \Delta T = 0) \)

4. The specific heat of a substance when it undergoes adiabatic changes is zero.

   As \( \frac{Q}{m\Delta T} = \frac{0}{m\Delta T} = 0 \) \( (\text{As } Q = 0) \)

5. Specific heat of a substance can also be negative.

   Negative specific heat means that in order to raise the temperature, a certain quantity of heat is to be withdrawn from the body.

   For example, specific heat of saturated vapours.

**Illustration 1.5**

A 60 kg boy running at 5.0 m/s while playing basketball falls down on the floor and skids along on his leg until he stops. How many calories of heat are generated between his leg and the floor?

Assume that all this heat energy is confined to a volume of 2.0 cm\(^3\) of his flesh. What will be temperature change of the flesh? Assume \( c = 1.0 \text{ cal/g} \times ^\circ \text{C} \) and \( \rho = 950 \text{ kg/m}^3 \) for flesh.

**Sol.** The boy's kinetic energy is changed to heat energy.

Set

\[
Q = \left( \frac{mv^2}{2} \right) / 2 = \left( \frac{60(25)}{2} \right) = 750 \text{ J} = 179 \text{ cal},
\]

from

\[
Q = cp \Delta T, 179 \text{ cal} = (1.0 \text{ cal/g} \times ^\circ \text{C})
\]

\[
(0.950 \text{ g/cm}^3)(2.0 \text{ cm}^3) \Delta T, \text{ whence } \Delta T = 94^\circ \text{C}
\]

**Illustration 1.6**

An electric heater supplies 1.8 kW of power in the form of heat to a tank of water. How long will it take to heat the 200 kg of water in the tank from 10°C to 70°C? Assume heat losses to the surroundings to be negligible.

**Sol.** The heat added is (1.8 J/s) \( t \)

The heat absorbed is \( cm\Delta T = (4184 \text{ kJ/kg} \times ^\circ \text{C}) \times (200 \text{ kg}) \times (60 \text{ K}) = 5.0 \times 10^6 \text{ J} \)

Equation, \( t = 2.78 \times 10^4 \text{ s} = 7.75 \text{ h} \)

**Specific Heat of Solids**

When a solid is heated through a small range of temperature, its volume remains more or less constant. Therefore specific heat of a solid may be called its specific heat at constant volume \( C_v \).

From the graph it is clear that at \( T = 0 \), \( C_v \) tends to zero.

With rise in temperature, \( C_v \) increases and becomes constant \( = 3R = 6 \text{ cal/mole-}^\circ \text{K} = 25 \text{ J/mole-}^\circ \text{K} \) at some particular temperature (Debye temperature).

For most of the solids, Debye temperature is close to room temperature.

**Specific Heat of Water**

The variation of specific heat with temperature for water is shown in Fig. 1.2. Usually this temperature dependence of specific heat is neglected.

From the graph:

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>0</th>
<th>15</th>
<th>35</th>
<th>50</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific heat (cal/g ( \times ^\circ \text{C} ))</td>
<td>1.008</td>
<td>1.000</td>
<td>0.997</td>
<td>0.998</td>
<td>1.006</td>
</tr>
</tbody>
</table>

As specific heat of water is very large, by absorbing or releasing large amount of heat, its temperature changes by small amount. This is why it is used in hot water bottles or as coolant in radiators.
Note: When specific heats are measured, the values obtained are also found to depend on the conditions of the experiment. In general, measurements made at constant pressure are different from those at constant volume. For solids and liquids, this difference is very small and usually neglected. The specific heats of gases are quite different under constant pressure condition \( c_p \) and constant volume \( c_v \). In the chapter ‘Kinetic Theory of Gases and First Law of Thermodynamics’ we have discussed this topic in detail.

**Illustration 1.7** What is wrong with the following statement: ‘Given any two bodies, the one with the higher temperature contains more heat.’

**Sol.** The statement shows a misunderstanding of the concept of heat. Heat is a process by which energy is transferred, not a form of energy that is held or contained. If you wish to speak of energy that is ‘contained’, you speak of internal energy, not heat.

Further, even if the statement used the term ‘internal energy’, it would still be incorrect, since the effects of specific heat and mass are both ignored. A 1 kg mass of water at 20°C has more internal energy than a 1 kg mass of air at 30°C.

Similarly, the earth has far more internal energy than a drop of molten titanium metal.

Correct statements would be: 1. ‘Given any two bodies in thermal contact, the one with the higher temperature will transfer energy to the other by heat’. 2. ‘Given any two bodies of equal mass, the one with the higher product of absolute temperature and specific heat contains more internal energy’. All to say is that internal energy depends not only on temperature but also on mass and nature of body.

**Illustration 1.8** Two bodies have the same heat capacity. If they are combined to form a single composite body, show that the equivalent specific heat of this composite body is independent of the masses of the individual bodies.

**Sol.** Let the two bodies have masses \( m_1, m_2 \) and specific heats \( s_1 \) and \( s_2 \), then

\[
m_1s_1 = m_2s_2 \quad \text{or} \quad \frac{m_1}{m_2} = \frac{s_2}{s_1}
\]

Let \( s = \text{specific heat of the composite body} \), then

\[
(m_1 + m_2)s = m_1s_1 + m_2s_2 = 2m_1s_1
\]

**Illustration 1.9** The temperature of a silver bar rises by 10.0°C when it absorbs 1.23 kJ of energy by heat. The mass of bar is 525 g. Determine the specific heat of silver.

**Sol.** We find its specific heat from the definition, which is contained in the equation \( Q = mc \Delta T \) for energy input by heat to produce a temperature change. Solving we have

\[
c_{silver} = \frac{Q}{m \Delta T} = \frac{1.23 \times 10^3 J}{(0.525 kg)(10.0°C)} = 234 \text{ J/kg°C}
\]

**Illustration 1.10** The air temperature above coastal areas is profoundly influenced by the large specific heat of water. One reason is that the energy released when 1 m³ of water cools by 1°C will raise the temperature of a much larger volume of air by 1°C. Find this volume of air. The specific heat of air is approximately 1 kJ/kg°C. Take the density of air to be 1.3 kg/m³.

**Sol.** The mass of 1 m³ of water is specified by its density,

\[
m = \rho V = (1.00 \times 10^3 \text{ kg/m³})(1 \text{ m}^3) = 1 \times 10^3 \text{ kg}
\]

When 1 m³ of water cools by 1°C, it releases energy

\[
Q_v = mc \Delta T = (1 \times 10^3 \text{ kg})(4186 \text{ J/kg°C})(-1°C) = -4 \times 10^6 \text{ J}
\]

Where the negative sign represents heat output. When \( +4 \times 10^6 \text{ J} \) is transferred to the air, raising its temperature by 1°C, the volume of the air is given by

\[
V = \frac{Q}{\rho c \Delta T} = \frac{4 \times 10^6 \text{ J}}{(1.3 \text{ kg/m}^3)(1 \times 10^3 \text{ J/kg°C})(1°C)} = 3 \times 10^3 \text{ m}^3
\]

The volume of the air is a thousand times larger than the volume of the water.

**Illustration 1.11** James Joule tested the conversion of mechanical energy into internal energy by measuring temperatures of falling water. If water at the top of a Swiss waterfall has a temperature of 10.0°C and then falls 50.0 m, what maximum temperature at the bottom would Joule expect? He did not succeed in measuring the temperature change, partly because evaporation cooled the falling water and also because his thermometer was not sufficiently sensitive.

**Sol.** The temperature change can be found from the potential energy that is converted to internal energy. The final temperature is this change added to the initial temperature of the water.

The gravitational energy that can change into internal energy is \( \Delta E_{me} = mgy \). It will produce the same temperature
change as the same amount of heat entering the water from a stove, as described by \( Q = mc \Delta T \). Thus, \( mgv = mc \Delta T \).

Isolating \( \Delta T \),

\[
\Delta T = \frac{gy}{c} = \frac{(9.80 \text{ m/s}^2)(50.0 \text{ m})}{4.186 \times 10^3 \text{ J/kg} \cdot \text{K}} = 0.117^\circ \text{C}
\]

\[
T_f = T_i + \Delta T = 10.0^\circ \text{C} + 0.117^\circ \text{C} = 10.1^\circ \text{C}
\]

The final temperature might be less than we calculated since this solution does not account for cooling of the water due to evaporation as it falls.

**Latent Heat**

- When a substance changes from one state to another state (say from solid to liquid or liquid to gas or from liquid to solid or gas to liquid) then energy is either absorbed or liberated. This heat energy is called latent heat.
- No change in temperature is involved when the substance changes its state. That is, phase transformation is an isothermal change. Ice at 0\(^\circ\)C melts into water at 0\(^\circ\)C. Water at 100\(^\circ\)C boils to form steam at 100\(^\circ\)C.
- The amount of heat required to change the state of the mass \( m \) of the substance is written as: \( \Delta Q = mL \), where \( L \) is the latent heat. Latent heat is also called as heat of transformation.
- Unit: cal/g or J/kg and dimension: \( L^2 T^{-2} \).
- Any material has two types of latent heats.

i. **Latent heat of fusion**: The latent heat of fusion is the heat energy required to change 1 kg of the material in its solid state at its melting point to 1 kg of the material in its liquid state. It is also the amount of heat energy released when at melting point 1 kg of liquid changes to 1 kg of solid. For water at its natural freezing temperature or melting point (0\(^\circ\)C), the latent heat of fusion (or latent heat of ice) is

\[
L_f = L_{wate} = 80 \text{ cal/g} = 6 \text{ kJ/mol} = 336 \text{ kJ/kg}
\]

ii. **Latent heat of vapourization**: The latent heat of vapourization is the heat energy required to change 1 kg of the material in its liquid state at its boiling point to 1 kg of the material in its gaseous state. It is also the amount of heat energy released when 1 kg of vapour changes into 1 kg of liquid. For water at its normal boiling point or condensation temperature (100\(^\circ\)C), the latent heat of vapourization (latent heat of steam) is

\[
L_v = L_{steam} = 540 \text{ cal/g} = 40.8 \text{ kJ/mol} = 2260 \text{ kJ/kg}
\]

- It is more painful to get burnt by steam rather than by boiling water at the same temperature. This is because when 1 g of steam at 100\(^\circ\)C gets converted to water at 100\(^\circ\)C, then it gives out 536 cal of heat. So, it is clear that steam at 100\(^\circ\)C has more internal energy than water at 100\(^\circ\)C (i.e., boiling of water).
- In case of change of state if the molecules come closer, energy is released and if the molecules move apart, energy is absorbed.
- Latent heat of vapourization is more than the latent heat of fusion. This is because when a substance gets converted from liquid to vapour, there is a large increase in volume. Hence, more amount of heat is required. But when a solid gets converted to a liquid, then the increase in volume is negligible. Hence, very less amount of heat is required. So, latent heat of vapourization is much more than the latent heat of fusion.
- After snow falls, the temperature of the atmosphere becomes very low. This is because the snow absorbs the heat from the atmosphere to melt down. So, in the mountains, when snow falls, one does not feel too cold, but when ice melts, he feels too cold.
- There is more viscerous effect of ice cream on teeth as compared to that of water (obtained from ice). This is because when ice cream melts down, it absorbs large amount of heat from teeth.

**Illustration 1.12**: Some water at 0\(^\circ\)C is placed in a large insulated enclosure (vessel). The water vapour formed is pumped out continuously. What fraction of the water will ultimately freeze, if the latent heat of vapourization is seven times the latent heat of fusion?

**Sol.** Let us learn the application of theory this illustration.

Let

\[
m = \text{mass of water}, f = \text{fraction which freezes}
\]

\[
L_f = \text{latent heat of vapourization}
\]

\[
L_i = \text{latent heat of fusion}
\]

Mass of water frozen = \( mf \)

Heat lost by freezing water = \( mL_i \)

Mass of vapour formed = \( m(1 - f) \)

Heat gained by vapours = \( m(1 - f)L_v \)

Now heat loss = heat gain:

\[
mL_i = m(1 - f) \times 7L_v
\]

\[
f = \frac{7 - 7f}{7f} \quad \text{or} \quad f = \frac{7}{8}
\]

**Illustration 1.14**: How many calories are required to change exactly 1 g of ice at -10\(^\circ\)C to steam at atmospheric pressure and 120\(^\circ\)C? [Assume the specific heat of steam at a constant pressure of 1 atm is 0.481 cal/(g°C).]

**Sol.** The first stage is the warming of the ice from -10\(^\circ\)C to the melting point (0\(^\circ\)C).

The specific heat capacity of ice is 0.50 cal/(g°C). Therefore, the heat required in the first stage is given by \( \Delta H_i = mc_i \)

\[
\Delta H_i = (1.00 \text{ g})[0.50 \text{ cal/(g°C)}](10 \text{ °C}) = 5.0 \text{ cal}
\]
The second stage is the melting of the ice at 0°C and 1 atm of pressure. The latent heat for the melting of ice is 79.8 cal/g. \( \Delta H_2 = 79.8 \text{ cal} \). The third stage is the heating of the water from 0°C to 100°C, the boiling point, so the heat required is given by 
\[
\Delta H_3 = m_c \Delta T_3 = (1.00 \text{ g})(1.000 \text{ cal/g°C})(100°C) = 100 \text{ cal}.
\]

The fourth stage is the boiling of the water at a temperature of 100 °C and at a constant pressure of 1.00 atm. According to Table the latent heat for the boiling water at 1.00 atm is 540 cal/g, so 
\[
\Delta H_4 = m L_e = (1.00 \text{ g})(540 \text{ cal/g}) = 540 \text{ cal}.
\]

The fifth and final stage is the heating of the steam from 100°C to 120°C (at a constant pressure of 1.00 atm).

Assuming that between 100°C and 120°C the specific heat capacity of steam is constant and has the value 0.481 cal/(g°C) given, we find 
\[
\Delta H_5 = m_c \Delta T_5 = (1 \text{ g})(0.481 \text{ cal/g°C})(20°C) = 9.62 \text{ cal}.
\]

The total heat requirement \( \Delta H = \Delta H_1 + \Delta H_2 + \Delta H_3 + \Delta H_4 + \Delta H_5 = (5.0 + 79.8 + 100 + 540 + 9.62) = 734.4 \text{ cal} \).

**Principle of Calorimetry**

When two bodies (one being solid and other liquid or both being liquid) at different temperatures are mixed, heat will be transferred from body at higher temperature to a body at lower temperature till both acquire same temperature. The body at higher temperature releases heat while body at lower temperature absorbs it, so that 

\[
\text{Heat lost} = \text{Heat gained}
\]

i.e., the principle of calorimetry represents the law of conservation of heat energy.

- Temperature of mixture \( T \) is always greater than lower temperature \( T_l \) and less than higher temperature \( T_h \), i.e., 
  \[
  T_l < T < T_h
  \]
  i.e., the temperature of mixture can never be lesser than lower temperatures (as a body cannot be cooled below the temperature of cooling body) and greater than higher temperature (as a body cannot be heated above the temperature of heating body). Furthermore usually rise in temperature of one body is not equal to the fall in temperature of the other body though heat gained by one body is equal to the heat lost by the other.

- When temperature of a body changes, the body releases heat if its temperature falls and absorbs heat when its temperature rises. The heat released or absorbed by a body of mass \( m \) is given by 
  \[
  Q = mc \Delta T,
  \]
  where \( c \) is specific heat of the body and \( \Delta T \) change in its temperature.

- When state of a body changes, change of state takes place at constant temperature (m.p. or b.p.) and heat released or absorbed is given by 
  \[
  Q = mL,
  \]
  where \( L \) is latent heat. Heat is absorbed if solid converts into liquid (at m.p.) or liquid converts into vapours (at b.p.) and is released if liquid converts into solid or vapours convert into liquid.

- If two bodies \( A \) and \( B \) of masses \( m_1 \) and \( m_2 \) at temperatures \( T_1 \) and \( T_2 \) (\( T_1 > T_2 \)) and having gram specific heat \( c_1 \) and \( c_2 \) are placed in contact, Heat lost by \( A = \text{Heat gained by} \ B \)

  or \[
  m_1c_1(T_1 - T) = m_2c_2(T - T_2)
  \]
  (where \( T = \text{temperature of equilibrium} \)
  \[
  \Rightarrow T = \frac{m_1c_1T_1 + m_2c_2T_2}{m_1c_1 + m_2c_2}
  \]
  \[
  \text{i. } \text{If bodies are of same material } c_1 = c_2 \text{ then } T = \frac{m_1T_1 + m_2T_2}{m_1 + m_2}
  \]
  \[
  \text{ii. } \text{If bodies are of same mass } m_1 = m_2 \text{ then } T = \frac{T_1c_1 + T_2c_2}{c_1 + c_2}
  \]
  \[
  \text{iii. If bodies are of same material and of equal masses } m_1 = m_2, c_1 = c_2 \text{ then } T = \frac{T_1 + T_2}{2}
  \]

**Illustration 1.4** Calculate the heat of fusion of ice from the following data for ice at 0°C added to water. Mass of calorimeter = 60 g, mass of calorimeter + water = 460 g, mass of calorimeter + water + ice = 618 g, initial temperature of water = 38°C, final temperature of the mixture = 5°C. The specific heat of calorimeter = 0.10 cal/g°C. Assume that the calorimeter was also at 0°C initially.

Sol. Mass of water = 460 - 60 = 400 g

 mass of ice = 618 - 460 = 158 g

 Heat lost by water = heat gained by ice to melt + heat gained by (water obtained from melting of ice + calorimeter) to reach 5°C

 \[
  \Rightarrow 400 \times 1 \times (38 - 5) = 158 \times L + 158 \times 1 \times 5 + 60 \times 0.1 \times 5
  \]

 (where \( L \) is the latent heat of fusion of ice)

 \[
  \Rightarrow L = 78.35 \text{ cal/g}
  \]

**Illustration 1.5** A lump of ice of 0.1 kg at -10°C is put in 0.15 kg of water at 20°C. How much water and ice will be found in the mixture when it has reached thermal equilibrium? Specific heat of ice = 0.5 kcal/kg/K and its latent heat of melting = 80 kcal/kg.

Sol. Heat released by 0.15 kg of water in being cooled to 0°C = 0.15 × 1 × 20 = 3 kcal

 Heat absorbed by ice from -10°C to 0°C = 0.1 × 0.5 × 10 = 0.5 kcal

 The balance heat is available for melting ice. Let \( m \) kg of ice melt.

 Then \( m \times 80 = 2.5 \) or \( m = 0.03 \) kg

 Thus the final temperature is 0°C with 0.07 kg of ice and 0.18 kg of water.

**Illustration 1.6** How should 1 kg of water at 5°C be divided into two parts so that if one part turned into ice at
0°C, it would release enough heat to vaporize the other part? Latent heat of steam = 540 cal/g and latent heat of ice = 80 cal/g.

**Sol.** Let the mass be divided into x grams (for ice) and (1000 - x) grams (for vapor).

Heat released by x grams of water = \(x \times 1 \times 5 + x \times 80\)

Heat absorbed by (1000 - x) grams of water

\[= (1000 - x) \times 1 \times 95 + (1000 - x) \times 540\]

Assuming that the conversion of the other part takes place at 100°C.

\[85x = 95 (1000 - x) + 540 (1000 - x) \quad \text{or} \quad x = 882 \text{ g}\]

Thus the mass is to be divided into 882 g for conversion into ice and 118 g for conversion into vapor.

**Illustration 1.19** When a block of metal of specific heat 0.1 cal/g°C and weighing 110 g is heated to 100°C and then quickly transferred to a calorimeter containing 200 g of a liquid at 10°C, the resulting temperature is 18°C. On repeating the experiment with 400 g of the same liquid in the same calorimeter at the same initial temperature, the resulting temperature is 14.5°C. Find

a. Specific heat of the liquid.
b. The water equivalent of calorimeter.

**Sol.** Let s be the specific heat of the liquid and W be the water equivalent of the calorimeter.

Heat lost by the block = heat gained by (liquid + calorimeter)

For the first case:

\[
110 \times 0.1 \times (100 - 18) = 200 \times s \times (18 - 10) + W \times 1 \times (18 - 10) \\
\Rightarrow 16000 + 8W = 902 \quad (i)
\]

For the second case:

\[
110 \times 0.1 \times (100 - 14.5) = 400 \times s \times (14.5 - 10) + W \times 1 \times (14.5 - 10) \\
\Rightarrow 18000 + 4.5W = 9405 \quad (ii)
\]

On solving Eqs. (i) and (ii), we get \(s = 0.48 \text{ cal/g°C} \) and \(W = 16.6 \text{ g} \).

**Illustration 1.20** Determine the final result when 200 g of water and 20 g of ice at 0°C are in a calorimeter having a water equivalent of 30 g and 50 g of steam is passed into it at 100°C.

**Sol.** When steam is passed, the final temperature can be 0°C, between 0°C and 100°C, or 100°C. We will consider all three possibilities.

**Case I**

Final temperature = 0°C

In this case, all the steam condenses and then cools down to 0°C.

Heat given out by steam

\[50 \times 540 + 50 \times 1 \times (100 - 0) = 32000 \text{ cal} \]

Mass of ice which will melt by this heat \(\frac{32000}{80} = 400 \text{ g} \)

But there is only 20 g of ice in the calorimeter. Hence final temperature cannot be 0°C.

**Case II**

Final temperature = \(\theta\) and \(0 < \theta < 100\)

Heat lost by steam = heat gained by (ice + water + calorimeter)
\[ 50 \times 540 + 50 \times 1 \times (100 - \theta) = 20 \times 80 + (20 + 20 + 30) \times 1 \times (\theta - 0) \Rightarrow \theta = 101.3^\circ C \]

The assumption \((0 < \theta < 100)\) is proved to be wrong. Hence, the final temperature cannot be between 0°C and 100°C.

The final temperature will be 100°C.

**Case III**

Let \(m = \) mass of steam condensed.

Heat lost by steam = heat gained by ice to melt + heat gained by (water + water + calorimeter) to reach 100°C.

\[ m \times 540 = 20 \times 80 + (20 + 20 + 30) \times (100 - 0) \]

\[ m = 26600/540 \approx 49 \text{ g} \]

49 g of steam gets condensed and the final temperature is 100°C.

**Illustration 1.24**

What will be the final temperature when 150 g of ice at 0°C is mixed with 300 g of water at 50°C. Specific heat of water = 1 cal/g°C. Latent heat of fusion of ice = 80 cal/g.

Sol. Let us assume that \(T > 0°C\)

Heat lost by water = heat gained by ice to melt + heat gained by water formed from ice.

\[ 300 \times 1 \times (50 - T) = 150 \times 80 + 150 \times 1 \times (T - 0) \]

\[ T = 6.7°C \]

Hence our assumption that \(T > 0°C\) is correct.

**Illustration 1.25**

In a calorimeter (water equivalent = 40 g) are 200 g of water and 50 g of ice all at 0°C. 30 g of water at 90°C is poured into it. What will be the final condition of the system?

Sol. Let us assume that all ice melts and temperature of water rises beyond 0°C. Thus we will assume that \(T > 0°C\).

Heat lost by water added = heat gained by ice to melt + heat to warm water formed from ice and water added + heat gained by calorimeter can.

\[ 30 \times 1 \times (90 - T) = 50 \times 80 + (50 + 200) \times 1 \times (T - 0) + 40 \times 1 \times T \]

\[ 2700 - 30T = 4000 + 250T + 40T \]

\[ T = -4.1°C \]

Hence our assumption that \(T > 0°C\) is wrong, since hot water added is not able to melt all of the ice.

Therefore the final temperature will be 0°C.

Let \(m = \) mass of ice finally left in the can.

Heat lost by water = heat gained by melting ice.

\[ 30 \times 1 \times (90 - 0) = (50 - m) \times 80 \Rightarrow m = 16.25 \text{ g} \]

Finally there is 16.25 g of ice and \((200 + 30 + 33.75) = 266.75 \text{ g of water at 0°C.}\)

**Heating Curve**

If heat is supplied at constant rate to a given mass \(m\) of a solid, \(P\) and a graph is plotted between temperature and time, the graph is as shown in Fig. 1.3 and is called heating curve. From this curve it is clear that:

- In the region \(OA\) temperature of solid is changing with time, so,

\[ Q = mc_t \Delta T \]

or

\[ P \Delta t = mc_t \Delta T \]  \(\text{as } Q = P \Delta t\)

But as \((\Delta T/\Delta t)\) is the slope of temperature-time curve

\[ c_t \propto 1/\text{slope of line } OA \]

**Fig. 1.3**

i.e., specific heat (or thermal capacity) is inversely proportional to the slope of temperature-time curve.

- In the region \(AB\) temperature is constant, so it represents change of state, i.e., melting of solid with melting point \(T_m\) at \(A\) melting starts and at \(B\) all solid is converted into liquid. So between \(A\) and \(B\) substance is partly solid and partly liquid. If \(L_f\) is the latent heat of fusion.

\[ Q = mL_f \]

or

\[ L_f = \frac{P(t_2 - t_1)}{m} \]

(as \(Q = P(t_2 - t_1)\))

- In the region \(BC\) temperature of liquid increases so specific heat (or thermal capacity) of liquid will be inversely proportional to the slope of line \(BC\), i.e.,

\[ c_l \propto 1/\text{slope of line } BC \]

- In the region \(CD\) temperature is constant, so it represents the change of state, i.e., boiling with boiling point \(T_B\). At \(C\) all substance is in liquid state while at \(D\) in vapour state and between \(C\) and \(D\) partly liquid and partly gas. The length of line \(i\) is proportional to latent heat of vapourization, i.e.,

\[ L_v \propto \text{Length of line } CD \]

(In this region specific heat \(\rightarrow \infty\))

- The line \(DE\) represents gaseous state of substance with its temperature increasing linearly with time. The reciprocal of slope of line will be proportional to specific heat or thermal capacity of substance in vapour state.
A substance is in the solid form at 0°C. The amount of heat added to this substance and its temperature are plotted in the following graph.

Fig. 1.4

If the relative specific heat capacity of the solid substance is 0.5, find from the graph (i) the mass of the substance; (ii) the specific latent heat of the melting process and (iii) the specific heat of the substance in the liquid state.

**Specific heat capacity of water = 1000 cal/kg/K**

**Sol.** 1000 cal of heat raises the temperature of the substance from 0°C to 80°C.

\[ 1000 = m \times (1000 \times 0.5) \times 80 \]

\( \therefore \) specific heat = relative sp. heat × of water

or

\( m = 0.025 \text{ kg} \)

Latent heat = 200 × 5 = 1000 cal (\( \because \) 1 div reads 200 cal) = 0.025 × \( L \)

\( \therefore \) \( L = 40000 \text{ cal/kg} \)

In the liquid state temperature rises from 80°C to 120°C, that is, by 40°C after absorbing 600 cal.

\( \therefore \) 0.025 × 40 = 600 or \( s = 600 \text{ cal/kg/K} \)

**Illustration 1.2** Two bodies of equal masses are heated at a uniform rate under identical conditions. The change in temperature in the two cases is shown graphically. What are their melting points? Find the ratio of their specific heats and latent heats.

**Sol.** The melting points of liquids I and II are 60°C and 40°C, respectively. Let \( R \) be the rate of supply of heat. We note from the graph that liquid I is heated through 60°C in 2 units of time and that liquid II is heated through 40°C in 4 units of time.

\( \therefore \) \( 2R = m \times c_1 \times 60 \) and \( 4R = m \times c_2 \times 40 \)

Hence, \( \frac{c_1}{c_2} = \frac{1}{3} \)

We note further that the temperature of I remains constant for 4 units of time and that of II for 2 units of time.

\( 4R = mL_1 \) and \( 2R = mL_2 \) so \( L_1 = L_2 = \frac{L_1}{L_2} \)

**Concept Application Exercise 1.1**

1. The greater the mass of a body, the greater is its heat capacity. Is this true or false?
2. The greater the mass of a body, the greater is its latent heat capacity. Is this true or false?
3. The greater the mass of a body, the greater is its specific heat capacity. True or false?
4. Can heat be added to a substance without causing the temperature of the body to rise? If so, does this contradict the concept of heat as energy in the process of transfer because of a temperature difference?
5. Can heat be considered to be a form of stored energy?
6. Give an example of a process in which no heat is transferred to or from a system but the temperature of the system changes.
7. The latent heat of fusion of a substance is always less than the latent heat of vapourization or latent heat of sublimation of the same substance. Explain.
8. Suppose an astronaut on the surface of the moon took some water at about 20°C out of his thermos and poured it into a glass beaker. What would happen to the water?
9. Heat is added to a body. Does its temperature necessarily increase?
10. When a hot body warms a cool one, are their temperature changes equal in magnitude?
11. Steam at 100°C is passed into a calorimeter of water equivalent 10 g containing 74 cc of water and 10 g of ice at 0°C. If the temperature of the calorimeter and its contents rises to 5°C, calculate the amount of steam passed. Latent heat of steam = 540 kcal/kg, latent heat of fusion = 80 kcal/kg.
12. Ice of mass 600 g and at a temperature of -10°C is placed in a copper vessel heated to 350°C. The resultant mixture is 550 g of ice and water. Find the mass of the vessel. The specific heat capacity of copper (c) = 100 cal/kg-K.
13. When a small ice crystal is placed in overcooled water it begins to freeze instantaneously.
   i. What amount of ice is formed from 1 kg of water overcooled to -8°C? L of water = 336 × 10³ J/kg and s of water = 4200 J/kg.
   ii. What should be the temperature of the overcooled water in order that all of it be converted into ice at 0°C?
14. An electric heater whose power is 54 W is immersed in 650 cm³ water in a calorimeter. In 3 min the water is heated by 3.4°C. What part of the energy of the heater passes out of the calorimeter in the form of radiant energy?

15. An ice cube whose mass is 50 g is taken from a refrigerator where its temperature was -10°C. If no heat is gained or lost from outside, how much water will freeze onto the cube if it is dropped into a beaker containing water at 0°C? Latent heat of fusion = 80 kcal/kg, specific heat capacity of ice = 500 cal/kg.

16. Equal volumes of three liquids of densities \( \rho_1, \rho_2 \) and \( \rho_3 \), specific heat capacities \( c_1, c_2 \) and \( c_3 \) and temperatures \( t_1, t_2 \) and \( t_3 \), respectively, are mixed together. What is the temperature of the mixture? Assume no changes in volume on mixing.

17. Victoria Falls in Africa is 122 m in height. Calculate the rise in temperature of the water if all the potential energy lost in the fall is converted into heat.

18. Equal masses of three liquids \( A, B \) and \( C \) are taken. Their initial temperatures are \( 10°C, 25°C \) and \( 40°C \), respectively. When \( A \) and \( B \) are mixed the temperature of the mixture is \( 19°C \). When \( B \) and \( C \) are mixed, the temperature of the mixture is \( 35°C \). Find the temperature if all three are mixed.

19. An earthenware vessel loses 1 g of water per second due to evaporation. The water equivalent of the vessel is 0.5 kg and the vessel contains 9.5 kg of water. Find the time required for the water in the vessel to cool to \( 28°C \) from \( 30°C \). Neglect radiation losses. Latent heat of vapourisation of water in this range of temperature is 540 cal/g.

20. A certain amount of ice is supplied heat at a constant rate for 7 min. For the first 1 min, the temperature rises uniformly with time; then it remains constant for the next 4 min and again rises at a uniform rate for the last 2 min. Explain physically these observations and calculate the final temperature. \( L \) of ice = \( 336 \times 10^3 \) J/kg and \( c_{water} = 4200 \) J/kg/K.

21. 1 g steam at 100°C is passed in an insulated vessel having 1 g ice at 0°C. Find the equilibrium temperature of the mixture. Neglect heat capacity of the vessel.

**THERMAL EXPANSION**

Figures 1.6 (a), (b) and (c) show molecules of solid, liquid and gas, respectively, in their random motions. The atoms are essentially in contact with one another. A rock is an example of a solid. It can stand alone because of the forces holding its atoms together.

Atoms in a liquid are also in close contact but can slide over one another. Forces between them strongly resist attempts to push them closer together and also hold them in close contact. Water is an example of a liquid. It can flow, but it is also in close contact. It can flow, but it also remains in an open container because of the forces between its atoms.

Atoms in a gas are separated by distances that are considerably larger than their diameters. Gas must be held in a closed container to prevent it from moving out freely.

Most substances expand when their temperature is raised and contract when cooled. There is an exception to this statement: water contracts when its temperature is increased from 0°C to 4°C. Thus water has its minimum volume, and hence maximum density, at 4°C.

Atoms in solids are in close contact; the forces between them allow the atoms to vibrate but not to move freely. These forces can be thought of as springs that can be stretched or compressed. An individual molecule's motion can be modelled as a point-like particle oscillating in a parallel well caused by the inter-atomic forces, which is parabolic for a Hooke's law spring \( U(x) = \frac{1}{2} kx^2 \). The mass oscillates in simple harmonic motion between maximum and minimum positions.

The potential energy curve is not symmetrical as shown in Fig. 1.7. The variable \( r \) is the separation between a particle and its nearest neighbour. At temperature \( T \), the total energy is \( E \), and its separation from its nearest neighbour lies between \( r_{min} \) and \( r_{max} \). The average separation is \( r_{\text{ave}} \). The \( U(r) \) is not symmetrical, it is flatter to the right at larger \( r \) values. At higher temperature the total energy \( E \) is higher; the particle spends more time at \( r \) values towards less steep portion of the curve.

The average separation \( r_{\text{ave}} \) increases at higher temperatures. Because \( r_{\text{ave}} > r_{\text{min}} \), the average separation of the atoms or molecules in the solid increases with increasing temperature.

When matter is heated without any change in state, it usually expands. According to atomic theory of matter, asymmetry in potential energy curve is responsible for thermal expansion. As with rise in temperature the amplitude of vibration and hence energy of atoms increases, hence the average distance between the atoms increases. So the matter as a whole expands.
Thermal Properties of Matter 1.13

- Thermal expansion is minimum in case of solids but maximum in case of gases because intermolecular force is maximum in solids but minimum in gases.
- Solids can expand in one dimension (linear expansion), two dimension (superficial expansion) and three dimension (volume expansion) while liquids and gases usually suffer change in volume only.
- The coefficient of linear expansion of the material of a solid is defined as the increase in its length per unit length per unit rise in its temperature.

$$\alpha = \frac{\Delta L}{L} \times \frac{1}{\Delta T}$$

Similarly, the coefficient of superficial expansion

$$\beta = \frac{\Delta A}{A} \times \frac{1}{\Delta T}$$

and coefficient of volume expansion

$$\gamma = \frac{\Delta V}{V} \times \frac{1}{\Delta T}$$

The value of $\alpha$, $\beta$ and $\gamma$ depends upon the nature of material. All have dimension $[\theta^{-1}]$ and unit per $^\circ$C.

- As $\alpha = \frac{\Delta L}{L} \times \frac{1}{\Delta T}$, $\beta = \frac{\Delta A}{A} \times \frac{1}{\Delta T}$ and $\gamma = \frac{\Delta V}{V} \times \frac{1}{\Delta T}$

$$\Delta L = \alpha L \Delta T, \Delta A = \beta A \Delta T \text{ and } \Delta V = \gamma V \Delta T$$

Final length

$$L' = L + \Delta L = L(1 + \alpha \Delta T)$$

(i)

Final area $A' = A + \Delta A = A(1 + \beta \Delta T)$

(ii)

Final volume $V' = V + \Delta V = V(1 + \gamma \Delta T)$

(iii)

- If $L$ is the side of square plate and it is heated by temperature $\Delta T$, then its side becomes $L'$.

The initial surface area $A = L^2$ and final surface area $A' = L'^2$

$$\frac{A'}{A} = \left(\frac{L'}{L}\right)^2 = \left(\frac{L(1 + \alpha \Delta T)}{L}\right)^2 = (1 + \alpha \Delta T)^2 = (1 + 2\alpha \Delta T)$$

(Using Binomial theorem)

or $A' = A(1 + 2\alpha \Delta T)$

Comparing with Eq. (ii), we get $\beta = 2\alpha$

Similarly, for volumetric expansion

$$\frac{V'}{V} = \left(\frac{L'}{L}\right) = \left(\frac{L(1 + \alpha \Delta T)}{L}\right) = (1 + \alpha \Delta T)^3 = (1 + 3\alpha \Delta T)$$

(Using Binomial theorem)

or $V' = V(1 + \gamma \Delta T)$

Comparing with Eq. (iii), we get $\gamma = 3\alpha$

So $\alpha : \beta : \gamma = 1 : 2 : 3$

### Some Important Points to Note

1. For the same rise in temperature, percentage change in area = $2 \times$ percentage change in length.
2. Percentage change in volume = $3 \times$ percentage change in length.
3. The three coefficients of expansion are not constant for a given solid. Their values depend on the temperature range in which they are measured.
4. The values of $\alpha$, $\beta$ and $\gamma$ are independent of the units of length, area and volume, respectively.
5. For anisotropic solids $\gamma = \alpha_x + \alpha_y + \alpha_z$, where $\alpha_x$, $\alpha_y$ and $\alpha_z$ represent the mean coefficients of linear expansion along three mutually perpendicular directions.

### Hint: Example 6.1

The rectangular plate shown in Fig. 1.8 has an area $A$. If the temperature increases by $\Delta T$, each dimension increases according to $\Delta L = \alpha L \Delta T$, where $\alpha$ is the average coefficient of linear expansion. Show that the increase in area is $\Delta A = 2\alpha A \Delta T$. What approximation does this expansion assume?

![Fig. 1.8](image)

**Sol.** We expect the area to increase in thermal expansion. It is neat that the coefficient of area expansion is just twice the coefficient of linear expansion.

![Fig. 1.9](image)
We will use the definitions of coefficients of linear and area expansion.

From the diagram in Fig. 1.9, we see that the change in area is

\[ \Delta A = l \Delta w + w \Delta l + \Delta w \Delta l \]

Since \( \Delta l \) and \( \Delta w \) are each small quantities, the product \( \Delta w \Delta l \) will be very small as compared to the original or final area.

Therefore, we assume \( \Delta w \Delta l \approx 0 \)

Since \( \Delta w = l \alpha \Delta T \) and \( \Delta l = l \alpha \Delta T \)

We then have \( \Delta A = l w \alpha \Delta T + w l \alpha \Delta T \)

Finally, since \( A = l w \), we have \( \Delta A = 2 \alpha \Delta T \)

**Illustration 1.26**  A mercury thermometer is constructed as shown in Fig. 1.10. The capillary tube has a diameter of 0.004 cm, and the bulb has a diameter of 0.250 cm. Neglecting the expansion of the glass, find the change in height of the mercury column with a temperature change of 30.0°C.

**Sol.** For an easy-to-read thermometer, the column should rise by a few centimetres.

We use the definition of the coefficient of expansion.

Neglecting the expansion of the glass, the volume of liquid in the capillary will be \( \Delta V = \Delta h A \) where \( A \) is the cross-sectional area of the capillary. Let \( V_1 \) represent the volume of the bulb.

\[ \Delta V = V_1 \gamma \Delta T \]

\[ \Delta h = \frac{(V_1)}{A} \gamma \Delta T = \left[ \frac{4}{3} \pi R_{cm}^3 \right] \gamma \Delta T \]

\[ \Delta h = \frac{4}{3} \left( \frac{0.125 \text{ cm}}{0.004 \text{ cm}} \right)^3 \times (1.82 \times 10^{-4} \text{ cm}) (30.0^\circ \text{C}) = 3.55 \text{ cm} \]

This is a practical thermometer. Glass expands so little as compared to mercury that only the third digit of the answer would be affected by including the expansion of the glass in our analysis.

**Illustration 1.27**  A metal rod \( A \) of 25 cm length expands by 0.05 cm when its temperature is raised from 0°C to 100°C. Another rod \( B \) of a different metal of length 40 cm expands by 0.04 cm for the same rise in temperature. A third rod \( C \) of 50 cm length made up of pieces of rods \( A \) and \( B \) placed end to end expands by 0.03 cm on heating from 0°C to 50°C. Find the length of each portion of composite rod \( C \).

**Sol.** From the given data for rod \( A \), we have

\[ \Delta L = \alpha_A L_1 \Delta T \]

or

\[ \alpha_A = \frac{\Delta L}{L_1 \Delta T} = \frac{0.05}{25 \times 100} = 2 \times 10^{-5} \text{C} \]

For rod \( B \), we have

\[ \Delta L = \alpha_B L_2 \Delta T \]

or

\[ \alpha_B = \frac{\Delta L}{L_2 \Delta T} = \frac{0.04}{40 \times 100} = 10^{-5} \text{C} \]

If rod \( C \) is made of segments of rod \( A \) and \( B \) of lengths \( l_1 \) and \( l_2 \) respectively, then we have at 0°C

\[ l_1 + l_2 = 50 \text{ cm} \]

At \( T = 50^\circ \text{C} \), \( l_1 + l_2 = 50.03 \text{ cm} \)

Thus

\[ \alpha_A l_1 \Delta T + \alpha_B l_2 \Delta T = 0.03 \text{ cm} \]

or

\[ 2 \times 10^{-5} \times l_1 \times 50 + 10^{-5} \times l_2 \times 50 = 0.03 \text{ cm} \]

or

\[ 2l_1 + l_2 = 0.03 \times \frac{50}{50} = 0.6 \text{ cm} \]

Solving Eqs. (i) and (ii), we get \( l_1 = 10 \text{ cm} \) and \( l_2 = 40 \text{ cm} \).

**Illustration 1.28**  Determine the lengths of an iron rod and a copper ruler at 0°C if the difference in their lengths at 50°C and 450°C is the same and is equal to 2 cm. The coefficient of linear expansion of iron = \( 12 \times 10^{-5} / \text{K} \) and that of copper = \( 17 \times 10^{-5} / \text{K} \).

**Sol.** Let \( x \) be the length of the iron rod at 0°C, \( y \) that of the copper rod at 0°C, and \( l \) the difference in lengths at \( t_1 \) and \( t_2 \)°C.

Then

\[ l = x (1 + \alpha_{t_1}) - y (1 + \alpha_{t_2}) \]

and

\[ l = x (1 + \alpha_{t_2}) - y (1 + \alpha_{t_1}) \]

Taking +1

\[ l = x (1 + \alpha_{t_2}) - y (1 + \alpha_{t_1}) \]

from Eqs. (i) and (iii), we get

\[ x\alpha_{t_1} = y\alpha_{t_2} \]

from Eqs. (i) and (iv), we get

\[ y = \frac{l\alpha_x}{\alpha_z - \alpha_x} = \frac{2 \times 12 \times 10^{-5}}{(17 - 12) \times 10^{-5}} = 4.8 \text{ cm} \]

and

\[ x = \frac{l\alpha_y}{\alpha_z - \alpha_y} \]

\[ x = \frac{2 \times 17 \times 10^{-5}}{(17 - 12) \times 10^{-5}} \text{ cm} = 6.8 \text{ cm} \]

Taking -1
\[
y = \frac{2t + 1; x = \frac{2t + 1}{(t_1 + t_2)}
\]
\[
x = \frac{2t + 1}{(t_1 - t_2)(\alpha_2 - \alpha_1)}
\]
\[
y' = \frac{2 \times 2 \times 12 \times 10^{-6} \times (450 + 50)}{(450 - 50)(17 - 12) \times 10^{-6}}
\]
\[
= 2006 \text{ cm} = 20.06 \text{ m}
\]
\[
x' = \frac{2 \times 2 \times 17 \times 10^{-6} \times (450 + 50)}{(450 - 50)(17 - 12) \times 10^{-6}}
\]
\[
= 2008.5 \text{ cm} = 20.08 \text{ m}
\]

**Illustration 1.39** A steel ball initially at a pressure of 10^5 Pa is heated from 20°C to 120°C keeping its volume constant. Find the final pressure inside the ball. Given that coefficient of linear expansion of steel is 1.1 \times 10^{-5}/°C and Bulk modulus of steel is 1.6 \times 10^{11} \text{ N/m}^2.

**Sol.** On increasing temperature of ball by 100°C (from 20°C to 120°C), the thermal expansion in its volume can be given as
\[
\Delta V = \gamma V \Delta T = 3 \alpha V \Delta T.
\]  
Here it is given that no change of volume is allowed. This implies that the volume increment by thermal expansion is compressed elastically by external pressure. Thus elastic compression in the sphere must be equal to that given in Eq. (i). Bulk modulus of a material is defined as
\[
B = \text{inCREASE in PRESSure} \div \text{VOLUME STRAIN} = \frac{\Delta P}{\Delta V/V}.
\]
Here the externally applied pressure to keep the volume of ball constant is given as
\[
\Delta P = B \times \Delta V = B(3 \alpha \Delta T)
\]
\[
= 1.6 \times 10^{11} \times 3 \times 1.1 \times 10^{-5} \times 100
\]
\[
= 5.28 \times 10^8 \text{ N/m}^2 = 5.28 \times 10^8 \text{ Pa}
\]
Thus this must be the excess pressure inside the ball at 120°C to keep its volume constant during heating.

**Illustration 1.40** A steel rail 30 m long is firmly attached to the roadbed only at its ends. The sun raises the temperature of the rail by 5°C, causing the rail to buckle. Assuming that the buckled rail consists of two straight parts meeting in the centre, calculate how much the centre of the rail rises. Coefficient of linear expansion of steel is 12 \times 10^{-6}/K.

**Sol.** As indicated in Fig. 1.11, we let the initial length be 2s and the final total length be 2(s + \Delta s).

The height of the centre of the buckled rail is denoted by \(y\). Assuming that the standard coefficient \(\alpha\) of linear expansion can be used (in spite of the fact that the ends are anchored), we have
\[
\Delta s = \alpha x \Delta T.
\]
By the Pythagorean theorem
\[
y = \sqrt{(s + \Delta s)^2 - s^2} = \sqrt{2s \Delta s + (\Delta s)^2} = s \sqrt{2 \alpha \Delta T + (\alpha \Delta T)^2}
\]

**Illustration 1.41** A small quantity of a liquid which does not mix with water sinks to the bottom at 20°C, the densities of the liquid and water being 1021 and 998 kg/m³, respectively. To what temperature must the mixture be uniformly heated in order that the liquid forms globules which just float on water? The cubical expansion of the liquid and water over the temperature ranges is 85 \times 10^{-6}/K and 45 \times 10^{-5}/K respectively.

**Sol.** The liquid will float on water at the temperature at which both of them have the same densities.
\[
D_{\text{common}} = \frac{1021}{1 + 85 \times 10^{-3} \Delta \theta} = \frac{998}{1 + 45 \times 10^{-5} \Delta \theta}
\]
\[
\Rightarrow 1021(1 + 45 \times 10^{-5} \Delta \theta) = 998(1 + 85 \times 10^{-3} \Delta \theta)
\]
1.16 Waves & Thermodynamics

\[ \Rightarrow \Delta \theta = 59^\circ C \Rightarrow \theta_2 - \theta_1 = 59^\circ C \]
\[ \Rightarrow \theta_2 = 59 + 20 = 79^\circ C \]

**Expansion of Liquid**

Liquids also expand on heating just like solids. Since liquids have no shape of their own, they suffer only volume expansion. If the liquid of volume \( V \) is heated and its temperature is raised by \( \Delta \theta \) then

\[ V'_v = V(1 + \gamma_v \Delta \theta) \]

\( (\gamma_v = \text{coefficient of real expansion or coefficient of volume expansion of liquid}) \)

As liquid is always taken in a vessel for heating, if a liquid is heated, the vessel also gets heated and it also expands.

\[ V'_s = V(1 + \gamma_s \Delta \theta) \]

\( (\gamma_s = \text{coefficient of volume expansion for solid vessel}) \)

So, the change in volume of liquid relative to vessel

\[ V'_v - V'_s = V(\gamma_v - \gamma_s) \Delta \theta \]

\[ \Delta V_{app} = V \gamma_{app} \Delta \theta \quad (\gamma_{app} = \gamma_v - \gamma_s = \text{Apparent coefficient of volume expansion for liquid}) \]

<table>
<thead>
<tr>
<th>( \gamma_v )</th>
<th>( \gamma_{app} )</th>
<th>( \Delta V_{app} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( &gt; 0 )</td>
<td>( &gt; 0 )</td>
<td>positive</td>
</tr>
<tr>
<td>( \gamma_s )</td>
<td>( \gamma_v )</td>
<td>( &gt; 0 )</td>
</tr>
<tr>
<td>( &lt; 0 )</td>
<td>( &lt; 0 )</td>
<td>negative</td>
</tr>
<tr>
<td>( \gamma_v, \gamma_s )</td>
<td>( 0 )</td>
<td>( 0 )</td>
</tr>
</tbody>
</table>

We will use the definition of the volume expansion coefficient. Both the liquid and the container expand. We will need to reason carefully about original, intermediate and final volumes of each.

When the temperature is increased from 20.0°C to 80.0°C, both the cylinder and the turpentine increase in volume by \( \Delta V = \gamma V \Delta T \):

a. The overflow is \( V_{over} = \Delta V_{cyl} - \Delta V_{t} \)

\[ V_{over} = (\gamma V \Delta T)_{cyl} - (\gamma V \Delta T)_{t} = V(\gamma_v \Delta T - 3 \alpha_t) \]

\[ V_{over} = (2.000 \text{L}) (60.0^\circ C) (9.00 \times 10^{-4}^\circ C^{-1} - 0.720 \times 10^{-3}^\circ C^{-1}) \]

\[ = 0.0994 \text{L} \]

b. After warming, the whole volume of the turpentine is

\[ V = 2000 \text{cm}^3 \times (9.00 \times 10^{-4}^\circ C^{-1})(2000 \text{cm}^3)(60.0^\circ C) = 2108 \text{cm}^3 \]

The fraction lost is \( \frac{99.4 \text{cm}^3}{2108 \text{cm}^3} = 0.0471 \)

This also is the fraction of the cylinder that will be empty after cooling. Therefore, change in level

\[ \Delta h = (0.0471 \times 20.0 \text{cm}) = 0.943 \text{cm} \]

The change in volume of the container is not negligible, but is 8% of the change in volume of the turpentine.

**Illustration 1.34**

A 1-L flask contains some mercury.

It is found that at different temperatures, the volume of air inside the flask remains the same. What is the volume of mercury in the flask, given that the coefficient of linear expansion of glass is \( 9 \times 10^{-6}^\circ C^{-1} \) and the coefficient of volume expansion of Hg is \( 1.8 \times 10^{-4}^\circ C^{-1} \)?

Sol. Since the volume above mercury remains the same at all temperatures, the expansion of the glass vessel must be the same as that of mercury in the vessel. Also, \( \gamma_e = 3 \alpha_e = 27 \times 10^{-6}^\circ C^{-1} \). Let \( V \) be the volume of the mercury. Then, from \( \Delta V = V \gamma \Delta T \)

\[ V' = (1.8 \times 10^{-4}) \Delta T = 10^{-7} \times 27 \times 10^{-4} \Delta T \]

\[ (\because 1 \text{L} = 10^{-3} \text{m}^3 \text{and } \gamma = 3 \alpha_e) \text{ or } V = 150 \times 10^{-6} \text{m}^3 \]

**Illustration 1.35**

A hollow aluminium cylinder 20.0 cm deep has an internal capacity of 2.000 L at 20.0°C. It is completely filled with turpentine and then slowly warmed to 80.0°C. a. How much turpentine overflows? b. If the cylinder is then cooled back to 20.0°C, how far below the cylinder's rim does the turpentine's surface recede?

Sol. We guess that vertical cubic centimetres of turpentine will overflow, and that the liquid level will drop about a centimetre.

The water runs over if:

a. the expansion of the bottle is neglected;

b. the expansion of the bottle is included? Given the coefficient of areal expansion of glass \( \beta = 1.2 \times 10^{-4}^\circ K^{-1} \) and \( \gamma_{water} = 60 \times 10^{-6}^\circ C^{-1} \).

Sol. Water overflow = (final volume of water) - (final volume of bottle)
a. If the expansion of bottle is neglected:

\[ \text{Water overflow} = 250(1 + \gamma \theta) - 250 = 250 \times 60 \times 10^{-3} \times 10 \]

\[ \Rightarrow \text{water overflow} = 1.5 \text{ cm}^3 \]

b. If the bottle (glass) expands:

\[ \text{Water overflow} = (\text{final volume of water}) - (\text{final volume of glass}) \]

\[ = 250(1 + \gamma_1 \theta) - 250(1 + \gamma_2 \theta) \]

\[ = 250(\gamma_1 - \gamma_2)\theta, \text{ where } \gamma_2 = 3/2\beta = 1.8 \times 10^{-5} / ^\circ\text{C} \]

\[ = 250(\gamma_1 - \gamma_2)\theta \times (60 - 50) \]

\[ \text{Water overflow} = 1.455 \text{ cm}^3 \]

**Effect of Temperature on Upthrust**

The thrust on volume \( V \) of a body in a liquid of density \( \sigma \) is given by \( \text{Th} = V \sigma g \)

Now, with rise in temperature by \( \Delta \theta ^\circ\text{C} \), due to expansion, volume of the body will increase while density of liquid will decrease according to the relations \( V' = V(1 + \gamma_1 \Delta \theta) \) and \( \sigma' = \sigma / (1 + \gamma_2 \Delta \theta) \)

So the thrust will become \( \text{Th}' = V \sigma' g \)

\[ \text{Th} = \frac{V \sigma g}{\left(1 + \gamma_2 \Delta \theta\right)} \]

and apparent weight of the body \( W_{\text{app}} = \text{actual weight} - \text{thrust} \)

As \( \gamma_2 < \gamma_1 \), therefore, \( \text{Th} < \text{Th}' \) with rise in temperature thrust also decreases and apparent weight of body increases.

**Illustration**

A solid floats in a liquid at 20°C with 75% of it immersed. When the liquid is heated to 100°C, the same solid floats with 80% of it immersed in the liquid. Calculate the coefficient of expansion of the liquid. Assume the volume of the solid to be constant.

**Solution**

Let \( m \) be the mass of the solid and \( V \) its volume. By the law of flotation

- Weight of floating object = Buoyant force

- In Case I: \( mg = \frac{3}{4} V \rho_{\text{sol}} g \)
  where \( \rho_{\text{sol}} \) = density of liquid at 20°C

- In Case II: \( mg = \left( \frac{80}{100} V \right) \rho_{\text{sol}} g \)
  where \( \rho_{\text{sol}} \) = density of liquid at 100°C

Considering both the cases

\[ \Rightarrow \frac{3}{4} \rho_{\text{sol}} = \frac{4}{5} \rho_{\text{sol}} = \frac{3}{41} \rho_{\text{sol}} \times 20 = \frac{4}{51} \rho_{\text{sol}} \times 100 \]

After solving we get

\[ \gamma = \frac{1}{1180} = 8.47 \times 10^{-4} / ^\circ\text{C} \]

**Anomalous Expansion of Water**

1. Generally matter expands on heating and contracts on cooling. In case of water, it expands on heating if its temperature is greater than 4°C. In the range 0°C to 4°C, water contracts on heating and expands on cooling, i.e., \( \gamma \) is negative. This behaviour of water in the range from 0°C to 4°C is called anomalous expansion.

2. The anomalous behaviour of water arises due to the fact that water has three types of molecules, viz., \( H_2O, (H_2O)_2 \) and \( (H_2O)_3 \), having different volume per unit mass values and at different temperatures their properties in water are different.

3. At 4°C, density of water is maximum while its specific volume is minimum.

During winter when the water on the surface of a lake cools below 4°C by cold air, it expands and becomes lighter than water below. Therefore, the water cooled below 4°C stays on the surface and freezes when the temperature of surroundings falls below 0°C. Thus the lake freezes first on the surface and water in contact with ice has temperature 0°C while at the bottom of the lake 4°C (as density of water at 4°C is maximum) and fish and other aquatic animals remain alive in this water.
Expansion of Gases
Gases have no definite shape; therefore, gases have only volume expansion. Since the expansion of container is negligible in comparison to the gases, gases have only real expansion.

Coefficient of Volume Expansion
At constant pressure, increase in volume per unit volume per unit degree rise of temperature is called coefficient of volume expansion.

\[ \alpha = \frac{\Delta V}{V} \times \frac{1}{\Delta T} \]
\[ \therefore \ \text{Final volume} \ V' = V(1 + \alpha \Delta T) \]

Coefficient of Pressure Expansion
\[ \beta = \frac{\Delta P}{P} \times \frac{1}{\Delta T} \]
\[ \therefore \ \text{Final pressure} \ P' = P(1 + \beta \Delta T) \]

For an ideal gas, coefficient of volume expansion is equal to the coefficient of pressure expansion.

i.e. \[ \alpha = \beta = \frac{1}{273} \ \text{°C} \]

Application of Thermal Expansion
1. Bi-metallic strip: Two strips of equal lengths but of different materials (different coefficient of linear expansion) when join together, is called 'bi-metallic strip' and can be used in thermostats to break or make electrical contact. This strip has the characteristic property of bending on heating due to unequal linear expansion of the two metals. The strip will bend with metal of greater \( \alpha \) on outer side, i.e., convex side.

Illustration 1.38
A copper and a tungsten plate having a thickness \( \delta = 2 \text{ mm} \) each are riveted together so that at 0°C they form a flat bimetallic plate. Find the average radius of curvature of this plate at \( r = 200^\circ C \). The coefficients of linear expansion for copper and tungsten are \( \alpha_c = 1.7 \times 10^{-5}/K \) and \( \alpha_w = 0.4 \times 10^{-5}/K \), respectively.

Sol. The average length of copper plate at a temperature \( T = 200^\circ C \) is
\[ l_c = l_0 (1 + \alpha_c T) \]
where \( l_0 \) is the length of copper plate at 0°C. The length of the tungsten plate is
\[ l_t = l_0 (1 + \alpha_w T) \]

We shall assume that the edges of plates are not displaced during deformation and that an increase in the plate thickness due to heating can be neglected.

From Fig. 1.16 we have
\[ l_c = \phi (R + \delta / 2) \Rightarrow l_t = \phi (R - \delta / 2) \]

Consequently,
\[ \phi (R + \delta / 2) = l_0 (1 + \alpha_c T) \quad (i) \]
\[ \phi (R - \delta / 2) = l_0 (1 + \alpha_w T) \quad (ii) \]

To eliminate the unknown quantities, \( \phi \) and \( l_0 \), we divide Eq. (i) by Eq. (ii) term-wise:
\[ \frac{R + \delta / 2}{R - \delta / 2} = \frac{(1 + \alpha_c T)}{(1 + \alpha_w T)} \Rightarrow R = \frac{\delta}{(\alpha_c - \alpha_w) T} \]

\[ \Rightarrow R = \frac{\delta}{2(\alpha_c + \alpha_w) T} \]

\[ \Rightarrow R = \frac{\delta}{(\alpha_c - \alpha_w) T} \]
neglecting \((\alpha_c + \alpha_s)\) in numerator. Substituting the values in above relation, we get: \(R = 0.769\) m.

2. Effect of temperature on the time period of a simple pendulum: A pendulum clock keeps proper time at temperature \(\theta\). If temperature is increased to \(\theta' (> \theta)\) then due to linear expansion, length of pendulum and hence its time period will increase. If \(L_0\) be the length of the pendulum, at \(\theta^\circ\text{C}\), then its time period

\[
T_0 = 2\pi \sqrt{\frac{L_0}{g}}
\]

At any temperature increment \(\Delta \theta\), the time period of the pendulum is given by

\[
T = 2\pi \sqrt{\frac{L}{g}} = T_0 (1 + \alpha L \Delta \theta)^\frac{1}{2}
\]

\[
= T_0 (1 + \alpha \Delta \theta)^\frac{1}{2}
\]

\[
= T_0 \left(1 + \frac{\alpha \Delta \theta}{2}\right) \text{ or } \frac{T}{T_0} = 1 + \frac{\alpha \Delta \theta}{2}
\]

\[
T - T_0 = \frac{\alpha \Delta \theta}{2} \text{ or } \Delta T = \frac{\alpha \Delta \theta}{2}
\]

\(\therefore\) 

\[
\Delta T = \left(\frac{\alpha \Delta \theta}{2}\right) T_0
\]

Note:
- Due to increment in its time period, a pendulum clock becomes slow in summer and will lose time.
- Loss of time in a time period \(\Delta t\) is \(\Delta t = \frac{1}{2} \alpha \Delta \theta T_0\).
- Loss of time in any given time interval \(t\) can be given by

\[
\Delta t = \frac{1}{2} \alpha \Delta \theta t
\]

- The clock will lose time, i.e., it will become slow if \(\theta' > \theta\) (in summer).
- It will gain time, i.e., it will become fast if \(\theta' < \theta\) (in winter).
- The gain or loss in time is independent of time period \(T\) and depends on the time interval \(t\).
- Time lost by the clock in a day \(= 86400\) s

\[
\Delta t = \frac{1}{2} \alpha \Delta \theta t = \frac{1}{2} \alpha \Delta \theta (86400) = 43200 \alpha \Delta \theta \text{ s}
\]
- Since coefficient of linear expansion \((\alpha)\) is very small for invar, pendulums are made of invar to show the correct time in all seasons.

Illustration 1.40
A clock with a brass pendulum shaft keeps correct time at a certain temperature.

a. How closely must the temperature be controlled if the clock is not to gain or lose more than 1 s a day? Does the answer depend on the period of the pendulum?
b. Will an increase of temperature cause the clock to gain or lose? \((\alpha_{\text{Brass}} = 2 \times 10^{-5} /\text{C})\)

Sol.
a. Number of seconds lost or gained per day = \(\frac{1}{2} \alpha \theta \times 86400\),

where \(\theta = \text{rise or drop in temperature; } \alpha = \text{coeff. of linear expansion of shaft.}\)

We want that

\[
\frac{1}{2} \alpha \theta \times 86400 < 1
\]

\[
\Rightarrow \theta < \frac{2}{2 \times 10^{-5} \times 86400} \Rightarrow \theta < 1.157^\circ\text{C}
\]

Hence temperature should not increase or decrease by more than \(1.157^\circ\text{C}\). It does not depend upon time period.

b. An increase in temperature makes the pendulum slow and hence clock loses time.

Illustration 1.40
A pendulum clock loses 12 s a day if the temperature is \(40^\circ\text{C}\) and goes fast by 4 s a day if the temperature is \(20^\circ\text{C}\). Find the temperature at which the clock will show correct time and the coefficient of linear expansion of the metal of the pendulum shaft.

Sol.
Let \(T\) be the temperature at which the clock is correct.

Time lost per day = \(\frac{1}{2} \alpha \theta\) (rise in temperature) \(\times 86400\)

\[
12 = \frac{1}{2} \alpha \theta (40 - T) \times 86400
\]

Time gained per day = \(\frac{1}{2} \alpha \theta\) (drop in temperature) \(\times 86400\)

\[
4 = \frac{1}{2} \alpha \theta (T - 20) \times 86400
\]

Adding Eqs. (i) and (ii), we get

\[
32 = 86400\alpha (40 - 20) \Rightarrow \alpha = 1.85 \times 10^{-5} \text{C}
\]

Dividing Eq. (i) by Eq. (ii), we get

\[
12(T - 20) = 4 (40 - T) \Rightarrow T = 25^\circ\text{C}
\]

\(\Rightarrow\) Clock shows correct time at \(25^\circ\text{C} \)

3. Thermal stress in a rigidly fixed rod: When a rod whose ends are rigidly fixed such as to prevent expansion or contraction, undergoes a change in temperature, a compressive or tensile stress is developed in it. Due to this thermal stress the rod will exert a large force on the supports.

![Fig. 1.17](https://example.com/fig1.17)
If temperature of rod is increased by $\Delta T$, then change in length
\[ \Delta l = \alpha l \Delta T \]
strain = \[ \frac{\Delta l}{l} = \alpha \Delta T \]
But due to rigid support, there is no strain. Supports provide force or stresses to keep the length of rod same
\[ Y = \text{stress} \]
thermal stress = \[ Y \text{ strain} = Y \alpha \Delta T \]
\[ \frac{F}{A} = Y \alpha \Delta T \]
\[ F = A Y \alpha \Delta T \]

**Illustration 1.41:** A rod of length 2 m is at a temperature of 20°C. Find the free expansion of the rod, if the temperature is increased to 50°C, then find stress produced when the rod is (i) fully prevented to expand, (ii) permitted to expand by 0.4 mm. $Y = 2 \times 10^{11}$ N/m²; $\alpha = 15 \times 10^{-6}$/°C.

**Sol.** Free expansion of the rod = $\alpha L \Delta \theta$
\[ = 15 \times 10^{-6}$/°C \times 2 \times (50 - 20)°C \]
\[ = 9 \times 10^{-4} \text{ m} = 0.9 \text{ mm} \]

(i) If the expansion is fully prevented,
then strain = \[ \frac{9 \times 10^{-4}}{2} \Rightarrow 4.5 \times 10^{-4} \]
:. Temperature stress = strain $\times Y$
\[ = 4.5 \times 10^{-4} \times 2 \times 10^{11} = 9 \times 10^7 \text{ N/m}^2 \]

(ii) If 0.4 mm expansion is allowed, then length restricted to expand = 0.9 - 0.4 = 0.5 mm

:. Strain = \[ \frac{5 \times 10^{-4}}{2} = 2.5 \times 10^{-4} \]

:. Temperature stress = strain $\times Y$
\[ = 2.5 \times 10^{-4} \times 2 \times 10^{11} = 5 \times 10^7 \text{ N/m}^2 \]

**Illustration 1.42:** Two rods of different metals having the same area of cross section $A$ are placed between the two massive walls as shown in Fig. 1.18. The first rod has a length $l_1$, coefficient of linear expansion $\alpha_1$ and Young's modulus $Y_1$. The corresponding quantities for second rod are $l_2$, $\alpha_2$, and $Y_2$. The temperature of both the rods is now raised by $\theta$°C.

![Fig. 1.18](image)

i. Find the force with which the rods act on each other (at higher temperature) in terms of given quantities.
ii. Also find the length of the rods at higher temperature.

**Sol.**

i. Let $\theta$°C = increase in the temperature.
Increase in length of first rod = $l_1 \alpha_1 \theta$
Increase in length of second rod = $l_2 \alpha_2 \theta$

:. Total extension in length due to rise in temperature
\[ = l_1 \alpha_1 \theta + l_2 \alpha_2 \theta = (l_1 \alpha_1 + l_2 \alpha_2) \theta \]  \hspace{1cm} (i)

Since the walls are rigid, this increase in length will not happen. This will be compensated by equal and opposite forces $F_1$, $F_2$ producing decrease in the lengths of the rods due to elasticity.

:. Decrease in length of first rod = $\frac{F_1 \times l_1}{Y_1 \times A}$
And decrease in length of second rod = $\frac{F_2 \times l_2}{Y_2 \times A}$

:. Total decrease in length due to elastic force
\[ = \frac{F}{A} \left( \frac{l_1}{Y_1} + \frac{l_2}{Y_2} \right) \]

\hspace{1cm} \hspace{1cm} (ii)

From Eqs. (i) and (ii), we have
\[ \frac{F}{A} \left( \frac{l_1}{Y_1} + \frac{l_2}{Y_2} \right) = (l_1 \alpha_1 + l_2 \alpha_2) \theta \]

or
\[ F = \frac{A (l_1 \alpha_1 + l_2 \alpha_2) \theta}{l_1 + l_2} \]
\[ \text{or} \quad F = \frac{A \left( l_1 \alpha_1 + l_2 \alpha_2 \right) \theta}{l_1 + l_2} \]

\hspace{1cm} \hspace{1cm} (iii)

**Illustration 1.43:** Two rods of equal cross sections, one of copper and the other of steel, are joined to form a composite rod of length 2.0 m at 20°C; the length of the copper rod is 0.5 m. When the temperature is raised to 120°C, the length of composite rod increases to 2.002 m. If the composite rod is fixed between two rigid walls and thus not allowed to expand, it is found that the lengths of the component rods also do not change with increase in temperature. Calculate Young's modulus of steel. (The coefficient of linear expansion of copper, $\alpha_c = 1.6 \times 10^{-5}$/°C and Young's modulus of copper is $1.3 \times 10^{11}$ N/m².)
\textbf{Sol. Change in length:}

For \textit{Cu} rod

\[
\alpha_{\text{Cu}}(t_2 - t_1) = 0.5 \times \alpha_{\text{Cu}} \times (120 - 20) = 50 \alpha_{\text{Cu}}
\]

For steel rod

\[
\alpha_{\text{Steel}}(t_2 - t_1) = 1.5 \alpha_{\text{Steel}} = 150 \alpha_{\text{Cu}}
\]

\[\therefore \text{Total change in length} = 50 \alpha_{\text{Cu}} + 150 \alpha_{\text{Cu}} = 0.002 \text{ m}\]

\[
\Rightarrow \alpha_{\text{Cu}} = \frac{4 \times 10^{-5} - \alpha_{\text{Cu}}}{3} = \frac{4 \times 10^{-5} - 1.6 \times 10^{-5}}{3} = 0.8 \times 10^{-5} \degree \text{C}
\]

Stress in steel rod, \(f = \sigma \times \text{strain} = \sigma \times \Delta l/l\]

\[
= \frac{Y \alpha_{\text{Steel}}(t_2 - t_1)}{100} = \frac{100 Y \alpha_{\text{Cu}}}{100} = 100 \frac{Y \alpha_{\text{Cu}}}{100}
\]

There is no change in the length of individual rod, because the length change due to stress is balanced by length change due to thermal expansion.

Similarly, stress in copper rod, \(f = \sigma / 100 = 100 \sigma / 100\)

Now stress is same in both:

\[
\frac{Y \alpha_{\text{Cu}}}{100} = \frac{1.3 \times 10^{-3} \times 1.6 \times 10^{-3}}{0.8 \times 10^{-5}} = 2.6 \times 10^3 \text{ N/m}^2
\]

4. \textbf{Error in scale reading due to expansion or contraction: If a scale gives correct reading at temperature } \(\theta, \text{ at temperature } \theta'(>\theta) \text{ due to linear expansion of scale, the scale will expand and scale reading will be lesser than true value so that}\)

\[
\text{True value} = \text{scale reading} [1 + \alpha(\theta' - \theta)]
\]

i.e., \(TV = SR[1 + \alpha\Delta \theta] \text{ with } \Delta \theta = (\theta' - \theta)\)

However, if \(\theta' < \theta, \text{ due to contractions of scale, scale reading will be more than true value, so true value will be lesser than scale reading and will still be given by same equation with } \Delta \theta = (\theta' - \theta) \text{ being negative.}\)

\textbf{Illustration 4.25: A barometer with a brass scale reads 755 mm on a day when the temperature is 25\degree C. If the scale is correctly graduated at 0\degree C, find the true pressure at 0\degree C \textbf{(in terms of height of Hg)} \text{ given that the coefficient of linear expansion of brass is } 18 \times 10^{-6}/\degree \text{C. Coefficient of cubical expansion of mercury } = 182 \times 10^{-9}/\degree \text{K}.\textbf{Sol. Given that } 1 \text{ mm at 0\degree C } = 1 \text{ mm\ud{0}})

\[
\therefore 755 \text{ mm at 25\degree C} = 755(1 + 18 \times 10^{-6} \times 25) \text{ mm} = 755.34 \text{ mm}
\]

Let \(P\) be the value of the atmospheric pressure.

Then \(P = 755.34 \rho_{\text{Hg}} = \rho_{\text{Hg}}\rho_{\text{Hg}}\), where \(\rho_{\text{Hg}}, \rho_{\text{Hg}}\) are densities of mercury at 0\degree C and 25\degree C, respectively.

or \(h = 755.34 \frac{\rho_{\text{Hg}}}{\rho_{\text{Hg}}} = 755.34 \rho_{\text{Hg}} \rho_{\text{Hg}}(1 + 182 \times 10^{-9} \times 25)
\]

or \(h = 755.19 \text{ mm}\)

\textbf{Illustration 4.26: At room temperature (25\degree C) the length of a steel rod is measured using a brass centimetre scale. The measured length is 20 cm. If the scale is calibrated to read accurately at temperature 0\degree C, find the actual length of steel rod at room temperature}\)

Sol. The brass scale is calibrated to read accurately at 0\degree C. This means at 0\degree C, each division of scale has exact 1 cm length. Thus at higher temperature the division length of scale will be more than 1 cm due to thermal expansion. Thus at higher temperature the scale reading for length measurement is not appropriate and as at higher temperature the division length is more, the length this scale reads will be lesser than the actual length to be measured. For illustration in this case the length of each division on brass scale at 25\degree C is

\[
l_{\text{div}} = (1 \text{ cm})[1 + \alpha_{\text{Br}}(25 - 0)]
\]

\[
= 1 + \alpha_{\text{Br}}(25)
\]

It is given that at 25\degree C the length of steel rod measured is 20 cm. Actually it is not 20 cm, it is 20 divisions on the brass scale. Now we can find the actual length of the steel rod at 25\degree C as

\[
l_{\text{25\degree C}} = (20 \text{ cm}) \times l_{\text{div}}
\]

or

\[
l_{\text{25\degree C}} = 20[1 + \alpha_{\text{Br}}(25)]
\]

The above expression is a general relation using which you can find the actual lengths of the objects of which lengths are measured by a metallic scale at some temperature other than the graduation temperature of the scale.

5. \textbf{Expansion of cavity: Thermal expansion of an isotropic object may be imagined as a photographic enlargement. So if there is a hole } A \text{ in a plate } C \text{ (or cavity } A \text{ inside a body}
The area of hole (or volume of cavity) will increase when body expands on heating, just as if the hole (or cavity) were solid of the same material. Also the expansion of area (or volume) of the body will be independent of shape and size of hole (or cavity), i.e., will be equal to that of D.

![Diagram showing expansion of A, B, C, and D](image)

**Fig. 1.20**

**Note:** For a solid and hollow sphere of same radius and material, heated to the same temperature, expansion of both will be equal because thermal expansion of isotropic solids is similar to true photographic enlargement. It means the expansion of cavity is same as if it has been a solid body of the same material. But if same heat is given to the two spheres, due to lesser mass, rise in temperature of hollow sphere will be more \[ A \Delta \theta = \frac{a}{mc} \]. Hence its expansion will be more.

### Practical Application:

1. When rails are laid down on the ground, space is left between the ends of two rails to allow for expansion.
2. The transmission cables are not tightly fixed to the poles.
3. Pendulum of wall clock and balance wheel of wrist watch are made of invar (an alloy which has very low value of coefficient of expansion).
4. Test tubes, beakers and crucibles are made of pyrex-glass or silica because they have very low value of coefficient of linear expansion.
5. The iron rim to be put on a cart wheel is always of slightly smaller diameter than that of wheel to ensure tight fit.
6. A glass stopper jammed in the neck of a glass bottle can be taken out by warming the neck of the bottle.

### Concept Application Exercise 1.2

1. Does the change in volume of a body when its temperature is raised depend on whether the body has cavities inside, other things being equal?
2. Explain why some rubber-like substances contract with rising temperature.
3. Two large holes are cut in a metal sheet. If this is heated, will their diameters increase or decrease?
4. In the above question, will the distance between the holes increase or decrease on heating?

---

5. A long metal rod is bent to form a ring with a small gap. If this is heated, will this gap increase or decrease?
6. Two iron spheres of the same diameter are heated to the same temperature. One is solid, and the other is hollow. Which will expand more?
7. A steel rod is 3.000 cm at 25°C. A brass ring has an interior diameter of 2.992 cm at 25°C. At what common temperature will the ring just slide on to the rod?
8. A clock with a metallic pendulum gains 5 s each day at a temperature of 15°C and loses 10 s each day at a temperature of 30°C. Find the coefficient of thermal expansion of the pendulum metal.
9. The design of some physical instrument requires that there be a constant difference in length of 10 cm between an iron rod and a copper cylinder laid side by side at all temperatures. Find their lengths.

\[ \alpha_{\text{iron}} = 11 \times 10^{-6} \text{C}^{-1} \]
\[ \alpha_{\text{copper}} = 17 \times 10^{-6} \text{C}^{-1} \]

10. A metal rod of 30 cm length expands by 0.075 cm when its temperature is raised from 0°C to 100°C. Another rod of a different metal of length 45 cm expands by 0.045 cm for the same rise in temperature. A composite rod C made by joining A and B end to end expands by 0.040 cm when its length is 45 cm and it is heated from 0°C to 50°C. Find the length of each portion of the composite rod.

11. A brass scale is graduated at 10°C. What is the true length of a zinc rod which measures 60.00 cm on this scale at 30°C?

Coefficient of linear expansion of brass = \( 18 \times 10^{-6} \text{K}^{-1} \).

12. A long horizontal glass capillary tube open at both ends contains a mercury thread 1 m long at 0°C. Find the length of the mercury thread, as read on this scale, at 100°C.

13. A mercury-in-glass thermometer has a stem of internal diameter 0.06 cm and contains 43 g of mercury. The mercury thread expands by 10 cm when the temperature changes from 0°C to 50°C. Find the coefficient of cubical expansion of mercury. Relative density of mercury = 13.6 and \( \alpha_{\text{mercury}} = 9 \times 10^{-4} \text{K}^{-1} \).

14. A sphere of diameter 7 cm and mass 266.5 g floats in a bath of liquid. The temperature is raised, and the sphere begins to sink at 35°C. If the density of the liquid is 1.527 at 0°C, find the coefficient of cubical expansion of the liquid. Neglect the expansion of the sphere.

15. A mercury thermometer is to be made with glass tubing of internal bore 0.5 mm diameter and the distance between the fixed point is to be 20 cm. Estimate the volume of the bulb below the lower fixed point, given that the coefficient of cubical expansion of mercury is 0.00018/K and the coefficient of linear expansion of glass is 0.000009/K.
16. On a Celsius thermometer the distance between the readings 0°C and 100°C is 30 cm and the area of cross section of the narrow tube containing mercury is $15 \times 10^{-4}$ cm$^2$. Find the total volume of mercury in the thermometer at 0°C. $\alpha$ of glass = $9 \times 10^{-6}$/K and the coefficient of real expansion of mercury = $18 \times 10^{-6}$/K.

17. The height of a mercury column measured with a brass scale, which is correct and equal to $H_0$ at 0°C, is $H_1$ at $\rho$°C? The coefficient of linear expansion of brass is $\alpha$ and the coefficient of volume expansion of mercury is $\gamma$. Relate $H_0$ and $H_1$.

18. A glass bulb contains air and mercury. What fraction of the bulb must be occupied by mercury if the volume of air in the bulb is to remain constant at all temperatures? The coefficient of linear expansion of glass is $9 \times 10^{-6}$/K and the coefficient of expansion of mercury is $1.8 \times 10^{-2}$/K.

19. When composite rod is free, composite length increases to 2.002 m when temperature increases from 20°C to 120°C. When composite rod is fixed between the support, there is no change in component length. Find $\gamma$ and $\alpha$ of steel if $\gamma_{Cu} = 1.5 \times 10^{6}$ N/m$^2$, $\alpha_{Cu} = 1.6 \times 10^{-5}$/°C.

**TRANSMISSION OF HEAT**

Heat energy transfers from a body at higher temperature to a body at lower temperature. The transfer of heat from one body to another may take place by any of the following modes.

<table>
<thead>
<tr>
<th>Conduction</th>
<th>Convection</th>
<th>Radiation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heat flows from hot end to cold end. Particles of the medium simply oscillate but do not leave their place.</td>
<td>Each particle absorbing heat is mobile.</td>
<td>Heat flows without any intervening medium in the form of electromagnetic waves.</td>
</tr>
<tr>
<td>Medium is necessary for conduction.</td>
<td>Medium is necessary for convection.</td>
<td>Medium is not necessary for radiation.</td>
</tr>
<tr>
<td>It is a slow process.</td>
<td>It is also a slow process.</td>
<td>It is a very fast process.</td>
</tr>
<tr>
<td>Path of heat flow may be zig-zag.</td>
<td>Path may be zig-zag or curved.</td>
<td>Path is a straight line.</td>
</tr>
</tbody>
</table>

**Conduction**

The process of transmission of heat energy in which the heat is transferred from one particle to other without dislocation of the particles from their equilibrium position is called conduction.

i. Conduction is a process which is possible in all states of matter.

ii. In solids only conduction takes place.

iii. In non-metallic solids and fluids the conduction takes place only due to vibrations of molecules; therefore, they are poor conductors.

iv. In metallic solids free electrons carry the heat energy; therefore, they are good conductors of heat.

1. **Variable and steady state:** When one end of a metallic rod is heated, heat flows by conduction from the hot end to the cold end. In the process of conduction each cross section of the rod receives heat from the adjacent cross section towards the hot end. A part of this heat is absorbed by the cross section itself whose temperature increases, another part is lost into atmosphere by convection and radiation and the rest is conducted away to the next cross section.

Because in this state temperature of every cross section of the rod goes on increasing; hence, rod is said to exist in variable state.

After some time, a state is reached when the temperature of every cross section of the rod becomes constant. In this state, no heat
is absorbed by the rod. The heat that reaches any cross section is transmitted to the next except that a small part of heat is lost to surrounding from the sides by convection and radiation. This state of the rod in which no part of rod absorbs heat is called steady state.

2. Isothermal surface: Any surface (within a conductor) having its all points at the same temperature is called isothermal surface. The direction of flow of heat through a conductor at any point is perpendicular to the isothermal surface passing through that point.

i. If the material is rectangular or cylindrical rod, the isothermal surface is a plane surface.
ii. If a point source of heat is situated at the centre of a sphere the isothermal surface will be spherical.
iii. If a stream passes along the axis of the hollow cylinder, heat will flow through the walls of the cylinder so that in this condition the isothermal surface will be cylindrical.

![Isothermal Surfaces](image)

Fig. 1.24

3. Temperature gradient: The rate of change of temperature with distance between two isothermal surfaces is called temperature gradient.

If the temperature of two isothermal surfaces be \( \theta \) and \( (\theta - \Delta \theta) \) and the perpendicular distance between them be \( \Delta x \), then the temperature gradient is \( \frac{\theta - (\theta - \Delta \theta)}{\Delta x} \).

![Temperature Gradient](image)

Fig. 1.25

The negative sign shows that temperature \( \theta \) decreases as the distance \( x \) increases in the direction of heat flow.

Unit: \( K/m \) (SI); dimension: \([L^{-1} \theta] \)

4. Coefficient of thermal conductivity: If \( L \) be the length of the rod, \( A \) the area of cross section and \( \theta_1 \) and \( \theta_2 \) be the temperatures of its two faces, then the amount of heat flowing from one face to the other face in time \( t \) is

\[
Q = \frac{A}{(\theta_1 - \theta_2)} \frac{\theta_1 - \theta_2}{t}
\]

In combined form

\[
Q = \frac{KA(\theta_1 - \theta_2)}{l}
\]

or

\[
Q = \frac{KA(\theta_1 - \theta_2)l}{1}
\]

where \( K \) is coefficient of thermal conductivity of material of rod. It is the measure of the ability of a substance to conduct heat through it.

This relation can also be expressed as

\[
\Delta Q = K\frac{(\theta_1 - \theta_2)}{l}
\]

If \( A = 1 \text{ m}^2, (\theta_1 - \theta_2) = 1^\circ C, t = 1 \text{ s}, \) and \( l = 1 \text{ m}, \) then

\[
Q = K
\]

Thus, thermal conductivity of a material is the amount of heat flowing per second during steady state through its rod of length \( 1 \text{ m} \) and cross section \( 1 \text{ m}^2 \) with a unit temperature difference between the opposite faces.

i. Units: cal/cm·s·°C (in CGS), kcal/m·s·°K (in MKS) and W/m·K (in SI)

ii. Dimension: \([MLT^{-2} \theta^{-1}] \)

iii. The magnitude of \( K \) depends only on nature of the material.

iv. For perfect conductors, \( K = \infty \) and for perfect insulators, \( K = 0 \)

v. Substances in which heat flows quickly and easily are known as good conductors of heat. They possess large thermal conductivity due to large number of free electrons. Example: silver, brass, etc.

vi. Substances which do not permit easy flow of heat are called bad conductors. They possess low thermal conductivity due to very few free electrons. Example: glass, wood, etc.

vii. The thermal conductivity of pure metals decreases with rise in temperature but for alloys thermal conductivity increases with increase of temperature.

viii. Human body is a bad conductor of heat (but it is a good conductor of electricity).

Illustration 1.47: A refrigerator door is 150 cm high, 80 cm wide, and 6 cm thick. If the coefficient of conductivity is 0.0005 cal/cm·°C, and the inner and outer surfaces are at 0°C and 30°C, respectively, what is the heat loss per minute through the door, in calories?
Sol. Apply the equation of thermal conductivity
\[ Q = \frac{kA(t_a - t_c)(\text{time})}{d} = \frac{0.005\times150\times80\times(30^\circ - 0^\circ)}{4} \]
\[ = 1800 \text{ cal} \]

**Illustration 1.24** An ordinary refrigerator is thermally equivalent to a box of corkboard 90 mm thick and 5.6 m² in inner surface area. When the door is closed, the inside wall is kept, on the average, 22.2°C below the temperature of the outside wall. If the motor of the refrigerator runs 15% of the time while the door is closed, at what rate must heat be taken from the interior while the motor is running? The thermal conductivity of corkboard is \( k = 0.05 \text{ W/mK} \).

Sol. Consider a time interval \( \Delta t \) during which the door is closed. As approximation, take the heat conduction to be steady over \( \Delta t \).

Then the rate of heat into the box is
\[ \frac{\Delta Q}{\Delta t} = kA \left( \frac{\Delta T}{\Delta x} \right) = \frac{(0.05)(5.6)(22.2)}{0.090} = 69.1 \text{ W} \]

To remove this heat, the motor must, since it runs only for a time (0.15) \( \Delta t \), cause heat to leave at the rate \( 69.1/0.15 = 460 \text{ W} \).

**Illustration 1.25** Water is being boiled in flat bottom kettle placed on a stove. The area of the bottom is 3000 cm² and the thickness is 2 mm. If the amount of steam produced is 1 g/min, calculate the difference of temperature between the inner and outer surface of the bottom. \( K \) for the material of kettle is 0.5 cal°C/s/cm, and the latent heat of steam is 540 cal/g.

Sol. Mass of steam produced = \( \frac{dm}{dt} = \frac{1}{60} \text{ g/s} \)

Heat transferred per second
\[ \frac{dH}{dt} = L \frac{dm}{dt} \Rightarrow \frac{dH}{dt} = 540 \times \frac{1}{60} = 9 \text{ cal/s} \]

Area = 3000 cm²; \( K = 0.5 \text{ cal°C/s/cm} \)
\( \theta = \text{temperature difference} \)
\( d = \text{thickness} = 2 \text{ mm} = 0.2 \text{ cm} \)

\[ \frac{dH}{dt} = \frac{K\theta}{d} \Rightarrow \frac{dH}{dt} = \frac{KA}{d} \]
\[ \Rightarrow 9 = \frac{0.5 \times 3000 \times \theta}{0.2} \Rightarrow \theta = 1.2 \times 10^3 \text{°C} \]

**Illustration 1.26** A closed cubical box made of perfectly insulating material has walls of thickness 8 cm and the only way for the heat to enter or leave the box is through the solid, cylindrical, metallic plugs each of cross-sectional area 12 cm² and length 8 cm fixed in the opposite walls of the box as shown in Fig. 1.27. The outer surface \( A \) is kept at 100°C while the outer surface \( B \) of other plug is kept at 4°C.

\( K \) of the material of the plugs is 0.5 cal/s/C/cm. A source of energy generating 36 cal/s is enclosed inside the box. Find the equilibrium temperature of the inner surface of the box assuming that it is same at all points on the inner surface.

Sol. Let \( \theta \) be the temperature of inner surface of box.

Heat transfer per second through \( A \) = heat produced by source per second = Heat transfer per second through \( B \)

\[ \Rightarrow \frac{dH}{dt}_A + 36 \text{ cal/s} = \frac{dH}{dt}_B \]

\[ \Rightarrow \frac{KA(100 - \theta)}{d} + 36 = \frac{KA(\theta - 4)}{d} \]

\[ \Rightarrow KA(\theta - 4 - 100 + \theta) = 36 \times d \]

Now, \( d = 8 \text{ cm} \), \( A = 12 \text{ cm}^2 \), \( K = 0.5 \text{ cal/s°C/cm} \).

\[ \Rightarrow 2\theta - 104 = \frac{36 \times 8}{12 \times 0.5} \Rightarrow \theta = 76°C \]

**Illustration 1.27** Two metal cubes \( A \) and \( B \) of same size are arranged as shown in Fig. 1.28. The extreme ends of the combination are maintained at the identical temperatures. The arrangement is thermally insulated. The coefficients of thermal conductivity of \( A \) and \( B \) are 300 W/m°C and 200 W/m°C, respectively. After steady state is reached, what will be the temperature \( T \) of the interface?

(IIT-JEE-1996)

Sol. In steady state, rate of flow of heat through \( A = \text{Rate of flow of heat through } B \)

or \( K_1A \left( \frac{100 - T}{x} \right) = K_2A \left( \frac{T - 0}{x} \right) \) or \( 300 - 3T = 2T \)

\[ \therefore T = 60°C \]

5. Relation between temperature gradient and thermal conductivity: In steady state, rate of flow of heat
\[ \frac{dQ}{dt} = -KA \frac{d\theta}{dx} = -KA \text{ (temperature gradient)} \]
If \( \frac{dQ}{dt} \) is constant then temperature gradient \( \propto \frac{1}{K} \).

Temperature difference between the hot end and the cold end in steady state is inversely proportional to \( K \), i.e., in case of good conductors temperature of the cold end will be very near to hot end.

In ideal conductor where \( K = \infty \), temperature difference in steady state will be zero.

6. Thermometric conductivity or diffusivity: It is a measure of rate of change of temperature (with time) when the body is not in steady state (i.e., in variable state).

The thermometric conductivity or diffusivity is defined as the ratio of the coefficient of thermal conductivity to the thermal capacity per unit volume of the material.

Thermal capacity per unit volume = \( \frac{mc}{V} = \rho c \) (as \( \rho \) is density of substance)

\[ \therefore \text{ Diffusivity (D)} = \frac{K}{\rho c} \]

Unit: m/s; dimension: \( [L^{-1}T^{-1}] \)

7. Thermal resistance: The thermal resistance of a body is a measure of its opposition to the flow of heat through it.

It is defined as the ratio of temperature difference to the heat current (= rate of flow of heat).

Now, temperature difference = \( (\theta_1 - \theta_2) \) and heat current, \( H = \frac{Q}{t} \).

\[ \therefore \text{ Thermal resistance} \]

\[ R = \frac{\theta_1 - \theta_2}{H} = \frac{\theta_1 - \theta_2}{\frac{Q}{t}} = \frac{\theta_1 - \theta_2}{KA(\theta_1 - \theta_2)/l} = \frac{l}{NK} \]

Unit: \( ^\circ \text{C} \times \text{s}/\text{cal} \) or \( K \times \text{s}/\text{kcal} \); dimension:

\[ [M^{-1}L^{-1}T^{-2}T] \]

**COMBINATION OF CONDUCTORS**

1. Series combination: Let \( n \) slabs each of cross-sectional area \( A \), lengths \( l_1, l_2, l_3, \ldots, l_n \) and conductivities \( K_1, K_2, K_3, \ldots, K_n \) respectively, be connected in the series.

Heat current is the same in all the conductors.

\[
\begin{array}{cccccccc}
\theta_1 & \theta_2 & \theta_3 & \theta_4 & \cdots & \theta_{n-1} & \theta_n \\
-\frac{1}{l_1} & -\frac{1}{l_2} & -\frac{1}{l_3} & -\frac{1}{l_4} & \cdots & -\frac{1}{l_{n-1}} & -\frac{1}{l_n}
\end{array}
\]

Fig. 1.29

\[ i.e., \quad \frac{Q}{t} = H_1 = H_2 = H_3 = \cdots = H_n \]

\[ \frac{K_1A(\theta_1 - \theta_2)}{l_1} = \frac{K_2A(\theta_2 - \theta_3)}{l_2} = \frac{K_3A(\theta_3 - \theta_4)}{l_3} = \cdots \]

\[ \frac{K_nA(\theta_{n-1} - \theta_n)}{l_n} \]

i. Equivalent resistance \( R = R_1 + R_2 + R_3 + \cdots + R_n \)

ii. If \( K_r \) is equivalent conductivity, then from relation

\[ R = \frac{l}{KA} \]

\[ \frac{l_1 + l_2 + l_3 + \cdots + l_n}{K_1} = \frac{l_1 + l_2 + l_3 + \cdots + l_n}{K_1A} \]

\[ \therefore K_r = \frac{l_1 + l_2 + l_3 + \cdots + l_n}{l_1 + l_2 + l_3 + \cdots + l_n} \]

\[ K_r = \frac{l_1 + l_2 + l_3 + \cdots + l_n}{k_1 + k_2 + k_3 + \cdots + k_n} \]

\[ K_r = \frac{l_1 + l_2 + l_3 + \cdots + l_n}{k_1 + K_2 + K_3 + \cdots + K_n} \]

\[ i.e., \quad K_r = \frac{K_1A}{\sum k_i} \]

iii. Equivalent thermal conductivity for \( n \) slabs of equal length

\[ K = \frac{1}{n} \left( \frac{K_1}{k_1} + \frac{K_2}{k_2} + \frac{K_3}{k_3} + \cdots + \frac{K_n}{k_n} \right) \]

For two slabs of equal length, \( K = \frac{2K_1K_2}{K_1 + K_2} \)

iv. Temperature of interface of composite bar: Let the two bars be arranged in series as shown in Fig. 1.30.

Then heat current is same in the two conductors.

\[ \begin{array}{c}
\theta_1 \leftarrow l_1 \rightarrow l_2 \rightarrow \theta_2
\end{array} \]

Fig. 1.30

\[ i.e., \quad \frac{Q}{t} = K_1A(\theta_1 - \theta) = K_2A(\theta - \theta_2) \]

By solving we get

\[ \theta = \frac{K_1\theta_1 + K_2\theta_2}{K_1 + K_2} \]

If \( (l_1 = l_2 = l) \) then \( \theta = \frac{K_1\theta_1 + K_2\theta_2}{K_1 + K_2} \)

2. Parallel combination: Let \( n \) slabs each of length \( l \), areas \( A_1, A_2, A_3, \ldots, A_n \) and thermal conductivities \( K_1, K_2, K_3, \ldots, K_n \) be connected in parallel. Then

i. Equivalent resistance \( \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n} \)

ii. Temperature gradient across each slab will be same.

\[ \text{ii. Heat current in each slab will be different. Net heat current} \]

\[ \text{will be the sum of heat currents through individual slabs.} \]

\[ i.e., \quad H = H_1 + H_2 + H_3 + \cdots + H_n \]

\[ = \frac{K_1A_1(\theta_1 - \theta_2)}{l_1} + \frac{K_2A_2(\theta_2 - \theta_3)}{l_2} + \frac{K_3A_3(\theta_3 - \theta_4)}{l_3} + \cdots + \frac{K_nA_n(\theta_n - \theta_{n-1})}{l_n} \]
\[ K = \frac{K_A + K_B + K_C + \cdots + K_n A_n}{A_1 + A_2 + A_3 + \cdots + A_n} \]

For \( n \) slabs of equal area \( K = \frac{K_1 + K_2 + K_3 + \cdots + K_n}{n} \)

**Equivalent thermal conductivity for two slabs of equal area**

\[ K = \frac{K_1 + K_2}{2} \]

**Electrical Analogy for Thermal Conduction**

It is an important fact to appreciate that there exists an exact similarity between thermal and electrical conductivities of a conductor.

<table>
<thead>
<tr>
<th>Electrical conduction</th>
<th>Thermal conduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electric charge flows from higher potential to lower potential</td>
<td>Heat flows from higher temperature to lower temperature</td>
</tr>
<tr>
<td>The rate of flow of charge is called the electric current, ( I = \frac{dQ}{dt} )</td>
<td>The rate of flow of heat may be called heat current, ( I = \frac{dQ}{dt} )</td>
</tr>
<tr>
<td>The relation between the electric current and the potential difference is given by Ohm's law, ( I = \frac{V_1 - V_2}{R} )</td>
<td>Similarly, the heat current may be related with the temperature difference as ( H = \frac{\theta_1 - \theta_2}{R} ) where ( R ) is the thermal resistance of the conductor</td>
</tr>
<tr>
<td>The electrical resistance is defined as ( R = \frac{\rho l}{A} )</td>
<td>The thermal resistance may be defined as ( R = \frac{l}{KA} )</td>
</tr>
<tr>
<td>( \rho = \text{resistivity} ) and ( \sigma = \text{electrical conductivity} )</td>
<td>where ( K = \text{Thermal conductivity of conductor} )</td>
</tr>
</tbody>
</table>

\[ \begin{align*}
\frac{dQ}{dt} &= I = \frac{V_1 - V_2}{R} \\
\frac{dQ}{dt} &= H = \frac{\theta_1 - \theta_2}{R} \\
\frac{dQ}{dt} &= \frac{KA}{l} (\frac{V_1 - V_2}{R})
\end{align*} \]

**Illustration 1.52** Three cylindrical rods \( A, B \) and \( C \) of equal lengths and equal diameters are joined in series as shown in Fig. 1.32. Their thermal conductivities are \( 2K, K \) and \( 0.5K \), respectively. In steady state, if the free ends of rods \( A \) and \( C \) are at \( 100^\circ C \) and \( 0^\circ C \), respectively, calculate the temperature at the two junction points. Assume negligible loss by radiation through the curved surface. What will be the equivalent thermal conductivity?

<table>
<thead>
<tr>
<th>Temperature</th>
<th>Rods</th>
</tr>
</thead>
<tbody>
<tr>
<td>100°C</td>
<td>( A )</td>
</tr>
<tr>
<td></td>
<td>( B )</td>
</tr>
<tr>
<td></td>
<td>( C )</td>
</tr>
<tr>
<td>0°C</td>
<td></td>
</tr>
</tbody>
</table>

Sol. As the rods are in series, \( R_{eq} = R_A + R_B + R_C \) with \( R = (L/K) \)

\[ \begin{align*}
R_{eq} &= \frac{L}{2KA} + \frac{L}{KA} + \frac{L}{0.5KA} = \frac{7L}{2KA} \\
H &= \frac{dQ}{dt} = \frac{\Delta \theta}{R} = \frac{(100 - 0)}{(7L/2KA)} = \frac{200KA}{7L}
\end{align*} \]

And hence:

\[ \theta_{eq} = 100 - \left( \frac{\theta_{eq} - 0}{(7L/2KA)} \right) = \frac{200KA}{7L} \]

Now in series, rate of flow of heat remains same, i.e., \( H = H_A = H_B = H_C \).

So for rod \( A \):

\[ \left[ \frac{dQ}{dt} \right]_A = \left[ \frac{dQ}{dt} \right] \]

\[ \frac{(100 - \theta_{eq})2KA}{L} = \frac{200KA}{7L} \]

or.

\[ \theta_{eq} = 100 - \left( \frac{\theta_{eq} - 0}{(7L/2KA)} \right) = \frac{(600/7)}{7} = 87.7^\circ C \]

And for rod \( C \):

\[ \left[ \frac{dQ}{dt} \right]_C = \left[ \frac{dQ}{dt} \right] \]

\[ \frac{(\theta_{eq} - 0) \times 0.5KA}{L} = \frac{200KA}{7L} \]

or.

\[ \theta_{eq} = \frac{(400L/7)}{7} = 57.1^\circ C \]

Furthermore, if \( K_{eq} \) is equivalent thermal conductivity,

\[ R_{eq} = \frac{l + L + L}{K_{eq}A} = \frac{7L}{2KA} \quad \text{[from Eq. (i)]} \]

\[ R_{eq} = \frac{l}{KA} \]

\[ K_{eq} = \frac{(6/7)K}{l} \]

**Illustration 1.53** Two walls of thickness in the ratio \( 1:3 \) and thermal conductivities in the ratio \( 3:2 \) form a composite wall of a building. If the free surfaces of the wall