

# PHYSICS

## Waves & Thermodynamics

**B.M. Sharma**

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**Physics for JEE/ISEET**  
**Waves &**  
**Thermodynamics**

**B.M. Sharma**

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# Preface

While the paper-setting pattern and assessment methodology have been revised many times over and newer criteria devised to help develop more aspirant-friendly engineering entrance tests, the need to standardize the selection processes and their outcomes at the national level has always been felt. A combined national-level engineering entrance examination has finally been proposed by the Ministry of Human Resource Development, Government of India. The Joint Entrance Examination (JEE) to India's prestigious engineering institutions (IITs, IIITs, NITs, ISM, IISERs, and other engineering colleges) aims to serve as a common national-level engineering entrance test, thereby eliminating the need for aspiring engineers to sit through multiple entrance tests.

While the methodology and scope of an engineering entrance test are prone to change, there are two basic objectives that any test needs to serve:

1. The objective to test an aspirant's caliber, aptitude, and attitude for the engineering field and profession.
2. The need to test an aspirant's grasp and understanding of the concepts of the subjects of study and their applicability at the grassroot level.

Students appearing for various engineering entrance examinations cannot bank solely on conventional shortcut measures to crack the entrance examination. Conventional techniques alone are not enough as most of the questions asked in the examination are based on concepts rather than on just formulae. Hence, it is necessary for students appearing for joint entrance examination to not only gain a thorough knowledge and understanding of the concepts but also develop problem-solving skills to be able to relate their understanding of the subject to real-life applications based on these concepts.

This series of books is designed to help students to get an all-round grasp of the subject so as to be able to make its useful application in all its contexts. It uses a right mix of fundamental principles and concepts, illustrations which highlight the application of these concepts, and exercises for practice. The objective of each book in this series is to help students develop their problem-solving skills/accuracy, the ability to reach the crux of the matter, and the speed to get answers in limited time. These books feature all types of problems asked in the examination—be it MCQs (one or more than one correct), assertion-reason type, matching column type, comprehension type, or integer type questions. These problems have skillfully been set to help students develop a sound problem-solving methodology.

Not discounting the need for skilled and guided practice, the material in the books has been enriched with a number of fully solved concept application exercises so that every step in learning is ensured for the understanding and application of the subject. This whole series of books adopts a multi-faceted approach to mastering concepts by including a variety of exercises asked in the examination. A mix of questions helps stimulate and strengthen multi-dimensional problem-solving skills in an aspirant.

It is imperative to note that this book would be as profound and useful as you want it to be. Therefore, in order to get maximum benefit from this book, we recommend the following study plan for each chapter.

Step 1: Go through the entire opening discussion about the fundamentals and concepts.

Step 2: After learning the theory/concept, follow the illustrative examples to get an understanding of the theory/concept.

Overall the whole content of the book is an amalgamation of the theme of physics with ahead-of-time problems, which equips the students with the knowledge of the field and paves a confident path for them to accomplish success in the JEE.

With best wishes!

**B.M. Sharma**

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# UNIT I

## THERMAL PHYSICS

CHAPTER 1: THERMAL PROPERTIES OF MATTER

CHAPTER 2: KINETIC THEORY OF GASES AND FIRST LAW OF  
THERMODYNAMICS

CHAPTER 3: ARCHIVES ON CHAPTERS 1 AND 2

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## CHAPTER

# 1

## Thermal Properties of Matter

- Heat
- Calorimetry
- Thermal Expansion
- Transmission of Heat
- Combination of Conductors
- Determination of Thermal Conductivity
- Growth of Ice on Lake
- Convection
- Radiation
- Prevost's Theory of Heat Exchange
- Kirchoff's Law
- Stefan's Law
- Newton's Law of Cooling
- Distribution of Energy in the Spectrum of Black Body
- Wien's Displacement Law
- Temperature of the Sun and Solar Constant

## HEAT

Heat is a form of energy which appears when two bodies at different temperatures come into contact and flows from the body at higher temperature to that at lower temperature. It is energy in motion or energy in transit. Heat is not a property of a system. A system can give out or absorb heat, but does not contain heat. It is the form of energy which determines the change in thermal state of a body and is defined as the flow of energy from one body to the other due to difference in the degree of hotness of two bodies (temperature). It flows from the body which is at a high temperature to the other at low temperature.

The energy associated with configuration and random motion of the atoms and molecules within a body is called internal energy and the part of this internal energy which is transferred from one body to the other due to temperature difference is called heat.

One calorie is defined as the amount of heat energy required to raise the temperature of 1 g of water through  $1^\circ\text{C}$  (more specifically from  $14.5^\circ\text{C}$  to  $15.5^\circ\text{C}$ ).

As heat is a form of energy it can be transformed into others and vice versa. For example, thermocouple converts heat energy into electrical energy, resistor converts electrical energy into heat energy. Friction converts mechanical energy into heat energy. Heat engine converts heat energy into mechanical energy.

Here it is important that whole of mechanical energy, i.e., work can be converted into heat but whole of heat can never be converted into work.

## Temperature

Temperature is defined by zeroth law of thermodynamics, which states that when two bodies  $A$  and  $B$  are separately in thermal equilibrium with a third body  $C$ , then  $A$  and  $B$  are also in thermal equilibrium with each other (thermal equilibrium implies equality of temperature). Temperature is a scalar quantity which is a property of all thermodynamic systems such that the equality of temperature is necessary and sufficient for thermal equilibrium.

1. Temperature is one of the seven fundamental quantities with dimension  $[\theta]$ .
2. It is a scalar physical quantity with SI unit kelvin.
3. When heat is given to a body and its state does not change, the temperature of the body rises and if heat is taken from a body its temperature falls, i.e., temperature can be regarded as the effect of cause 'heat'.
4. According to the kinetic theory of gases, temperature (macroscopic physical quantity) is a measure of average translational kinetic energy of a molecule (microscopic physical quantity).

$$\text{Temperature} \propto \text{kinetic energy} \left[ \text{As } E = \frac{3}{2} RT \right]$$

5. Although the temperature of a body can be raised without limit, it cannot be lowered without limit and theoretically

limiting low temperature is taken to be zero of the kelvin scale.

6. Highest possible temperature achieved in laboratory is about  $10^8 \text{ K}$ , while the lowest possible temperature attained is  $10^{-8} \text{ K}$ .
7. Branch of physics dealing with production and measurement of temperatures close to  $0 \text{ K}$  is known as cryogenics, while that dealing with the measurement of very high temperature is called as pyrometry.
8. Temperature of the core of the sun is  $10^7 \text{ K}$  while that of its surface is  $6000 \text{ K}$ .
9. Normal temperature of human body is  $310.15 \text{ K} = 37^\circ\text{C} = 98.6^\circ\text{F}$ .
10. NTP or STP implies  $273.15 \text{ K}$  ( $0^\circ\text{C} = 32^\circ\text{F}$ ).

As the temperature is measured by the value of the thermodynamic property of a substance, i.e., a property which varies linearly with the temperature, two fixed points are needed to define a temperature scale.

These two fixed points in modern thermometry are taken as

1. Triple point of water, i.e., the state of water where the liquid, solid and vapour phases of water coexist in equilibrium. It is characterized by unique values of temperature and pressure.
2. On this scale, the other fixed point may be taken as the absolute zero.

We then need to assign some numbers to these two fixed points. The lowest temperature may be taken as zero. The triple point of water on Celsius scale is  $0.01^\circ\text{C}$ . Thus the absolute temperature  $T$  for triple point of water will be given by

$$T = t_c + 273.15 = 0.01 + 273.15 = 273.16 \text{ K}$$

$$T = 273.16 \text{ K (triple point of water)}$$

## Thermometry

An instrument used to measure the temperature of a body is called a thermometer.

The linear variation in some physical property of a substance with change of temperature is the basic principle of thermometry and these properties are defined as thermometric property ( $x$ ) of the substance.

- $x$  may be
- (i) length of liquid in capillary;
  - (ii) pressure of gas at constant volume;
  - (iii) volume of gas at constant pressure and
  - (iv) resistance of a given platinum wire.

In old thermometry, two arbitrarily fixed points ice and steam point (freezing point and boiling point at 1 atm) are taken to define the temperature scale. In Celsius scale, freezing point of water is assumed to be  $0^\circ\text{C}$  while boiling point  $100^\circ\text{C}$  and the temperature interval between these is divided into 100 equal parts.

So, if the thermometric property at temperatures  $0^\circ\text{C}$ ,  $100^\circ\text{C}$  and  $T_c^\circ\text{C}$  is  $x_0$ ,  $x_{100}$  and  $x$ , respectively, then by linear variation ( $y = mx + c$ ) we can say that



$$O = ax_0 + b$$

$$100 = ax_{100} + b$$

$$T_c = ax + b$$

From these equations  $\frac{T_c - 0}{100 - 0} = \frac{x - x_0}{x_{100} - x_0}$

$$T_c = \frac{x - x_0}{x_{100} - x_0} \times 100^\circ\text{C}$$

In modern thermometry instead of two fixed points only one reference point is chosen (triple point of water 273.16 K at which ice, water and water vapours coexist).

So, if the values of thermometric property at 0 K, 273.16 K and  $T_K$  K are 0,  $x_{tr}$  and  $x$ , respectively, then by linear variation ( $y = mx + c$ ) we can say that

$$0 = a \times 0 + b$$

$$273.16 = a \times x_{tr} + b$$

$$T_K = a \times x + b$$

From these equations  $\frac{T_K}{273.16} = \frac{x}{x_{tr}}$

$$T_K = 273.16 \left[ \frac{x}{x_{tr}} \right] \text{ K}$$

## Measurement of Temperature

There are different systems of measurement of temperature. The lower fixed point (LFP) and the upper fixed point (UFP) in any system of units are corresponding to freezing point and boiling point of water at 1 atm.

For different system of units the LFP and UFP are given as

System of units	Units	Lower fixed point (LFP)	Upper fixed point (UFP)	Different UFP - LFP
Degree celsius (centigrade)	$^\circ\text{C}$	$0^\circ\text{C}$	$100^\circ\text{C}$	100
Kelvin scale (SI unit)	K	273.15 K	373.5 K	100
Fahrenheit	$^\circ\text{F}$	$32^\circ\text{F}$	$212^\circ\text{F}$	180

Temperature on one scale can be converted into other scale by using the following identity.

$$\frac{\text{Reading on any scale} - \text{lower fixed point (LFP)}}{\text{Upper fixed point (UFP)} - \text{lower fixed point (LFP)}} = \frac{\text{Constant}}{\text{all scales}}$$

The relation between Celsius (C), Kelvin (K), Fahrenheit (F) and any other new scale  $\theta$  is

$$\frac{C - 0}{100} = \frac{F - 32}{180} = \frac{K - 273}{100} = \frac{\theta - \theta_0}{n} \quad (\text{i})$$

where  $n$  is the number of divisions between ice point and steam point on the new scale and  $\theta_0$  is the ice point on it.

**Illustration 1.1** Liquid nitrogen has a boiling point of  $-195.81^\circ\text{C}$  at atmospheric pressure. Calculate this temperature (a) in degrees Fahrenheit and (b) in kelvin.

**Sol.** We can use Eq. (i) to convert degree celsius into Fahrenheit and kelvin.

a. Temperature in Fahrenheit is given by

$$T_F = \frac{9}{5}T_C + 32^\circ\text{F} = \frac{9}{5}(-195.81) + 32 = -320.46^\circ\text{F}$$

b. Temperature in Kelvin  $T_K = 273.15 \text{ K} - 195.81 \text{ K} = 77.3 \text{ K}$

A convenient way to change one scale to another is to remember the freezing and boiling points of water in each form:

$$T_{\text{freeze}} = 32.0^\circ\text{F} = 0^\circ\text{C} = 273.15 \text{ K}$$

$$T_{\text{boil}} = 212^\circ\text{F} = 100^\circ\text{C}$$

To convert from Fahrenheit to Celsius, subtract 32 (the freezing point) and then adjust the scale by the liquid range of the water

$$\text{Scale factor} = \frac{(100 - 0)^\circ\text{C}}{(212 - 32)^\circ\text{F}} = \frac{5^\circ\text{C}}{9^\circ\text{F}}$$

A Kelvin is the same size change as a degree celsius, but the Kelvin scale takes its zero point at absolute zero, instead of the freezing point of water. Therefore, to convert from Kelvin to Celsius, subtract 273.15 K from given Kelvin temperature.

**Illustration 1.2** Two ideal gas thermometers A and B use oxygen and hydrogen, respectively. The following observations are made:

Temperature	Pressure thermometer A	Pressure thermometer B
Triple point of water	$1.250 \times 10^5 \text{ Pa}$	$0.200 \times 10^5 \text{ Pa}$
Normal melting point of sulphur	$1.797 \times 10^5 \text{ Pa}$	$0.287 \times 10^5 \text{ Pa}$

- What is the absolute temperature of normal melting point of sulphur as read by thermometer A and B?
- What do you think is the reason for slightly different answers from A and B?

**Sol.**

a. For thermometer A,

$$T_{tr} = 273 \text{ K}, P_{tr} = 1.250 \times 10^5 \text{ Pa}$$

$$\begin{aligned} \text{We have } T &= \frac{P}{P_{tr}} \times T_{tr} \\ &= \frac{1.797 \times 10^5}{1.250 \times 10^5} \times 273 = 392.46 \text{ K} \end{aligned}$$

For thermometer B,

$$T_{tr} = 273 \text{ K}, P_{tr} = 0.200 \times 10^5 \text{ Pa}$$

$$\text{We have } T = \frac{P}{P_{tr}} \times T_{tr}$$

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$$= \frac{0.287 \times 10^5 \times 273}{0.200 \times 10^5} = 391.75 \text{ K}$$

- b. The slight difference in the temperatures as read by two thermometers is due to the fact that oxygen and hydrogen do not behave like an ideal gas.

**Illustration 1.3** What will be the following temperatures on the Kelvin scale: a.  $37^\circ\text{C}$ , b.  $80^\circ\text{F}$ , c.  $-196^\circ\text{C}$ ?

Sol.

- a. Temperature on Kelvin scale  $T_k$  is related to temperature  $T_c$  on Celsius scale as

$$T_k = T_c + 273$$

or

$$T_k = 37 + 273 = 310 \text{ K}$$

- b. Temperatures  $T_k$  on Kelvin scale and  $T_F$  on Fahrenheit scale are related as

$$\frac{T_k - 273}{373 - 273} = \frac{T_F - 32}{212 - 32}$$

or

$$\frac{T_k - 273}{100} = \frac{T_F - 32}{180}$$

or

$$T_k = \frac{5}{9}(T_F - 32) + 273$$

here

$$T_F = 80^\circ\text{F}; \text{ thus}$$

$$T_k = \frac{5}{9}(80 - 32) + 273 = 299.66 \text{ K}$$

- c. Again from relation used in part (a)

$$T_k = T_c + 273 = -196 + 273 = 77 \text{ K}$$

## CALORIMETRY

This is the branch of heat transfer that deals with the measurement of heat. The heat is usually measured in calories or kilocalories.

### One Calorie

One calorie is the quantity of heat required to raise the temperature of 1 g of water by  $1^\circ\text{C}$

### Mechanical Equivalent of Heat (J)

According to Joule, work may be converted into heat and vice versa. The ratio of work done ( $W$ ) to heat produced ( $Q$ ) by that work without any wastage is always constant.

$$W/Q = \text{constant}$$

This constant is called *mechanical equivalent of heat* ( $J$ ). The value of this constant is taken as 4.18 J/cal.

**Illustration 1.4** In the Joule experiment, a mass of 20 kg falls through 1.5 m at a constant velocity to stir the water in a calorimeter. If the calorimeter has a water equivalent of 2 g and contains 12 g of water, what is  $f$ , the mechanical equivalent of heat, for a temperature rise of  $5.0^\circ\text{C}$ ?

Sol. Expressing  $\Delta\text{PE}$  in Joules and  $Q$  in Calories, we have

$$f = \frac{\Delta\text{PE}}{Q} = \frac{mgy}{m_w c \Delta t} = \frac{20(9.8)(1.5)}{(12+2)(1)(5.0)} = 4.2 \text{ J/cal}$$

## Thermal Capacity and Water Equivalent

- Thermal capacity:** It is defined as the amount of heat required to raise the temperature of the whole body (mass  $m$ ) through  $1^\circ\text{C}$  or 1 K.

$$\text{Thermal capacity} = H = \frac{Q}{\Delta T}$$

The value of thermal capacity of a body depends upon the nature of the body and its mass.

Dimension:  $[ML^2T^{-2}\theta^{-1}]$ ; unit:  $\text{cal}/^\circ\text{C}$  (practical) J/K (SI)

- Water equivalent:** Water equivalent of a body is defined as the mass of water which would absorb or evolve the same amount of heat as is done by the body in rising or falling through the same range of temperature. It is represented by  $W$ .

If  $m$  = mass of the body,  $c$  = specific heat of body,  $\Delta T$  = rise in temperature.

Then heat given to body

$$\Delta Q = mc\Delta T \quad (i)$$

If same amount of heat is given to  $W$  grams of water and its temperature also rises by  $\Delta T$ .

Then heat given to water

$$\Delta Q = W \times 1 \times \Delta T \quad [\text{As } c_{\text{water}} = 1] \quad (ii)$$

From Eqs. (i) and (ii),  $\Delta Q = mc\Delta T = W \times 1 \times \Delta T$

$\therefore$  Water equivalent ( $W$ ) =  $mc$  grams

Unit: kg (SI); dimension:  $[ML^2T^{-2}]$ .

**Note:**

- Unit of thermal capacity is J/kg while unit of water equivalent is kg.
- Thermal capacity of the body and its water equivalent are numerically equal.
- If thermal capacity of a body is expressed in terms of mass of water it is called water equivalent of the body.

## Specific Heat

- Gram specific heat:** When heat is given to a body and its temperature increases, the heat required to raise the temperature of unit mass of a body through  $1^\circ\text{C}$  (or K) is called specific heat of the material of the body.



If  $Q$  heat changes the temperature of mass  $m$  by  $\Delta T$

$$\text{Specific heat } c = \frac{Q}{m\Delta T}$$

Units:  $\text{cal/g} \times ^\circ\text{C}$  (practical),  $\text{J/kg} \times \text{K}$  (SI); dimension:  $[L^2T^{-2}\theta^{-1}]$

2. **Molar specific heat:** Molar specific heat of a substance is defined as the amount of heat required to raise the temperature of 1 g mole of the substance through a unit degree; it is represented by  $C$ .

By definition, 1 mole of any substance is a quantity of the substance, whose mass  $M$  grams is numerically equal to the molecular mass  $M$ .

$\therefore$  Molar specific heat =  $M \times$  gram specific heat

or

$$C = M \frac{Q}{m\Delta T} = \frac{1}{\mu} \frac{Q}{\Delta T}$$

$$\left[ \text{As } c = \frac{Q}{m\Delta T} \text{ and } \mu = \frac{m}{M} \right]$$

$$\therefore C = \frac{Q}{\mu\Delta T}$$

Units:  $\text{cal/mol} \times ^\circ\text{C}$  (practical),  $\text{J/mol} \times \text{kelvin}$  (SI); dimension:  $[ML^2T^{-2}\theta^{-1}\mu^{-1}]$

### Important Points

1. Specific heat for hydrogen is maximum ( $3.5 \text{ cal/g} \times ^\circ\text{C}$ ) and for water, it is  $1 \text{ cal/g} \times ^\circ\text{C}$ .

For all other substances, the specific heat is less than  $1 \text{ cal/g} \times ^\circ\text{C}$  and it is minimum for radon and actinium ( $\approx 0.022 \text{ cal/g} \times ^\circ\text{C}$ ).

2. Specific heat of a substance also depends on the state of the substance, i.e., solid, liquid or gas.

For example,  $C_{\text{ice}} = 0.5 \text{ cal/g} \times ^\circ\text{C}$  (solid),  $C_{\text{water}} = 1 \text{ cal/g} \times ^\circ\text{C}$  (liquid) and  $C_{\text{steam}} = 0.47 \text{ cal/g} \times ^\circ\text{C}$  (gas)

3. The specific heat of a substance when it melts or boils at constant temperature is infinite.

$$\text{As } C = \frac{Q}{m\Delta T} = \frac{Q}{m \times 0} = \infty \quad (\text{As } \Delta T = 0)$$

4. The specific heat of a substance when it undergoes adiabatic changes is zero.

$$\text{As } C = \frac{Q}{m\Delta T} = \frac{0}{m\Delta T} = 0 \quad (\text{As } Q = 0)$$

5. Specific heat of a substance can also be negative. Negative specific heat means that in order to raise the temperature, a certain quantity of heat is to be withdrawn from the body.

For example, specific heat of saturated vapours.

**Illustration 8.5** A 60 kg boy running at 5.0 m/s while playing basketball falls down on the floor and skids along on his leg until he stops. How many calories of heat are generated between his leg and the floor?

Assume that all this heat energy is confined to a volume of  $2.0 \text{ cm}^3$  of his flesh. What will be temperature change of the flesh? Assume  $c = 1.0 \text{ cal/g} \times ^\circ\text{C}$  and  $\rho = 950 \text{ kg/m}^3$  for flesh.

**Sol.** The boy's kinetic energy is changed to heat energy.

$$\text{Set } Q = (mv^2)/2 = [60(25)]/2 = 750 \text{ J} = 179 \text{ cal.}$$

$$\text{from } Q = c\rho V \Delta T, 179 \text{ cal} = (1.0 \text{ cal/g} \times ^\circ\text{C})$$

$$(0.950 \text{ g/cm}^3)(2.0 \text{ cm}^3) \Delta T, \text{ whence } \Delta T = 94^\circ\text{C}$$

**Illustration 1.6** An electric heater supplies 1.8 kW of power in the form of heat to a tank of water. How long will it take to heat the 200 kg of water in the tank from  $10^\circ\text{C}$  to  $70^\circ\text{C}$ ? Assume heat losses to the surroundings to be negligible.

**Sol.** The heat added is  $(1.8 \text{ J/s}) t$

$$\text{The heat absorbed is } cm\Delta T = (4.184 \text{ kJ/kg K})(200 \text{ kg})(60 \text{ K}) = 5.0 \times 10^4 \text{ kJ}$$

$$\text{Equation heats, } t = 2.78 \times 10^4 \text{ s} = 7.75 \text{ h}$$

### Specific Heat of Solids

When a solid is heated through a small range of temperature, its volume remains more or less constant. Therefore specific heat of a solid may be called its specific heat at constant volume  $C_v$ .

From the graph it is clear that at  $T = 0$ ,  $C_v$  tends to zero

With rise in temperature,  $C_v$  increases and becomes constant  $= 3R = 6 \text{ cal/mole-kelvin} = 25 \text{ J/mole-kelvin}$  at some particular temperature (Debye temperature)

For most of the solids, Debye temperature is close to room temperature.

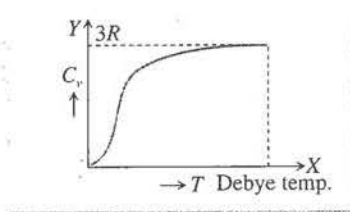


Fig. 1.1

### Specific Heat of Water

The variation of specific heat with temperature for water is shown in Fig. 1.2. Usually this temperature dependence of specific heat is neglected.

From the graph:

Temperature ( $^\circ\text{C}$ )	0	15	35	50	100
Specific heat ( $\text{cal/g} \times ^\circ\text{C}$ )	1.008	1.000	0.997	0.998	1.006

As specific heat of water is very large, by absorbing or releasing large amount of heat, its temperature changes by small amount. This is why it is used in hot water bottles or as coolant in radiators.



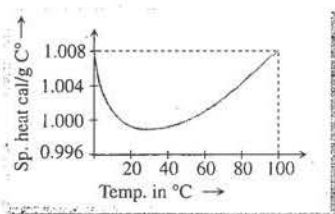


Fig. 1.2

**Note:** When specific heats are measured, the values obtained are also found to depend on the conditions of the experiment. In general, measurements made at constant pressure are different from those at constant volume. For solids and liquids, this difference is very small and usually neglected. The specific heats of gases are quite different under constant pressure condition ( $c_p$ ) and constant volume ( $c_v$ ). In the chapter 'Kinetic Theory of Gases and First Law of Thermodynamics' we have discussed this topic in detail.

**Illustration 1.7** What is wrong with the following statement: 'Given any two bodies, the one with the higher temperature contains more heat'.

**Sol.** The statement shows a misunderstanding of the concept of heat. Heat is a process by which energy is transferred, not a form of energy that is held or contained. If you wish to speak of energy that is 'contained', you speak of internal energy, not heat.

Further, even, if the statement used the term 'internal energy', it would still be incorrect, since the effects of specific heat and mass are both ignored. A 1 kg mass of water at 20°C has more internal energy than a 1 kg mass of air at 30°C.

Similarly, the earth has far more internal energy than a drop of molten titanium metal.

Correct statements would be: 1. 'Given any two bodies in thermal contact, the one with the higher temperature will transfer energy to the other by heat'. 2. 'Given any two bodies of equal mass, the one with the higher product of absolute temperature and specific heat contains more internal energy'. All to say is that internal energy depends not only on temperature but also on mass and nature of body.

**Illustration 1.8** Two bodies have the same heat capacity. If they are combined to form a single composite body, show that the equivalent specific heat of this composite body is independent of the masses of the individual bodies.

**Sol.** Let the two bodies have masses  $m_1$ ,  $m_2$  and specific heats  $s_1$  and  $s_2$ . Then

$$m_1 s_1 = m_2 s_2 \quad \text{or} \quad m_1/m_2 = s_2/s_1$$

Let  $s$  = specific heat of the composite body.

Then  $(m_1 + m_2)s = m_1 s_1 + m_2 s_2 = 2 m_1 s_1$

$$s = \frac{2m_1 s_1}{m_1 + m_2} = \frac{2m_1 s_1}{m_1 + m_1(s_2/s_1)} = \frac{2s_1 s_2}{s_2 + s_1}$$

**Illustration 1.9** The temperature of a silver bar rises by 10.0°C when it absorbs 1.23 kJ of energy by heat. The mass of bar is 525 g. Determine the specific heat of silver.

**Sol.** We find its specific heat from the definition, which is contained in the equation  $Q = mc_{\text{silver}} \Delta T$  for energy input by heat to produce a temperature change. Solving we have

$$c_{\text{silver}} = \frac{Q}{m \Delta T}$$

$$c_{\text{silver}} = \frac{1.23 \times 10^3 \text{ J}}{(0.525 \text{ kg})(10.0^\circ \text{C})} = 234 \text{ J/kg}^\circ \text{C}$$

**Illustration 1.10** The air temperature above coastal areas is profoundly influenced by the large specific heat of water. One reason is that the energy released when 1 m<sup>3</sup> of water cools by 1°C will raise the temperature of a much larger volume of air by 1°C. Find this volume of air. The specific heat of air is approximately 1 kJ/kg°C. Take the density of air to be 1.3 kg/m<sup>3</sup>.

**Sol.** The mass of 1 m<sup>3</sup> of water is specified by its density,

$$m = \rho V = (1.00 \times 10^3 \text{ kg/m}^3)(1 \text{ m}^3) = 1 \times 10^3 \text{ kg}$$

When 1 m<sup>3</sup> of water cools by 1°C, it releases energy

$$Q_c = mc \Delta T = (1 \times 10^3 \text{ kg})(4186 \text{ J/kg}^\circ \text{C})(-1^\circ \text{C}) = -4 \times 10^6 \text{ J}$$

where the negative sign represents heat output. When  $+4 \times 10^6 \text{ J}$  is transferred to the air, raising its temperature by 1°C, the volume of the air is given by  $Q_c = mc \Delta T = \rho V c \Delta T$

$$V = \frac{Q_c}{\rho c \Delta T} = \frac{4 \times 10^6 \text{ J}}{(1.3 \text{ kg/m}^3)(1 \times 10^3 \text{ J/kg}^\circ \text{C})(1^\circ \text{C})} = 3 \times 10^3 \text{ m}^3$$

The volume of the air is a thousand times larger than the volume of the water.

**Illustration 1.11** James Joule tested the conversion of mechanical energy into internal energy by measuring temperatures of falling water. If water at the top of a Swiss waterfall has a temperature of 10.0°C and then falls 50.0 m, what maximum temperature at the bottom would Joule expect? He did not succeed in measuring the temperature change, partly because evaporation cooled the falling water and also because his thermometer was not sufficiently sensitive.

**Sol.** The temperature change can be found from the potential energy that is converted to internal energy. The final temperature is this change added to the initial temperature of the water.

The gravitational energy that can change into internal energy is  $\Delta E_{\text{int}} = mgy$ . It will produce the same temperature



change as the same amount of heat entering the water from a stove, as described by  $Q = mc\Delta T$ . Thus,  $mgy = mc\Delta T$ .

$$\text{Isolating } \Delta T, \Delta T = \frac{gy}{c} = \frac{(9.80 \text{ m/s}^2)(50.0 \text{ m})}{4.186 \times 10^3 \text{ J/kg}^\circ\text{C}} = 0.117^\circ\text{C}$$

$$T_f = T_i + \Delta T = 10.0^\circ\text{C} + 0.117^\circ\text{C} = 10.1^\circ\text{C}$$

The final temperature might be less than we calculated since this solution does not account for cooling of the water due to evaporation as it falls.

## Latent Heat

- When a substance changes from one state to another state (say from solid to liquid or liquid to gas or from liquid to solid or gas to liquid) then energy is either absorbed or liberated. This heat energy is called latent heat.
- No change in temperature is involved when the substance changes its state. That is, phase transformation is an isothermal change. Ice at  $0^\circ\text{C}$  melts into water at  $0^\circ\text{C}$ . Water at  $100^\circ\text{C}$  boils to form steam at  $100^\circ\text{C}$ .
- The amount of heat required to change the state of the mass  $m$  of the substance is written as:  $\Delta Q = mL$ , where  $L$  is the latent heat. Latent heat is also called as heat of transformation.
- Unit: cal/g or J/kg and dimension:  $[L^2T^{-2}]$ .
- Any material has two types of latent heats.

i. **Latent heat of fusion:** The latent heat of fusion is the heat energy required to change 1 kg of the material in its solid state at its melting point to 1 kg of the material in its liquid state. It is also the amount of heat energy released when at melting point 1 kg of liquid changes to 1 kg of solid. For water at its normal freezing temperature or melting point ( $0^\circ\text{C}$ ), the latent heat of fusion (or latent heat of ice) is

$$L_F = L_{\text{ice}} \approx 80 \text{ cal/g} \approx 6 \text{ kJ/mol} \approx 336 \text{ kJ/kg}$$

ii. **Latent heat of vapourization:** The latent heat of vapourization is the heat energy required to change 1 kg of the material in its liquid state at its boiling point to 1 kg of the material in its gaseous state. It is also the amount of heat energy released when 1 kg of vapour changes into 1 kg of liquid. For water at its normal boiling point or condensation temperature ( $100^\circ\text{C}$ ), the latent heat of vapourization (latent heat of steam) is

$$L_V = L_{\text{steam}} \approx 540 \text{ cal/g} \approx 40.8 \text{ kJ/mol} \approx 2260 \text{ kJ/kg}$$

- In the process of melting or boiling, heat supplied is used to increase the internal potential energy of the substance and also in doing work against external pressure while internal kinetic energy remains constant. This is the reason that internal energy of steam at  $100^\circ\text{C}$  is more than that of water at  $100^\circ\text{C}$ .

- It is more painful to get burnt by steam rather than by boiling water at same temperature. This is so because when 1 g of steam at  $100^\circ\text{C}$  gets converted to water at  $100^\circ\text{C}$ , then it gives out 536 cal of heat. So, it is clear that steam at  $100^\circ\text{C}$  has more internal energy than water at  $100^\circ\text{C}$  (i.e., boiling of water).
- In case of change of state if the molecules come closer, energy is released and if the molecules move apart, energy is absorbed.
- Latent heat of vapourization is more than the latent heat of fusion. This is because when a substance gets converted from liquid to vapour, there is a large increase in volume. Hence, more amount of heat is required. But when a solid gets converted to a liquid, then the increase in volume is negligible. Hence, very less amount of heat is required. So, latent heat of vapourization is more than the latent heat of fusion.
- After snow falls, the temperature of the atmosphere becomes very low. This is because the snow absorbs the heat from the atmosphere to melt down. So, in the mountains, when snow falls, one does not feel too cold, but when ice melts, he feels too cold.
- There is more shivering effect of ice cream on teeth as compared to that of water (obtained from ice). This is because when ice cream melts down, it absorbs large amount of heat from teeth.

### Illustration 1.12

Some water at  $0^\circ\text{C}$  is placed in a large insulated enclosure (vessel). The water vapour formed is pumped out continuously. What fraction of the water will ultimately freeze, if the latent heat of vapourization is seven times the latent heat of fusion?

Sol. Let us learn the application of theory this illustration.

Let  $m$  = mass of water,  $f$  = fraction which freezes  
 $L_1$  = latent heat of vapourization  
 $L_2$  = latent heat of fusion

Mass of water frozen =  $mf$

Heat lost by freezing water =  $mfL_2$

Mass of vapour formed =  $m(1 - f)$

Heat gained by vapours =  $m(1 - f)L_1$

Now heat loss = heat gain:

$$mfL_2 = m(1 - f) \times 7L_2$$

$$f = 7 - 7f \quad \text{or} \quad f = 7/8$$

### Illustration 1.13

How many calories are required to change exactly 1 g of ice at  $-10^\circ\text{C}$  to steam at atmospheric pressure and  $120^\circ\text{C}$ ? [Assume the specific heat of steam at a constant pressure of 1 atm is  $0.481 \text{ cal/(g}^\circ\text{C)}$ .]

Sol. The first stage is the warming of the ice from  $-10^\circ\text{C}$  to the melting point ( $0^\circ\text{C}$ ).

The specific heat capacity of ice is  $0.50 \text{ cal/(g}^\circ\text{C)}$ . Therefore, the heat required in the first stage is given by  $\Delta H_1 = mc_1 \Delta t_1 = (1.00 \text{ g})(0.50 \text{ cal/(g}^\circ\text{C)})(10^\circ\text{C}) = 5.0 \text{ cal}$ .



The second stage is the melting of the ice at  $0^\circ\text{C}$  and 1 atm of pressure. The latent heat for the melting of ice is  $79.8 \text{ cal/g}$ .  $\Delta H_2 = 79.8 \text{ cal}$ . The third stage is the heating of the water from  $0^\circ\text{C}$  to  $100^\circ\text{C}$ , the boiling point, so the heat required is given by  $\Delta H_3 = mc_3 \Delta t_3 = (1.00 \text{ g})(1.000 \text{ cal/g}^\circ\text{C})(100^\circ\text{C}) = 100 \text{ cal}$ .

The fourth stage is the boiling of the water at a temperature of  $100^\circ\text{C}$  and at a constant pressure of 1.00 atm. According to Table, the latent heat for the boiling water at 1.00 atm is  $540 \text{ cal/g}$ , so  $\Delta H_4 = mL_4 = (1.00 \text{ g})(540 \text{ cal/g}) = 540 \text{ cal}$ .

The fifth and final stage is the heating of the steam from  $100^\circ\text{C}$  to  $120^\circ\text{C}$  (at a constant pressure of 1.00 atm).

Assuming that between  $100^\circ\text{C}$  and  $120^\circ\text{C}$  the specific heat capacity of steam is constant and has the value  $0.481 \text{ cal/(g}^\circ\text{C)}$  given, we find  $\Delta H_5 = mc_5 \Delta t_5 = (1 \text{ g})[0.481 \text{ cal/(g}^\circ\text{C)}](20^\circ\text{C}) = 9.62 \text{ cal}$

The total heat requirement  $\Delta H = \Delta H_1 + \Delta H_2 + \Delta H_3 + \Delta H_4 + \Delta H_5 = (5.0 + 79.8 + 100 + 540 + 9.62) = 734.4 \text{ cal}$ .

## Principle of Calorimetry

When two bodies (one being solid and other liquid or both being liquid) at different temperatures are mixed, heat will be transferred from body at higher temperature to a body at lower temperature till both acquire same temperature. The body at higher temperature releases heat while body at lower temperature absorbs it, so that

$$\text{Heat lost} = \text{Heat gained}$$

i.e., the principle of calorimetry represents the law of conservation of heat energy.

- Temperature of mixture ( $T$ ) is always  $\geq$  lower temperature ( $T_L$ ) and  $\leq$  higher temperature ( $T_H$ ), i.e.,  $T_L \leq T \leq T_H$

i.e., the temperature of mixture can never be lesser than lower temperatures (as a body cannot be cooled below the temperature of cooling body) and greater than higher temperature (as a body cannot be heated above the temperature of heating body). Furthermore usually rise in temperature of one body is not equal to the fall in temperature of the other body though heat gained by one body is equal to the heat lost by the other.

- When temperature of a body changes, the body releases heat if its temperature falls and absorbs heat when its temperature rises. The heat released or absorbed by a body of mass  $m$  is given by  $Q = mc \Delta T$ , where  $c$  is specific heat of the body and  $\Delta T$  change in its temperature.
- When state of a body changes, change of state takes place at constant temperature [m.pt. or b.pt.] and heat released or absorbed is given by  $Q = mL$ , where  $L$  is latent heat. Heat is absorbed if solid converts into liquid (at m.pt.) or liquid converts into vapours (at b.pt.) and is

released if liquid converts into solid or vapours convert into liquid.

- If two bodies  $A$  and  $B$  of masses  $m_1$  and  $m_2$  at temperatures  $T_1$  and  $T_2$  ( $T_1 > T_2$ ) and having gram specific heat  $c_1$  and  $c_2$  are placed in contact,

Heat lost by  $A$  = Heat gained by  $B$

$$\text{or } m_1 c_1 (T_1 - T) = m_2 c_2 (T - T_2) \quad (\text{where } T = \text{temperature of equilibrium})$$

$$\therefore T = \frac{m_1 c_1 T_1 + m_2 c_2 T_2}{m_1 c_1 + m_2 c_2}$$

- If bodies are of same material  $c_1 = c_2$  then  $T = \frac{m_1 T_1 + m_2 T_2}{m_1 + m_2}$
- If bodies are of same mass ( $m_1 = m_2$ ) then  $T = \frac{T_1 c_1 + T_2 c_2}{c_1 + c_2}$
- If bodies are of same material and of equal masses ( $m_1 = m_2$ ,  $c_1 = c_2$ ) then  $T = \frac{T_1 + T_2}{2}$

### Illustration 1.4

Calculate the heat of fusion of ice from the following data for ice at  $0^\circ\text{C}$  added to water. Mass of calorimeter = 60 g, mass of calorimeter + water = 460 g, mass of calorimeter + water + ice = 618 g, initial temperature of water =  $38^\circ\text{C}$ , final temperature of the mixture =  $5^\circ\text{C}$ . The specific heat of calorimeter =  $0.10 \text{ cal/g}^\circ\text{C}$ . Assume that the calorimeter was also at  $0^\circ\text{C}$  initially.

**Sol.** Mass of water =  $460 - 60 = 400 \text{ g}$

mass of ice =  $618 - 460 = 158 \text{ g}$

Heat lost by water = heat gained by ice to melt + heat gained by (water obtained from melting of ice + calorimeter) to reach  $5^\circ\text{C}$

$$\Rightarrow 400 \times 1 \times (38 - 5) = 158 \times L + 158 \times 1 \times 5 + 60 \times 0.1 \times 5$$

(where  $L$  is the latent heat of fusion of ice)

$$\Rightarrow L = 78.35 \text{ cal/g}$$

### Illustration 1.5

A lump of ice of  $0.1 \text{ kg}$  at  $-10^\circ\text{C}$  is put in  $0.15 \text{ kg}$  of water at  $20^\circ\text{C}$ . How much water and ice will be found in the mixture when it has reached thermal equilibrium? Specific heat of ice =  $0.5 \text{ kcal/kg}^\circ\text{C}$  and its latent heat of melting =  $80 \text{ kcal/kg}$ .

**Sol.** Heat released by  $0.15 \text{ kg}$  of water in being cooled to  $0^\circ\text{C} = 0.15 \times 1 \times 20 = 3 \text{ kcal}$

Heat absorbed by ice from  $-10^\circ\text{C}$  to  $0^\circ\text{C} = 0.1 \times 0.5 \times 10 = 0.5 \text{ kcal}$

The balance heat is available for melting ice. Let  $m \text{ kg}$  of ice melt.

$$\text{Then } m \times 80 = 2.5 \quad \text{or } m = 0.03 \text{ kg}$$

Thus the final temperature is  $0^\circ\text{C}$  with  $0.07 \text{ kg}$  of ice and  $0.18 \text{ kg}$  of water.

### Illustration 1.6

How should  $1 \text{ kg}$  of water at  $5^\circ\text{C}$  be divided into two parts so that if one part turned into ice at



0°C, it would release enough heat to vapourize the other part? Latent heat of steam = 540 cal/g and latent heat of ice = 80 cal/g.

**Sol.** Let the mass be divided into  $x$  grams (for ice) and  $(1000 - x)$  grams (for vapour).

Heat released by  $x$  grams of water =  $x \times 1 \times 5 + x \times 80$

Heat absorbed by  $(1000 - x)$  grams of water

$$= (1000 - x) \times 1 \times 95 + (1000 - x) \times 540$$

Assuming that the conversion of the other part takes place at 100°C.

$$85x = 95(1000 - x) + 540(1000 - x) \quad \text{or} \quad x = 882 \text{ g}$$

Thus the mass is to be divided into 882 g for conversion into ice and 118 g for conversion into vapour.

**Illustration 1.17** When a block of metal of specific heat 0.1 cal/g°C and weighing 110 g is heated to 100°C and then quickly transferred to a calorimeter containing 200 g of a liquid at 10°C, the resulting temperature is 18°C. On repeating the experiment with 400 g of same liquid in the same calorimeter at same initial temperature, the resulting temperature is 14.5°C. Find

a. Specific heat of the liquid.

b. The water equivalent of calorimeter.

**Sol.** Let  $s$  be the specific heat of the liquid and  $W$  be the water equivalent of the calorimeter.

Heat lost by the block = heat gained by (liquid + calorimeter)

For the first case:

$$\begin{aligned} \Rightarrow 110 \times 0.1 \times (100 - 18) &= 200 \times s \\ &\times (18 - 10) + W \times 1 \times (18 - 10) \\ \Rightarrow 1600s + 8W &= 902 \end{aligned} \quad (i)$$

For the second case:

$$\begin{aligned} \Rightarrow 110 \times 0.1 \times (100 - 14.5) &= 400 \times s \\ &\times (14.5 - 10) + W \times 1 \times (14.5 - 10) \\ \Rightarrow 1800s + 4.5W &= 940.5 \end{aligned} \quad (ii)$$

On solving Eqs. (i) and (ii), we get  $s = 0.48$  cal/g°C and  $W = 16.6$  g.

**Illustration 1.18** The temperatures of equal masses of three different liquids  $A$ ,  $B$  and  $C$  are 12°C, 19°C and 28°C, respectively. The temperature when  $A$  and  $B$  are mixed is 16°C, while when  $B$  and  $C$  are mixed, it is 23°C. What would be the temperature when  $A$  and  $C$  are mixed?

**Sol.** Let  $m$  = mass of each liquid, when  $A$  and  $B$  are mixed, Heat lost by  $B$  = heat gained by  $A$

$$\Rightarrow m s_B (19 - 16) = m s_A (16 - 12) \Rightarrow 3s_B = 4s_A \quad (i)$$

When  $B$  and  $C$  are mixed,

Heat lost by  $C$  = heat gained by  $B$

$$\Rightarrow m s_C (28 - 23) = m s_B (23 - 19) \Rightarrow 5s_C = 4s_B \quad (ii)$$

From Eqs. (i) and (ii), we get

$$16s_A = 12s_B = 15s_C \quad (iii)$$

When  $A$  and  $C$  are mixed, Let  $\theta$  = final temperature

Heat lost by  $C$  = heat gained by  $A$

$$\Rightarrow m s_C (28 - \theta) = m s_A (\theta - 12)$$

Using Eq. (iii), we get

$$\Rightarrow 15 s_C (28 - \theta) = 15 s_A (\theta - 12)$$

$$\Rightarrow 16s_A (28 - \theta) = 15s_A (\theta - 12)$$

On solving for  $\theta$ , we get  $\theta = \frac{16 \times 28 + 12 \times 15}{16 + 15}$

$$\Rightarrow \theta = 20.26^\circ\text{C}$$

**Illustration 1.19** A tube leads from a flask in which water is boiling under atmospheric pressure to a calorimeter. The mass of the calorimeter is 150 g, its specific heat capacity is 0.1 cal/g°C, and it contains originally 340 g of water at 15°C. Steam is allowed to condense in the calorimeter until its temperature increases to 71°C, after which total mass of calorimeter and contents are found to be 525 g. Compute the heat of condensation of steam.

**Sol.** Mass of calorimeter and contents before passing steam =  $(150 + 340) = 490$  g

mass after passing steam = 525 g

$$\Rightarrow \text{mass of steam which condenses} = (525 - 490) \text{ g} = 35 \text{ g}$$

Let  $L$  = latent heat of steam.

Heat lost by steam = heat gained by water +

heat gained by calorimeter

$$35L + 35 \times 1 (100 - 71) = 340 \times 1 \times (71 - 15) + 150 \times 0.1 \times (71 - 15)$$

$$\Rightarrow L = 539 \text{ cal/g}$$

**Illustration 1.20** Determine the final result when 200 g of water and 20 g of ice at 0°C are in a calorimeter having a water equivalent of 30 g and 50 g of steam is passed into it at 100°C

**Sol.** When steam is passed, the final temperature can be 0°C, between 0°C and 100°C, or 100°C.

We will consider all three possibilities.

#### Case I

Final temperature = 0°C

In this case, all the steam condenses and then cools down to 0°C.

Heat given out by steam

$$= 50 \times 540 + 50 \times 1 \times (100 - 0) = 32000 \text{ cal}$$

$$\text{Mass of ice which will melt by this heat} = \frac{32000}{80} = 400 \text{ g}$$

But there is only 20 g of ice in the calorimeter.

Hence final temperature cannot be 0°C.

#### Case II

Final temperature =  $\theta$  and  $0 < \theta < 100$

Heat lost by steam = heat gained by (ice + water + calorimeter)

$$\Rightarrow 50 \times 540 + 50 \times 1 \times (100 - \theta) = 20 \times 80 + (20 + 200 + 30) \times 1 \times (\theta - 0) \Rightarrow \theta = 101.3^\circ\text{C}$$

The assumption ( $0 < \theta < 100$ ) is proved to be wrong. Hence, the final temperature cannot be between  $0^\circ\text{C}$  and  $100^\circ\text{C}$

$\Rightarrow$  The final temperature will be  $100^\circ\text{C}$

### Case III

Let  $m$  = mass of steam condensed.

Heat lost by steam = heat gained by ice to melt + heat gained by (water + water + calorimeter) to reach  $100^\circ\text{C}$

$$\Rightarrow m(540) = 20 \times 80 + (20 + 200 + 30) \times (100 - 0)$$

$$\Rightarrow m = 26600/540 \approx 49 \text{ g}$$

$\Rightarrow$  49 g of steam gets condensed and the final temperature is  $100^\circ\text{C}$ .

### Illustration P21

What will be the final temperature when 150 g of ice at  $0^\circ\text{C}$  is mixed with 300 g of water at  $50^\circ\text{C}$ . Specific heat of water =  $1 \text{ cal/g}^\circ\text{C}$ . Latent heat of fusion of ice =  $80 \text{ cal/g}$ .

Sol. Let us assume that  $T > 0^\circ\text{C}$

Heat lost by water = heat gained by ice to melt + heat gained by water formed from ice

$$\Rightarrow 300 \times 1 \times (50 - T) = 150 \times 80 + 150 \times 1 \times (T - 0)$$

$$\Rightarrow T = 6.7^\circ\text{C}$$

Hence our assumption that  $T > 0^\circ\text{C}$  is correct.

### Illustration P22

In a calorimeter (water equivalent = 40 g) are 200 g of water and 50 g of ice all at  $0^\circ\text{C}$ . 30 g of water at  $90^\circ\text{C}$  is poured into it. What will be the final condition of the system?

Sol. Let us assume that all ice melts and temperature of water rises beyond  $0^\circ\text{C}$ . Thus we will assume that  $T > 0$ .

Heat lost by water added = heat gained by ice to melt + Heat to warm water formed from ice and water added + Heat gained by calorimeter can.

$$\Rightarrow 30 \times 1 \times (90 - T) = 50 \times 80 + (50 + 200) \times 1$$

$$\times (T - 0) + 40 \times 1 \times (T - 0)$$

$$\Rightarrow 2700 - 30T = 4000 + 250T + 40T$$

$$\Rightarrow T = -4.1^\circ\text{C}$$

Hence our assumption that  $T > 0$  is wrong, since hot water added is not able to melt all of the ice.

Therefore the final temperature will be  $0^\circ\text{C}$ .

Let  $m$  = mass of ice finally left in the can.

Heat lost by water = heat gained by melting ice

$$\Rightarrow 30 \times 1 \times (90 - 0) = (50 - m) \times 80 \Rightarrow m = 16.25 \text{ g}$$

Finally there is 16.25 g of ice and  $(200 + 30 + 33.75) = 266.75 \text{ g}$  of water at  $0^\circ\text{C}$ .

## Heating Curve

If heat is supplied at constant rate to a given mass  $m$  of a solid,  $P$  and a graph is plotted between temperature and time, the graph is as shown in Fig. 1.3 and is called heating curve. From this curve it is clear that

- In the region  $OA$  temperature of solid is changing with time, so,

$$Q = mc_s \Delta T$$

$$\text{or } P \Delta t = mc_s \Delta T \quad (\text{as } Q = P \Delta t)$$

But as  $(\Delta T / \Delta t)$  is the slope of temperature-time curve

$$c_s \propto (1/\text{slope of line } OA)$$

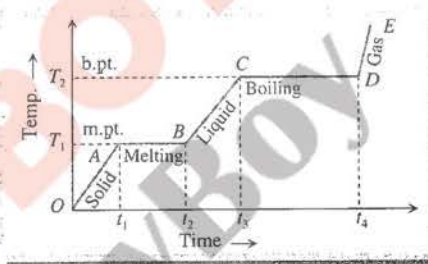


Fig. 1.3

i.e., specific heat (or thermal capacity) is inversely proportional to the slope of temperature-time curve.

- In the region  $AB$  temperature is constant, so it represents change of state, i.e., melting of solid with melting point  $T_1$ . At  $A$  melting starts and at  $B$  all solid is converted into liquid. So between  $A$  and  $B$  substance is partly solid and partly liquid. If  $L_F$  is the latent heat of fusion.

$$Q = mL_F \quad \text{or} \quad L_F = \frac{P(t_2 - t_1)}{m} \quad [\text{as } Q = P(t_2 - t_1)]$$

$$\text{or } L_F \propto \text{length of line } AB$$

i.e., latent heat of fusion is proportional to the length of line of zero slope. (In this region specific heat  $\rightarrow \infty$ )

- In the region  $BC$  temperature of liquid increases so specific heat (or thermal capacity) of liquid will be inversely proportional to the slope of line  $BC$ , i.e.,

$$c_L \propto (1/\text{slope of line } BC)$$

- In the region  $CD$  temperature is constant, so it represents the change of state, i.e., boiling with boiling point  $T_2$ . At  $C$  all substance is in liquid state while at  $D$  in vapour state and between  $C$  and  $D$  partly liquid and partly gas. The length of line  $CD$  is proportional to latent heat of vapourization, i.e.,

$$L_V \propto \text{Length of line } CD$$

(In this region specific heat  $\rightarrow \infty$ )

- The line  $DE$  represents gaseous state of substance with its temperature increasing linearly with time. The reciprocal of slope of line will be proportional to specific heat or thermal capacity of substance in vapour state.



A substance is in the solid form at  $0^\circ\text{C}$ . The amount of heat added to this substance and its temperature are plotted in the following graph.

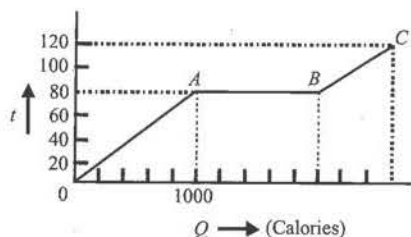


Fig. 1.4

If the relative specific heat capacity of the solid substance is 0.5, find from the graph (i) the mass of the substance; (ii) the specific latent heat of the melting process and (iii) the specific heat of the substance in the liquid state.

Specific heat capacity of water =  $1000 \text{ cal/kg/K}$

Sol. 1000 cal of heat raises the temperature of the substance from  $0^\circ\text{C}$  to  $80^\circ\text{C}$ .

$$\therefore 1000 = m (1000 \times 0.5) \times 80$$

$$(\because \text{specific heat} = \text{relative sp. heat} \times \text{of water})$$

$$\text{or } m = 0.025 \text{ kg}$$

$$\text{Latent heat} = 200 \times 5 = 1000 \text{ cal } (\because 1 \text{ div reads } 200 \text{ cal}) = 0.025 \times L$$

$$\therefore L = 40000 \text{ cal/kg}$$

In the liquid state temperature rises from  $80^\circ\text{C}$  to  $120^\circ\text{C}$ , that is, by  $40^\circ\text{C}$  after absorbing 600 cal.

$$\therefore 0.025 \times s \times 40 = 600 \quad \text{or } s = 600 \text{ cal/kg/K}$$

**Illustration 1.24** Two bodies of equal masses are heated at a uniform rate under identical conditions. The change in temperature in the two cases is shown graphically. What are their melting points? Find the ratio of their specific heats and latent heats.

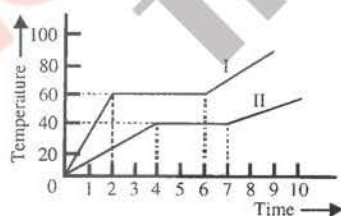


Fig. 1.5

Sol. The melting points of liquids I and II are  $60^\circ\text{C}$  and  $40^\circ\text{C}$ , respectively. Let  $R$  be the rate of supply of heat. We note from the graph that liquid I is heated through  $60^\circ\text{C}$  in 2 units of time and that liquid II is heated through  $40^\circ\text{C}$  in 4 units of time.

$$\therefore 2R = m \times c_1 \times 60 \quad \text{and} \quad 4R = m \times c_2 \times 40$$

$$\text{Hence, } \frac{c_1}{c_2} = \frac{1}{3}$$

We note further that the temperature of I remains constant for 4 units of time and that of II for 2 units of time.

$$\therefore 4R = mL_1 \quad \text{and} \quad 2R = mL_2 \Rightarrow \frac{L_1}{L_2} = 2$$

### Concept Application Exercise 1.1

- The greater the mass of a body, the greater is its heat capacity. Is this true or false?
- The greater the mass of a body, the greater is its latent heat capacity. Is this true or false?
- The greater the mass of a body, the greater is its specific heat capacity. True or false?
- Can heat be added to a substance without causing the temperature of the body to rise? If so, does this contradict the concept of heat as energy in the process of transfer because of a temperature difference?
- Can heat be considered to be a form of stored energy?
- Give an example of a process in which no heat is transferred to or from a system but the temperature of the system changes?
- The latent heat of fusion of a substance is always less than the latent heat of vapourization or latent heat of sublimation of the same substance. Explain.
- Suppose an astronaut on the surface of the moon took some water at about  $20^\circ\text{C}$  out of his thermos and poured it into a glass beaker. What would happen to the water?
- Heat is added to a body. Does its temperature necessarily increase?
- When a hot body warms a cool one, are their temperature changes equal in magnitude?
- Steam at  $100^\circ\text{C}$  is passed into a calorimeter of water equivalent 10 g containing 74 cc of water and 10 g of ice at  $0^\circ\text{C}$ . If the temperature of the calorimeter and its contents rises to  $5^\circ\text{C}$ , calculate the amount of steam passed. Latent heat of steam =  $540 \text{ kcal/kg}$ , latent heat of fusion =  $80 \text{ kcal/kg}$ .
- Ice of mass 600 g and at a temperature of  $-10^\circ\text{C}$  is placed in a copper vessel heated to  $350^\circ\text{C}$ . The resultant mixture is 550 g of ice and water. Find the mass of the vessel. The specific heat capacity of copper ( $c$ ) =  $100 \text{ cal/kg-K}$ .
- When a small ice crystal is placed in overcooled water it begins to freeze instantaneously.
  - What amount of ice is formed from 1 kg of water overcooled to  $-8^\circ\text{C}$ ?  $L$  of water =  $336 \times 10^3 \text{ J/kg}$  and  $s$  of water =  $4200 \text{ J/kg-K}$ .
  - What should be the temperature of the overcooled water in order that all of it be converted into ice at  $0^\circ\text{C}$ ?



14. An electric heater whose power is 54 W is immersed in  $650 \text{ cm}^3$  water in a calorimeter. In 3 min the water is heated by  $3.4^\circ\text{C}$ . What part of the energy of the heater passes out of the calorimeter in the form of radiant energy?
15. An ice cube whose mass is 50 g is taken from a refrigerator where its temperature was  $-10^\circ\text{C}$ . If no heat is gained or lost from outside, how much water will freeze onto the cube if it is dropped into a beaker containing water at  $0^\circ\text{C}$ ? Latent heat of fusion =  $80 \text{ kcal/kg}$ , specific heat capacity of ice =  $500 \text{ cal/kg/K}$ .
16. Equal volumes of three liquids of densities  $\rho_1, \rho_2$  and  $\rho_3$ , specific heat capacities  $c_1, c_2$  and  $c_3$  and temperatures  $t_1, t_2$  and  $t_3$ , respectively, are mixed together. What is the temperature of the mixture? Assume no changes in volume on mixing.
17. Victoria Falls in Africa is 122 m in height. Calculate the rise in temperature of the water if all the potential energy lost in the fall is converted into heat.
18. Equal masses of three liquids A, B and C are taken. Their initial temperatures are  $10^\circ\text{C}, 25^\circ\text{C}$  and  $40^\circ\text{C}$ , respectively. When A and B are mixed the temperature of the mixture is  $19^\circ\text{C}$ . When B and C are mixed, the temperature of the mixture is  $35^\circ\text{C}$ . Find the temperature if all three are mixed.
19. An earthenware vessel loses 1 g of water per second due to evaporation. The water equivalent of the vessel is 0.5 kg and the vessel contains 9.5 kg of water. Find the time required for the water in the vessel to cool to  $28^\circ\text{C}$  from  $30^\circ\text{C}$ . Neglect radiation losses. Latent heat of vapourization of water in this range of temperature is  $540 \text{ cal/g}$ .
20. A certain amount of ice is supplied heat at a constant rate for 7 min. For the first 1 min, the temperature rises uniformly with time; then it remains constant for the next 4 min and again rises at a uniform rate for the last 2 min. Explain physically these observations and calculate the final temperature.  $L$  of ice =  $336 \times 10^3 \text{ J/kg}$  and  $c_{\text{water}} = 4200 \text{ J/kg/K}$ .
21. 1 g steam at  $100^\circ\text{C}$  is passed in an insulated vessel having 1 g ice at  $0^\circ\text{C}$ . Find the equilibrium temperature of the mixture. Neglect heat capacity of the vessel.

## THERMAL EXPANSION

Figures 1.6 (a), (b) and (c) show molecules of solid, liquid and gas, respectively, in their random motions. The atoms are essentially in contact with one another. A rock is an example of a solid. It can stand alone because of the forces holding its atoms together.

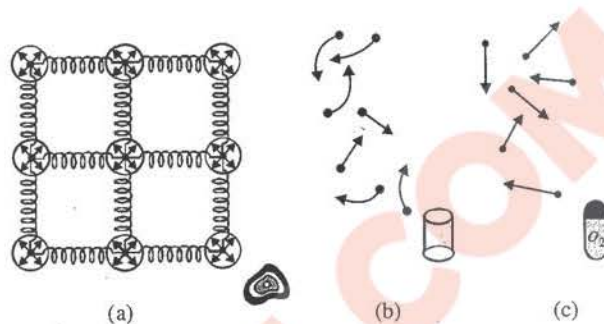


Fig. 1.6

Atoms in a liquid are also in close contact but can slide over one another. Forces between them strongly resist attempts to push them closer together and also hold them in close contact. Water is an example of a liquid. It can flow, but it is also in close contact. It can flow, but it also remains in an open container because of the forces between its atoms.

Atoms in a gas are separated by distances that are considerably larger than their diameters. Gas must be held in a closed container to prevent it from moving out freely.

Most substances expand when their temperature is raised and contract when cooled. There is an exception to this statement: water contracts when its temperature is increased from  $0^\circ\text{C}$  to  $4^\circ\text{C}$ . Thus water has its minimum volume, and hence maximum density, at  $4^\circ\text{C}$ .

Atoms in solids are in close contact; the forces between them allow the atoms to vibrate but not to move freely. These forces can be thought of as springs that can be stretched or compressed. An individual molecule's motion can be modelled as a point-like particle oscillating in a parallel well caused by the inter-atomic forces, which is parabolic for a Hooke's law spring  $\left( U(x) = \frac{1}{2} kx^2 \right)$ . The mass oscillates in simple harmonic motion between maximum and minimum positions. The potential energy curve is not symmetrical as shown in Fig. 1.7. The variable  $r$  is the separation between a particle and its nearest neighbour. At temperature  $T_1$  the total energy is  $E_1$  and its separation from its nearest neighbour lies between  $r_{1\text{min}}$  and  $r_{1\text{max}}$ , the average separation is  $r_{1\text{av}}$ . The  $U(r)$  is not symmetrical, it is flatter to the right at larger  $r$  values. At higher temperature the total energy  $E$  is higher; the particle spends more time at  $r$  values towards less steep portion of the curve. The average separation  $r_{2\text{av}}$  increases at higher temperatures. Because  $r_{2\text{av}} > r_{1\text{av}}$ , the average separation of the atoms or molecules in the solid increases with increasing temperature. When matter is heated without any change in state, it usually expands. According to atomic theory of matter, asymmetry in potential energy curve is responsible for thermal expansion. As with rise in temperature the amplitude of vibration and hence energy of atoms increases, hence the average distance between the atoms increases. So the matter as a whole expands.

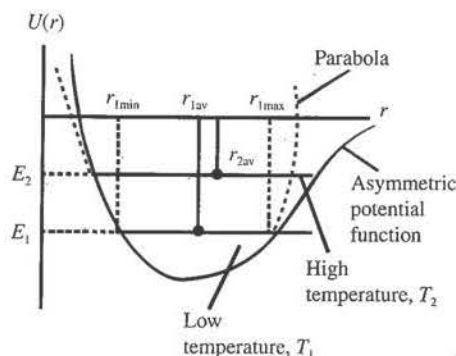


Fig. 1.7

- Thermal expansion is minimum in case of solids but maximum in case of gases because intermolecular force is maximum in solids but minimum in gases.
- Solids can expand in one dimension (linear expansion), two dimension (superficial expansion) and three dimension (volume expansion) while liquids and gases usually suffers change in volume only.
- The coefficient of linear expansion of the material of a solid is defined as the increase in its length per unit length per unit rise in its temperature.

$$\alpha = \frac{\Delta L}{L} \times \frac{1}{\Delta T}$$

Similarly, the coefficient of superficial expansion

$$\beta = \frac{\Delta A}{A} \times \frac{1}{\Delta T}$$

and coefficient of volume expansion

$$\gamma = \frac{\Delta V}{V} \times \frac{1}{\Delta T}$$

The value of  $\alpha$ ,  $\beta$  and  $\gamma$  depends upon the nature of material. All have dimension  $[\theta^{-1}]$  and unit per  $^{\circ}\text{C}$ .

- As  $\alpha = \frac{\Delta L}{L} \times \frac{1}{\Delta T}$ ,  $\beta = \frac{\Delta A}{A} \times \frac{1}{\Delta T}$  and  $\gamma = \frac{\Delta V}{V} \times \frac{1}{\Delta T}$   
 $\therefore \Delta L = L\alpha\Delta T$ ,  $\Delta A = A\beta\Delta T$  and  $\Delta V = V\gamma\Delta T$

$$\text{Final length } L' = L + \Delta L = L(1 + \alpha\Delta T) \quad (\text{i})$$

$$\text{Final area } A' = A + \Delta A = A(1 + \beta\Delta T) \quad (\text{ii})$$

$$\text{Final volume } V' = V + \Delta V = V(1 + \gamma\Delta T) \quad (\text{iii})$$

- If  $L$  is the side of square plate and it is heated by temperature  $\Delta T$ , then its side becomes  $L'$ .

The initial surface area  $A = L^2$  and final surface area  $A' = L'^2$

$$\therefore \frac{A'}{A} = \left(\frac{L'}{L}\right)^2 = \left(\frac{L(1 + \alpha\Delta T)}{L}\right)^2 = (1 + \alpha\Delta T)^2 = (1 + 2\alpha\Delta T)$$

(using Binomial theorem)

$$\text{or } A' = A(1 + 2\alpha\Delta T)$$

Comparing with Eq. (ii), we get  $\beta = 2\alpha$

Similarly, for volumetric expansion

$$\frac{V'}{V} = \left(\frac{L'}{L}\right)^3 = \left(\frac{L(1 + \alpha\Delta T)}{L}\right)^3 = (1 + \alpha\Delta T)^3 = (1 + 3\alpha\Delta T)$$

(using Binomial theorem)

$$\text{or } V' = V(1 + \gamma\Delta T)$$

Comparing with Eq. (iii), we get  $\gamma = 3\alpha$

$$\text{So } \alpha : \beta : \gamma = 1 : 2 : 3$$

### Some Important Points to Note

- For the same rise in temperature  
 Percentage change in area =  $2 \times$  percentage change in length.  
 Percentage change in volume =  $3 \times$  percentage change in length.
- The three coefficients of expansion are not constant for a given solid. Their values depend on the temperature range in which they are measured.
- The values of  $\alpha$ ,  $\beta$  and  $\gamma$  are independent of the units of length, area and volume, respectively.
- For anisotropic solids  $\gamma = \alpha_x + \alpha_y + \alpha_z$ , where  $\alpha_x$ ,  $\alpha_y$  and  $\alpha_z$  represent the mean coefficients of linear expansion along three mutually perpendicular directions.

### Illustration 1.25

The rectangular plate shown in Fig. 1.8 has an area  $A$ . If the temperature increases by  $\Delta T$ , each dimension increases according to  $\Delta L = \alpha L \Delta T$ , where  $\alpha$  is the average coefficient of linear expansion. Show that the increase in area is  $\Delta A = 2\alpha A \Delta T$ . What approximation does this expansion assume?

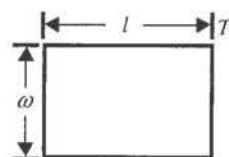


Fig. 1.8

**Sol.** We expect the area to increase in thermal expansion. It is neat that the coefficient of area expansion is just twice the coefficient of linear expansion.

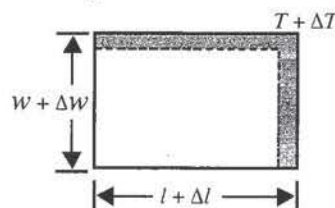


Fig. 1.9



## 1.14 Waves & Thermodynamics

We will use the definitions of coefficients of linear and area expansion.

From the diagram in Fig. 1.9, we see that the change in area is

$$\Delta V = l\Delta w + w\Delta l + \Delta w\Delta l$$

Since  $\Delta l$  and  $\Delta w$  are each small quantities, the product  $\Delta w\Delta l$  will be very small as compared to the original or final area.

Therefore, we assume  $\Delta w\Delta l \propto 0$

Since  $\Delta w = w\alpha\Delta T$  and  $\Delta l = l\alpha\Delta T$   
 We then have  $\Delta A = lw\alpha\Delta T + wl\alpha\Delta T$   
 Finally, since  $A = lw$ , we have  $\Delta A = 2\alpha A\Delta T$

**Illustration 1.26** A mercury thermometer is constructed as shown in Fig. 1.10. The capillary tube has a diameter of 0.004 00 cm, and the bulb has a diameter of 0.250 cm. neglecting the expansion of the glass, find the change in height of the mercury column with a temperature change of 30.0°C.

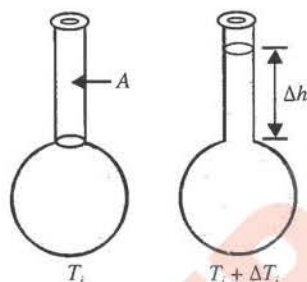


Fig. 1.10

**Sol.** For an easy-to-read thermometer, the column should rise by a few centimetres.

We use the definition of the coefficient of expansion.

Neglecting the expansion of the glass, the volume of liquid in the capillary will be  $\Delta V = A\Delta h$ , where  $A$  is the cross-sectional area of the capillary. Let  $V_i$  represent the volume of the bulb.

$$\Delta V = V_i\gamma\Delta T$$

$$\Delta h = \left(\frac{V_i}{A}\right)\gamma\Delta T = \left[\frac{\frac{4}{3}\pi R_{\text{bulb}}^3}{\pi R_{\text{cap}}^2}\right]\gamma\Delta T$$

$$\Delta h = \frac{4}{3} \frac{(0.125 \text{ cm})^3}{(0.002 00 \text{ cm})^2} (1.82 \times 10^{-4} / ^\circ\text{C})(30.0^\circ\text{C}) = 3.55 \text{ cm}$$

This is a practical thermometer. Glass expands so little as compared to mercury that only the third digit of the answer would be affected by including the expansion of the glass in our analysis.

**Illustration 1.27** A metal rod  $A$  of 25 cm length expands by 0.05 cm when its temperature is raised from

0°C to 100°C. Another rod  $B$  of a different metal of length 40 cm expands by 0.04 cm for the same rise in temperature. A third rod  $C$  of 50 cm length made up of pieces of rods  $A$  and  $B$  placed end to end expands by 0.03 cm on heating from 0°C to 50°C. Find the length of each portion of composite rod  $C$ .

**Sol.** From the given data for rod  $A$ , we have

$$\Delta L = \alpha_A L \Delta T$$

$$\text{or } \alpha_A = \frac{\Delta L}{L \Delta T} = \frac{0.05}{25 \times 100} = 2 \times 10^{-5} / ^\circ\text{C}$$

For rod  $B$ , we have  $\Delta L = \alpha_B L \Delta T$

$$\text{or } \alpha_B = \frac{\Delta L}{L \Delta T} = \frac{0.04}{40 \times 100} = 10^{-5} / ^\circ\text{C}$$

If rod  $C$  is made of segments of rod  $A$  and  $B$  of lengths  $l_1$  and  $l_2$ , respectively, then we have at 0°C

$$l_1 + l_2 = 50 \text{ cm} \quad (\text{i})$$

$$\text{At } T = 50^\circ\text{C} \quad l_1' + l_2' = 50.03 \text{ cm}$$

$$\text{Thus } \alpha_A l_1 \Delta T + \alpha_B l_2 \Delta T = 0.03 \text{ cm}$$

$$\text{or } 2 \times 10^{-5} \times l_1 \times 50 + 10^{-5} \times l_2 \times 50 = 0.03 \text{ cm}$$

$$\text{or } 2l_1 + l_2 = \frac{0.03}{50} \times 10^5 = 60 \text{ cm} \quad (\text{ii})$$

Solving Eqs. (i) and (ii), we get  $l_1 = 10 \text{ cm}$  and  $l_2 = 40 \text{ cm}$ .

**Illustration 1.28** Determine the lengths of an iron rod and a copper ruler at 0°C if the difference in their lengths at 50°C and 450°C is the same and is equal to 2 cm. The coefficient of linear expansion of iron =  $12 \times 10^{-6} / \text{K}$  and that of copper =  $17 \times 10^{-6} / \text{K}$ .

**Sol.** Let  $x$  be the length of the iron rod at 0°C,  $y$  that of the copper rod at 0°C, and  $I$  the difference in lengths at  $t_1$  and  $t_2^\circ\text{C}$ .

$$\text{Then } I = x(1 + \alpha_1 t_1) - y(1 + \alpha_2 t_1) \quad (\text{i})$$

$$\text{and } \pm I = x(1 + \alpha_1 t_2) - y(1 + \alpha_2 t_2) \quad (\text{ii})$$

Taking +1

$$I = x(1 + \alpha_1 t_2) - y(1 + \alpha_2 t_2) \quad (\text{iii})$$

from Eqs. (i) and (iii), we get

$$x\alpha_1 = y\alpha_2 \quad (\text{iv})$$

from Eqs. (i) and (iv), we get

$$y = \frac{l\alpha_1}{\alpha_2 - \alpha_1} = \frac{2 \times 12 \times 10^{-6}}{(17 - 12) \times 10^{-6}} = 4.8 \text{ cm}$$

and

$$x = \frac{l\alpha_2}{\alpha_2 - \alpha_1}$$

⇒

$$x = \frac{2 \times 17 \times 10^{-6}}{(17 - 12) \times 10^{-6}} \text{ cm} = 6.8 \text{ cm}$$

Taking -1

$$y = \frac{2l + l\alpha_1(t_1 + t_2)}{(t_2 - t_1)(\alpha_2 - \alpha_1)}, \quad x = \frac{2l + l\alpha_2(t_1 + t_2)}{(t_2 - t_1)(\alpha_2 - \alpha_1)}$$

$$\therefore y = \frac{2 \times 2 + 2 \times 12 \times 10^{-6}(450 + 50)}{(450 - 50)(17 - 12) \times 10^{-6}} \text{ cm} = 2006 \text{ cm} = 20.06 \text{ m}$$

$$x = \frac{2 \times 2 + 2 \times 17 \times 10^{-6}(450 + 50)}{(450 - 50)(17 - 12) \times 10^{-6}} \text{ cm} = 2008.5 \text{ cm} = 20.08 \text{ m}$$

**Illustration 1.29** A steel ball initially at a pressure of  $10^5 \text{ Pa}$  is heated from  $20^\circ\text{C}$  to  $120^\circ\text{C}$  keeping its volume constant. Find the final pressure inside the ball. Given that coefficient of linear expansion of steel is  $1.1 \times 10^{-5}/^\circ\text{C}$  and Bulk modulus of steel is  $1.6 \times 10^{11} \text{ Nt/m}^2$ .

**Sol.** On increasing temperature of ball by  $100^\circ\text{C}$  (from  $20^\circ\text{C}$  to  $120^\circ\text{C}$ ), the thermal expansion in its volume can be given as

$$\Delta V = \gamma_{st} V \Delta T = 3 \alpha_{st} V \Delta T \quad (\text{i})$$

Here it is given that no change of volume is allowed. This implies that the volume increment by thermal expansion is compressed elastically by external pressure. Thus elastic compression in the sphere must be equal to that given in Eq. (i). Bulk modulus of a material is defined as

$$B = \frac{\text{increase in pressure}}{\text{volume strain}} = \frac{\Delta P}{\Delta V/V}$$

Here the externally applied pressure to keep the volume of ball constant is given as

$$\begin{aligned} \Delta P &= B \times \frac{\Delta V}{V} = B(3\alpha_{st}\Delta T) \\ &= 1.6 \times 10^{11} \times 3 \times 1.1 \times 10^{-5} \times 100 \\ &= 5.28 \times 10^8 \text{ Nt/m}^2 = 5.28 \times 10^8 \text{ Pa} \end{aligned}$$

Thus this must be the excess pressure inside the ball at  $120^\circ\text{C}$  to keep its volume constant during heating.

**Illustration 1.30** A steel rail 30 m long is firmly attached to the roadbed only at its ends. The sun raises the temperature of the rail by  $5^\circ\text{C}$ , causing the rail to buckle. Assuming that the buckled rail consists of two straight parts meeting in the centre, calculate how much the centre of the rail rises. Coefficient of linear expansion of steel is  $12 \times 10^{-6}/^\circ\text{C}$ .

**Sol.** As indicated in Fig. 1.11, we let the initial length be  $2s$  and the final total length be  $2(s + \Delta s)$ .

The height of the centre of the buckled rail is denoted by  $y$ . Assuming that the standard coefficient  $\alpha$  of linear expansion can be used (in spite of the fact that the ends are anchored), we have  $\Delta s = \alpha s \Delta T$ .

By the Pythagorean theorem

$$y = \sqrt{(s + \Delta s)^2 - s^2} = \sqrt{2s\Delta s + (\Delta s)^2} = s\sqrt{2\alpha\Delta T + (\alpha\Delta T)^2}$$

With  $s = 15.0 \text{ m}$ ,  $\alpha = 12 \times 10^{-6}/^\circ\text{C}$ , and  $\Delta T = 50 \text{ K}$ , we obtain

$$y = (15.0 \text{ m})\sqrt{(12 \times 10^{-4}) + (6 \times 10^{-4})^2} = 0.52 \text{ m}$$

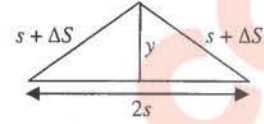


Fig. 1.11

### Variation of Density with Temperature

- i. Suppose  $\rho_0$  is the density of the substance at  $0^\circ\text{C}$  and at any temperature  $t$ , it becomes  $\rho_t$ . As mass of the substance remains constant at any temperature, we have

$$\rho_0 V_0 = \rho_t V_t$$

Here  $V_0$  and  $V_t$  are the volumes of the substance at  $0^\circ\text{C}$  and  $t^\circ\text{C}$ , respectively.

Also

$$V_t = V_0(1 + \gamma t)$$

$$\therefore \rho_0 V_0 = \rho_t \{V_0(1 + \gamma t)\}$$

or

$$\rho_t = \frac{\rho_0}{(1 + \gamma t)}$$

For small value of  $\gamma$ , we can approximate it as

$$\rho_t \approx \rho_0(1 - \gamma t)$$

- ii. If  $\rho_1$  and  $\rho_2$  are the densities at  $t_1$  and  $t_2$ , respectively, then we can write

$$\rho_1 V_1 = \rho_2 V_2$$

$$\text{or} \quad \rho_1 V_0(1 + \gamma t_1) = \rho_2 V_0(1 + \gamma t_2)$$

or

$$\rho_1 = \rho_2 \frac{(1 + \gamma t_2)}{(1 + \gamma t_1)}$$

$$= \rho_2(1 + \gamma t_2)(1 - \gamma t_1) = \rho_2[1 + \gamma(t_2 - t_1)]$$

(neglecting  $\gamma^2$  on being small)

$\therefore$

$$\gamma = \frac{\rho_1 - \rho_2}{\rho_2(t_2 - t_1)}$$

**Illustration 1.31** A small quantity of a liquid which does not mix with water sinks to the bottom at  $20^\circ\text{C}$ , the densities of the liquid and water being  $1021$  and  $998 \text{ kg/m}^3$ , respectively. To what temperature must the mixture be uniformly heated in order that the liquid forms globules which just float on water? The cubical expansion of the liquid and water over the temperature ranges is  $85 \times 10^{-5}/^\circ\text{C}$  and  $45 \times 10^{-5}/^\circ\text{C}$  respectively.

**Sol.** The liquid will float on water at the temperature at which both of them have the same densities.

$$D_{\text{common}} = \frac{1021}{1 + 85 \times 10^{-5} \Delta\theta} = \frac{998}{1 + 45 \times 10^{-5} \Delta\theta}$$

$$\Rightarrow 1021(1 + 45 \times 10^{-5} \Delta\theta) = 998(1 + 85 \times 10^{-5} \Delta\theta)$$



$$\Rightarrow \Delta\theta = 59^\circ\text{C} \Rightarrow \theta_2 - \theta_1 = 59^\circ\text{C}$$

$$\Rightarrow \theta_2 = 59 + 20 = 79^\circ\text{C}$$

### Expansion of Liquid

Liquids also expand on heating just like solids. Since liquids have no shape of their own, they suffer only volume expansion. If the liquid of volume  $V$  is heated and its temperature is raised by  $\Delta\theta$  then

$$V_L = V(1 + \gamma_L \Delta\theta)$$

( $\gamma_L$  = coefficient of real expansion or coefficient of volume expansion of liquid)

As liquid is always taken in a vessel for heating, if a liquid is heated, the vessel also gets heated and it also expands.

$$V_S = V(1 + \gamma_S \Delta\theta)$$

( $\gamma_S$  = coefficient of volume expansion for solid vessel)

So, the change in volume of liquid relative to vessel

$$V_L - V_S = V(\gamma_L - \gamma_S)\Delta\theta$$

$\Delta V_{app} = V\gamma_{app}\Delta\theta$  ( $\gamma_{app} = \gamma_L - \gamma_S$  = Apparent coefficient of volume expansion for liquid)

$\gamma_L > \gamma_S$	$\gamma_{app} > 0$	$\Delta V_{app} = \text{positive}$	Level of liquid in vessel will rise on heating.
$\gamma_L < \gamma_S$	$\gamma_{app} < 0$	$\Delta V_{app} = \text{negative}$	Level of liquid in vessel will fall on heating.
$\gamma_L = \gamma_S$	$\gamma_{app} = 0$	$\Delta V_{app} = 0$	Level of liquid in vessel will remain same.

**Illustration 1.32** A 1-L flask contains some mercury. It is found that at different temperatures, the volume of air inside the flask remains the same. What is the volume of mercury in the flask, given that the coefficient of linear expansion of glass =  $9 \times 10^{-6}/^\circ\text{C}$  and the coefficient of volume expansion of Hg =  $1.8 \times 10^{-4}/^\circ\text{C}$ ?

**Sol.** Since the volume above mercury remains the same at all temperatures, the expansion of the glass vessel must be the same as that of mercury in the vessel. Also,  $\gamma_g = 3\alpha_g = 27 \times 10^{-6}/^\circ\text{C}$ . Let  $V$  be the volume of the mercury. Then, from  $\Delta V = V\gamma\Delta T$

$$V \times (1.8 \times 10^{-4}) \times \Delta T = 10^{-3} \times 27 \times 10^{-6} \Delta T$$

$$(\because 1 \text{ L} = 10^{-3} \text{ m}^3 \text{ and } \gamma = 3\alpha) \text{ or } V = 150 \times 10^{-6} \text{ m}^3$$

**Illustration 1.33** A hollow aluminium cylinder 20.0 cm deep has an internal capacity of 2.000 L at  $20.0^\circ\text{C}$ . It is completely filled with turpentine and then slowly warmed to  $80.0^\circ\text{C}$ . a. How much turpentine overflows? b. If the cylinder is then cooled back to  $20.0^\circ\text{C}$ , how far below the cylinder's rim does the turpentine's surface recede?

**Sol.** We guess that vertical cubic centimetres of turpentine will overflow, and that the liquid level will drop about a centimetre.

We will use the definition of the volume expansion coefficient. Both the liquid and the container expand. We will need to reason carefully about original, intermediate and final volumes of each.

When the temperature is increased from  $20.0^\circ\text{C}$  to  $80.0^\circ\text{C}$ , both the cylinder and the turpentine increase in volume by  $\Delta V = \gamma V \Delta T$ :

a. The overflow is  $V_{\text{over}} = \Delta V_{\text{turp}} - \Delta V_{\text{Al}}$

$$V_{\text{over}} = (\gamma V \Delta T)_{\text{turp}} - (\gamma V \Delta T)_{\text{Al}} = V_i \Delta T (\gamma_{\text{turp}} - 3\alpha_{\text{Al}})$$

$$V_{\text{over}} = (2.000 \text{ L}) (60.0^\circ\text{C}) (9.00 \times 10^{-4}/^\circ\text{C} - 0.720 \times 10^{-4}/^\circ\text{C})$$

$$= 0.0994 \text{ L}$$

b. After warming, the whole volume of the turpentine is

$$V' = 2000 \text{ cm}^3 + (9.00 \times 10^{-4}/^\circ\text{C})(2000 \text{ cm}^3)(60.0^\circ\text{C}) = 2108 \text{ cm}^3$$

$$\text{The fraction lost is } \frac{99.4 \text{ cm}^3}{2108 \text{ cm}^3} = 4.71 \times 10^{-2}$$

This also is the fraction of the cylinder that will be empty after cooling. Therefore, change in level

$$\Delta h = (4.71 \times 10^{-2})(20.0 \text{ cm}) = 0.943 \text{ cm}$$

The change in volume of the container is not negligible, but is 8% of the change in volume of the turpentine.

**Illustration 1.34** A glass flask whose volume is exactly  $1000 \text{ cm}^3$  at  $0^\circ\text{C}$  is filled level full of mercury at this temperature. When the flask and mercury are heated to  $100^\circ\text{C}$ ,  $15.2 \text{ cm}^3$  of mercury overflows. The coefficient of cubical expansion of Hg is  $1.82 \times 10^{-4}/^\circ\text{C}$ . Compute the coefficient of linear expansion of glass.

**Sol.** As  $15.2 \text{ cm}^3$  of Hg overflows at  $100^\circ\text{C}$ ,

$$(\text{final volume of Hg}) - (\text{final volume of glass flask}) = 15.2 \text{ cm}^3$$

$$\Rightarrow 1000(1 + \gamma_g \theta) - 1000(1 + \gamma_s \theta) = 15.2,$$

where  $\theta$  = rise in temperature =  $100 - 0 = 100^\circ\text{C}$

$$\Rightarrow \gamma_g = \gamma_s - \frac{15.2}{1000 \theta} = 0.000182 - 0.000152$$

$$\Rightarrow \gamma_g = 3 \times 10^{-5}/^\circ\text{C} \Rightarrow \alpha = \frac{\gamma_g}{3} = 1 \times 10^{-5}/^\circ\text{C}$$

**Illustration 1.35** A  $250 \text{ cm}^3$  glass bottle is completely filled with water at  $50^\circ\text{C}$ . The bottle and water are heated to  $60^\circ\text{C}$ . How much water runs over if:

- the expansion of the bottle is neglected;
- the expansion of the bottle is included? Given the coefficient of areal expansion of glass  $\beta = 1.2 \times 10^{-5}/\text{K}$  and  $\gamma_{\text{water}} = 60 \times 10^{-5}/^\circ\text{C}$ .

**Sol.** Water overflow = (final volume of water) - (final volume of bottle)

- a. If the expansion of bottle is neglected:

$$\begin{aligned}\text{Water overflow} &= 250(1 + \gamma_l \theta) - 250 \\ &= 250 \times 60 \times 10^{-5} \times 10\end{aligned}$$

$$\Rightarrow \text{water overflow} = 1.5 \text{ cm}^3$$

- b. If the bottle (glass) expands:

Water overflow

$$\begin{aligned}&= (\text{final volume of water}) - (\text{final volume of glass}) \\ &= 250(1 + \gamma_l \theta) - 250(1 + \gamma_g \theta) \\ &= 250(\gamma_l - \gamma_g)\theta, \quad \text{where } \gamma_g = 3/2\beta = 1.8 \times 10^{-5} / ^\circ\text{C} \\ &= 250(58.2 \times 10^{-5}) \times (60 - 50)\end{aligned}$$

$$\text{Water overflow} = 1.455 \text{ cm}^3$$

### Effect of Temperature on Upthrust

The thrust on volume  $V$  of a body in a liquid of density  $\sigma$  is given by  $Th = V\sigma g$

Now with rise in temperature by  $\Delta\theta$  °C, due to expansion, volume of the body will increase while density of liquid will decrease according to the relations  $V' = V(1 + \gamma_s \Delta\theta)$  and  $\sigma' = \sigma / (1 + \gamma_L \Delta\theta)$

So the thrust will become  $Th' = V'\sigma'g$

$$\therefore \frac{Th'}{Th} = \frac{V'\sigma'g}{V\sigma g} = \frac{(1 + \gamma_s \Delta\theta)}{(1 + \gamma_L \Delta\theta)}$$

and apparent weight of the body  $W_{\text{app}} = \text{actual weight} - \text{thrust}$

As  $\gamma_s < \gamma_L$ , therefore,  $Th' < Th$  with rise in temperature thrust also decreases and apparent weight of body increases.

**Illustration 1.36** A solid floats in a liquid at 20°C with 75% of it immersed. When the liquid is heated to 100°C, the same solid floats with 80% of it immersed in the liquid. Calculate the coefficient of expansion of the liquid. Assume the volume of the solid to be constant.

**Sol.** Let  $m$  be the mass of the solid and  $V$  its volume. By the law of flotation

Weight of floating object = Buoyant force

$$\text{In Case I: } mg = \left(\frac{3}{4}V\right)\rho_{20}g$$

where  $\rho_{20}$  = density of liquid at 20°C

$$\text{In Case II: } mg = \left(\frac{80}{100}V\right)\rho_{100}g,$$

where  $\rho_{100}$  = density of liquid at 100°C

Considering both the cases

$$\Rightarrow \frac{3}{4}\rho_{20} = \frac{4}{5}\rho_{100} \Rightarrow \frac{3}{4} \frac{\rho_0}{1 + \gamma \times 20} = \frac{4}{5} \frac{\rho_0}{1 + \gamma \times 100}$$

After solving we get

$$\gamma = \frac{1}{1180} = 8.47 \times 10^{-4} / ^\circ\text{C}$$

**Illustration 1.37** A sinker of weight  $W_0$  has an apparent weight  $W_1$  when placed in a liquid at a temperature  $T_1$  and  $W_2$  when weighed in the same liquid at a temperature  $T_2$ . The coefficient of cubical expansion of the material of the sinker is  $\gamma_s$ . What is the coefficient of volume expansion of the liquid?

**Sol.** Let  $\theta = T_2 - T_1$  and  $\gamma$  = coefficient of volume expansion of liquid.

Let density of liquid at temperatures  $T_1$  and  $T_2$  be  $\rho_1$  and  $\rho_2$ , respectively.

$$\Rightarrow \rho_1 = \rho_2(1 + \gamma\theta) \quad (i)$$

Let  $V_1$  and  $V_2$  be the volumes of the sinker at temperatures  $T_1$  and  $T_2$ , respectively.

$$\Rightarrow V_2 = V_1(1 + \gamma_s\theta) \quad (ii)$$

$$\text{The loss in weight at } T_1 = V_1\rho_1g \Rightarrow W_0 - W_1 = V_1\rho_1g \quad (iii)$$

$$\text{The loss in weight at } T_2 = V_2\rho_2g \Rightarrow W_0 - W_2 = V_2\rho_2g \quad (iv)$$

$$\text{Dividing Eq. (iii) by Eq. (iv), } \frac{W_0 - W_1}{W_0 - W_2} = \frac{V_1\rho_1}{V_2\rho_2}$$

$$\text{Using Eqs. (i) and (ii), } \frac{W_0 - W_1}{W_0 - W_2} = \frac{1 + \gamma\theta}{1 + \gamma_s\theta}$$

$$\Rightarrow 1 + \gamma\theta = \frac{W_0 - W_1}{W_0 - W_2} + \gamma_s \left( \frac{W_0 - W_1}{W_0 - W_2} \right) \theta$$

$$\Rightarrow \gamma = \left( \frac{W_2 - W_1}{W_0 - W_2} \right) \frac{1}{T_2 - T_1} + \left( \frac{W_0 - W_1}{W_0 - W_2} \right) \gamma_s$$

### Anomalous Expansion of Water

- Generally matter expands on heating and contracts on cooling. In case of water, it expands on heating if its temperature is greater than 4°C. In the range 0°C to 4°C, water contracts on heating and expands on cooling, i.e.,  $\gamma$  is negative. This behaviour of water in the range from 0°C to 4°C is called anomalous expansion.
- The anomalous behaviour of water arises due to the fact that water has three types of molecules, viz.,  $\text{H}_2\text{O}$ ,  $(\text{H}_2\text{O})_2$  and  $(\text{H}_2\text{O})_3$ , having different volume per unit mass values and at different temperatures their properties in water are different.
- At 4°C, density of water is maximum while its specific volume is minimum.

During winter when the water on the surface of a lake cools below 4°C by cold air, it expands and becomes lighter than water below. Therefore, the water cooled below 4°C stays on the surface and freezes when the temperature of surroundings falls below 0°C. Thus the lake freezes first on the surface and water in contact with ice has temperature 0°C while at the bottom of the lake 4°C (as density of water at 4°C is maximum) and fish and other aquatic animals remain alive in this water.



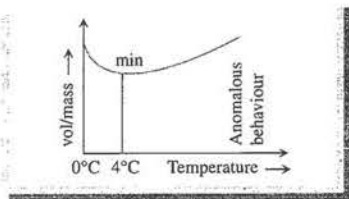


Fig. 1.12

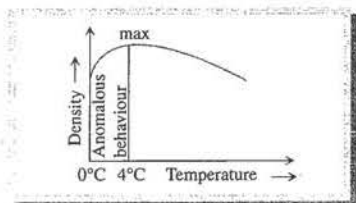


Fig. 1.13

## Expansion of Gases

Gases have no definite shape; therefore, gases have only volume expansion. Since the expansion of container is negligible in comparison to the gases, gases have only real expansion.

### Coefficient of Volume Expansion

At constant pressure, increase in volume per unit volume per unit degree rise of temperature is called coefficient of volume expansion.

$$\alpha = \frac{\Delta V}{V} \times \frac{1}{\Delta T} \quad \therefore \text{Final volume } V' = V(1 + \alpha \Delta T)$$

### Coefficient of Pressure Expansion

$$\beta = \frac{\Delta P}{P} \times \frac{1}{\Delta T}$$

$$\therefore \text{Final pressure } P' = P(1 + \beta \Delta T)$$

For an ideal gas, coefficient of volume expansion is equal to the coefficient of pressure expansion.

$$\text{i.e.} \quad \alpha = \beta = \frac{1}{273} / ^\circ\text{C}$$

## Application of Thermal Expansion

- Bi-metallic strip:** Two strips of equal lengths but of different materials (different coefficient of linear expansion) when join together, is called 'bi-metallic strip' and can be used in thermostat to break or make electrical contact. This strip has the characteristic property of bending on heating due to unequal linear expansion of the two metals. The strip will bend with metal of greater  $\alpha$  on outer side, i.e., convex side.

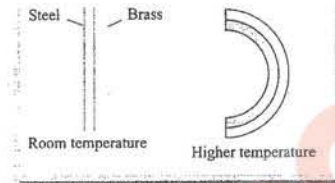


Fig. 1.14

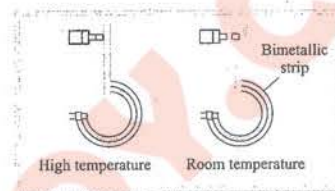


Fig. 1.15

**Illustration 1.38** A copper and a tungsten plate having a thickness  $\delta = 2$  mm each are riveted together so that at  $0^\circ\text{C}$  they form a flat bimetallic plate. Find the average radius of curvature of this plate at  $t = 200^\circ\text{C}$ . The coefficients of linear expansion for copper and tungsten are  $\alpha_{cu} = 1.7 \times 10^{-5}/\text{K}$  and  $\alpha_w = 0.4 \times 10^{-5}/\text{K}$ , respectively.

**Sol.** The average length of copper plate at a temperature  $T = 200^\circ\text{C}$  is  $l_c = l_0(1 + \alpha_c T)$ , where  $l_0$  is the length of copper plate at  $0^\circ\text{C}$ . The length of the tungsten plate is  $l_t = l_0(1 + \alpha_t T)$

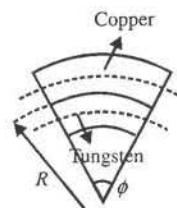


Fig. 1.16

We shall assume that the edges of plates are not displaced during deformation and that an increase in the plate thickness due to heating can be neglected.

From Fig. 1.16 we have

$$l_c = \phi(R + \delta/2) \Rightarrow l_t = \phi(R - \delta/2)$$

Consequently,

$$\phi(R + \delta/2) = l_0(1 + \alpha_c T) \quad (i)$$

$$\phi(R - \delta/2) = l_0(1 + \alpha_t T) \quad (ii)$$

To eliminate the unknown quantities,  $\phi$  and  $l_0$ , we divide Eq. (i) by Eq. (ii) term-wise:

$$\Rightarrow \frac{(R + \delta/2)}{(R - \delta/2)} = \frac{(1 + \alpha_c T)}{(1 + \alpha_t T)} \Rightarrow R = \delta \frac{[2 + (\alpha_c + \alpha_t)T]}{[2(\alpha_c - \alpha_t)T]}$$

$$\Rightarrow R = \frac{\delta}{(\alpha_c - \alpha_t)T}$$

neglecting  $(\alpha_c + \alpha_r)$  in numerator. Substituting the values in above relation, we get:  $R = 0.769$  m.

- 2. Effect of temperature on the time period of a simple pendulum:** A pendulum clock keeps proper time at temperature  $\theta$ . If temperature is increased to  $\theta'$  ( $> \theta$ ) then due to linear expansion, length of pendulum and hence its time period will increase.

If  $l_0$  be the length of the pendulum, at  $\theta^\circ\text{C}$ , then its time period

$$T_0 = 2\pi \sqrt{\frac{l_0}{g}} \quad (i)$$

At any temperature increment  $\Delta\theta$ , the time period of the pendulum is given by

$$T = 2\pi \sqrt{\frac{l}{g}}$$

Here,  $l = l_0(1 + \alpha\Delta\theta)$

$$\therefore T = 2\pi \sqrt{\frac{l_0(1 + \alpha\Delta\theta)}{g}} = 2\pi \sqrt{\frac{l_0}{g}} (1 + \alpha\Delta\theta)^{1/2}$$

$$= T_0 (1 + \alpha\Delta\theta)^{1/2} \quad (ii)$$

$$= T_0 \left( 1 + \frac{\alpha\Delta\theta}{2} \right) \quad \text{or} \quad \frac{T}{T_0} - 1 = \frac{\alpha\Delta\theta}{2}$$

$$\frac{T - T_0}{T_0} = \frac{\alpha\Delta\theta}{2} \quad \text{or} \quad \frac{\Delta T}{T_0} = \frac{\alpha\Delta\theta}{2}$$

$$\therefore \Delta T = \left( \frac{\alpha\Delta\theta}{2} \right) T_0$$

**Note:**

- Due to increment in its time period, a pendulum clock becomes slow in summer and will lose time.

- Loss of time in a time period  $\Delta T = \frac{1}{2} \alpha \Delta\pi T_0$

- Loss of time in any given time interval  $t$  can be given by

$$\Delta t = \frac{1}{2} \alpha \Delta\pi t$$

- The clock will lose time, i.e., it will become slow if  $\theta' > \theta$  (in summer).

- It will gain time, i.e., it will become fast if  $\theta' < \theta$  (in winter).

- The gain or loss in time is independent of time period  $T$  and depends on the time interval  $t$ .

Time lost by the clock in a day ( $t = 86400$  s)

$$\Delta t = \frac{1}{2} \alpha \Delta\pi t = \frac{1}{2} \alpha \Delta\pi (86400) = 43200 \alpha \Delta\pi \text{ s}$$

- Since coefficient of linear expansion ( $\alpha$ ) is very small for invar, pendulums are made of invar to show the correct time in all seasons.

**Illustration 1.9** A clock with a brass pendulum shaft keeps correct time at a certain temperature.

- How closely must the temperature be controlled if the clock is not to gain or lose more than 1 s a day? Does the answer depend on the period of the pendulum?
- Will an increase of temperature cause the clock to gain or lose? ( $\alpha_{\text{brass}} = 2 \times 10^{-5} / ^\circ\text{C}$ )

**Sol.**

- Number of seconds lost or gained per day =  $\frac{1}{2} \alpha \theta \times 86400$ ,

where  $\theta$  = rise or drop in temperature;  $\alpha$  = coeff. of linear expansion of shaft.

$$\text{We want that } \left| \frac{1}{2} \alpha \theta \times 86400 \right| < 1$$

$$\Rightarrow |\theta| < \frac{2}{2 \times 10^{-5} \times 86400} \Rightarrow |\theta| < 1.1574^\circ\text{C}$$

Hence temperature should not increase or decrease by more than  $1.1574^\circ\text{C}$ . It does not depend upon time period.

- An increase in temperature makes the pendulum slow and hence clock loses time.

**Illustration 1.10** A pendulum clock loses 12 s a day if the temperature is  $40^\circ\text{C}$  and goes fast by 4 s a day if the temperature is  $20^\circ\text{C}$ . Find the temperature at which the clock will show correct time and the coefficient of linear expansion of the metal of the pendulum shaft.

**Sol.** Let  $T$  be the temperature at which the clock is correct.

Time lost per day =  $1/2 \alpha$  (rise in temperature)  $\times 86400$

$$\Rightarrow 12 = 1/2 \alpha (40 - T) \times 86400 \quad (i)$$

Time gained per day =  $1/2 \alpha$  (drop in temperature)  $\times 86400$

$$\Rightarrow 4 = 1/2 \alpha (T - 20) \times 86400 \quad (ii)$$

Adding Eqs. (i) and (ii), we get

$$32 = 86400 \alpha (40 - 20) \Rightarrow \alpha = 1.85 \times 10^{-5} / ^\circ\text{C}$$

Dividing Eq. (i) by Eq. (ii), we get

$$12(T - 20) = 4(40 - T) \Rightarrow T = 25^\circ\text{C}$$

$\Rightarrow$  Clock shows correct time at  $25^\circ\text{C}$ .

- Thermal stress in a rigidly fixed rod:** When a rod whose ends are rigidly fixed such as to prevent expansion or contraction, undergoes a change in temperature, a compressive or tensile stress is developed in it. Due to this thermal stress the rod will exert a large force on the supports.

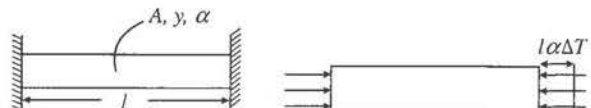


Fig. 1.17



## 1.20 Waves & Thermodynamics

If temperature of rod is increased by  $\Delta T$ , then change in length

$$\Delta l = l \alpha \Delta T$$

$$\text{strain} = \frac{\Delta l}{l} = \alpha \Delta T$$

But due to rigid support, there is no strain. Supports provide force or stresses to keep the length of rod same

$$Y = \frac{\text{stress}}{\text{strain}}$$

thermal stress =  $Y \text{ strain} = Y \alpha \Delta T$

$$\frac{F}{A} = Y \alpha \Delta T \quad F = A Y \alpha \Delta T$$

**Illustration 1.41** A rod of length 2 m is at a temperature of  $20^\circ\text{C}$ . Find the free expansion of the rod, if the temperature is increased to  $50^\circ\text{C}$ , then find stress produced when the rod is (i) fully prevented to expand, (ii) permitted to expand by 0.4 mm.  $Y = 2 \times 10^{11} \text{ N/m}^2$ ;  $\alpha = 15 \times 10^{-6}/^\circ\text{C}$ .

**Sol.** Free expansion of the rod =  $\alpha L \Delta \theta$

$$= 15 \times 10^{-6}/^\circ\text{C} \times 2 \text{ m} \times (50 - 20)^\circ\text{C}$$

$$= 9 \times 10^{-4} \text{ m} = 0.9 \text{ mm}$$

i. If the expansion is fully prevented,

$$\text{then strain} = \frac{9 \times 10^{-4}}{2} \Rightarrow 4.5 \times 10^{-4}$$

$\therefore$  Temperature stress = strain  $\times Y$

$$= 4.5 \times 10^{-4} \times 2 \times 10^{11} = 9 \times 10^7 \text{ N/m}^2$$

ii. If 0.4 mm expansion is allowed, then length restricted to expand =  $0.9 - 0.4 = 0.5 \text{ mm}$

$$\therefore \text{Strain} = \frac{5 \times 10^{-4}}{2} = 2.5 \times 10^{-4}$$

$$\therefore \text{Temperature stress} = \text{strain} \times Y = 2.5 \times 10^{-4} \times 2 \times 10^{11}$$

$$= 5 \times 10^7 \text{ N/m}^2$$

**Illustration 1.42** Two rods of different metals having the same area of cross section  $A$  are placed between the two massive walls as shown in Fig. 1.18. The first rod has a length  $l_1$ , coefficient of linear expansion  $\alpha_1$  and Young's modulus  $Y_1$ . The corresponding quantities for second rod are  $l_2$ ,  $\alpha_2$  and  $Y_2$ . The temperature of both the rods is now raised by  $t^\circ\text{C}$ .

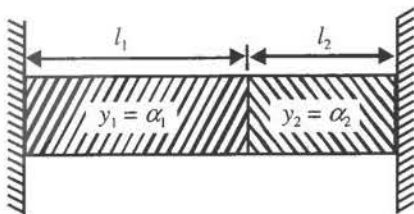


Fig. 1.18

- Find the force with which the rods act on each other (at higher temperature) in terms of given quantities.
- Also find the length of the rods at higher temperature.

**Sol.**

i. Let  $t^\circ\text{C}$  = increase in the temperature.

Increase in length of first rod =  $l_1 \alpha_1 t$

Increase in length of second rod =  $l_2 \alpha_2 t$

$\therefore$  Total extension in length due to rise in temperature

$$= l_1 \alpha_1 t + l_2 \alpha_2 t = (l_1 \alpha_1 + l_2 \alpha_2) t \quad (i)$$

Since the walls are rigid, this increase in length will not happen. This will be compensated by equal and opposite forces  $F$ ,  $F$  producing decrease in the lengths of the rods due to elasticity.

$$\therefore \text{Decrease in length of first rod} = \frac{F \times l_1}{Y_1 \times A}$$

$$\text{And decrease in length of second rod} = \frac{F \times l_2}{Y_2 \times A}$$

$\therefore$  Total decrease in length due to elastic force

$$= \frac{F}{A} \left( \frac{l_1}{Y_1} + \frac{l_2}{Y_2} \right) \quad (ii)$$

From Eqs. (i) and (ii), we have

$$\frac{F}{A} \left( \frac{l_1}{Y_1} + \frac{l_2}{Y_2} \right) = (l_1 \alpha_1 + l_2 \alpha_2) t$$

or

$$F = \frac{A(l_1 \alpha_1 + l_2 \alpha_2) t}{\left( \frac{l_1}{Y_1} + \frac{l_2}{Y_2} \right)} \quad (iii)$$

ii. Length of the first rod = original length + increase in length due to temperature – decrease in length due to force

$$= \left( l_1 + l_1 \alpha_1 t - \frac{F l_1}{A Y_1} \right)$$

$$\text{and length of second rod} = l_2 + l_2 \alpha_2 t - \frac{F l_2}{A Y_2}$$

$\therefore$  The total length is same =  $l_1 + l_2$  at all temperatures.

**Illustration 1.43** Two rods of equal cross sections, one of copper and the other of steel, are joined to form a composite rod of length 2.0 m at  $20^\circ\text{C}$ ; the length of the copper rod is 0.5 m. When the temperature is raised to  $120^\circ\text{C}$ , the length of composite rod increases to 2.002 m. If the composite rod is fixed between two rigid walls and thus not allowed to expand, it is found that the lengths of the component rods also do not change with increase in temperature. Calculate Young's modulus of steel. (The coefficient of linear expansion of copper,  $\alpha_c = 1.6 \times 10^{-5}/^\circ\text{C}$  and Young's modulus of copper is  $1.3 \times 10^{13} \text{ N/m}^2$ .)



**Sol.** Change in length:

For Cu rod

$$l_c \alpha_c [t_2 - t_1] = 0.5 \times \alpha_c \times (120 - 20) = 50 \alpha_c$$

For steel rod

$$l_s \alpha_s (t_2 - t_1) = 1.5 \alpha_s \times 100 = 150 \alpha_s$$

$\therefore$  Total change in length =  $50 \alpha_c + 150 \alpha_s = 0.002 \text{ m}$

$$\Rightarrow \alpha_s = \frac{4 \times 10^{-5} - \alpha_c}{3} = \frac{4 \times 10^{-5} - 1.6 \times 10^{-5}}{3} = 0.8 \times 10^{-5} / ^\circ\text{C}$$

Stress in steel rod,  $f_s = Y_s \times \text{strain} = Y_s \times \Delta l / l$

$$= Y_s \alpha_s (t_2 - t_1) = Y_s \times \alpha_s \times 100 = 100 Y_s \alpha_s$$

There is no change in the length of individual rod, because the length change due to stress is balanced by length change due to thermal expansion.

Similarly, stress in copper rod,  $f_c = Y_c \alpha_c \times 100 = 100 Y_c \alpha_c$

Now stress is same in both:

$$Y_s = \frac{Y_c \alpha_c}{\alpha_s} = \frac{1.3 \times 10^{13} \times 1.6 \times 10^{-5}}{0.8 \times 10^{-5}} = 2.6 \times 10^{13} \text{ N/m}^2$$

4. **Error in scale reading due to expansion or contraction:** If a scale gives correct reading at temperature  $\theta$ , at temperature  $\theta' (> \theta)$  due to linear expansion of scale, the scale will expand and scale reading will be lesser than true value so that

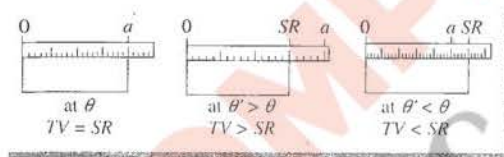


Fig. 1.19

True value = scale reading  $[1 + \alpha(\theta' - \theta)]$

i.e.,  $TV = SR[1 + \alpha\Delta\theta]$  with  $\Delta\theta = (\theta' - \theta)$

However, if  $\theta' < \theta$ , due to contractions of scale, scale reading will be more than true value, so true value will be lesser than scale reading and will still be given by same equation with  $\Delta\theta = (\theta' - \theta)$  being negative.

**Illustration 1.25** A surveyor's 30 m steel tape is correct at a temperature of  $20^\circ\text{C}$ . The distance between two points, as measured by this tape on a day when the temperature is  $35^\circ\text{C}$ , is 26 m. What is the true distance between the point? ( $\alpha_{\text{steel}} = 1.2 \times 10^{-5} / ^\circ\text{C}$ )

**Sol.** Let temperature rise above the correct temperature by  $\theta$ .

$$\Rightarrow \theta = 35 - 20 = 15^\circ\text{C}$$

Using the relation:

Correct length = measured length  $(1 + \alpha\theta) \Rightarrow$  true distance between the points

$$= 26(1 + 1.2 \times 10^{-5} \times 15) \Rightarrow \text{true distance} = 26.00468 \text{ m}$$

**Illustration 1.26**

A barometer with a brass scale reads 755 mm on a day when the temperature is  $25^\circ\text{C}$ . If the scale is correctly graduated at  $0^\circ\text{C}$ , find the true pressure at  $0^\circ\text{C}$  (in terms of height of Hg) given that the coefficient of linear expansion of brass is  $18 \times 10^{-6} / \text{K}$ . Coefficient of cubical expansion of mercury =  $182 \times 10^{-6} / \text{K}$ .

**Sol.** Given that 1 mm at  $0^\circ\text{C} = 1 \text{ mm}$

$$\therefore 755 \text{ mm at } 25^\circ\text{C} = 755(1 + 18 \times 10^{-6} \times 25) \text{ mm} = 755.34 \text{ mm}$$

Let  $P$  be the value of the atmospheric pressure.

Then  $P = 755.34 \rho_{25} g = h \rho_0 g$ , where  $\rho_0, \rho_{25}$  are densities of mercury at  $0^\circ\text{C}$  and  $25^\circ\text{C}$ , respectively.

$$\text{or } h = 755.34 \times \frac{\rho_{25}}{\rho_0} = 755.34 \times \frac{\rho_0}{\rho_0(1 + 182 \times 10^{-6} \times 25)}$$

$$\text{or } h = 751.19 \text{ mm}$$

**Illustration 1.30**

At room temperature ( $25^\circ\text{C}$ ) the length of a steel rod is measured using a brass centimetre scale. The measured length is 20 cm. If the scale is calibrated to read accurately at temperature  $0^\circ\text{C}$ , find the actual length of steel rod at room temperature

**Sol.** The brass scale is calibrated to read accurately at  $0^\circ\text{C}$ . This means at  $0^\circ\text{C}$ , each division of scale has exact 1 cm length. Thus at higher temperature the division length of scale will be more than 1 cm due to thermal expansion. Thus at higher temperature the scale reading for length measurement is not appropriate and as at higher temperature the division length is more, the length this scale reads will be lesser than the actual length to be measured. For illustration in this case the length of each division on brass scale at  $25^\circ\text{C}$  is

$$l_{\text{div}} = (1 \text{ cm})[1 + \alpha_{\text{br}}(25 - 0)] \\ = 1 + \alpha_{\text{br}}(25)$$

It is given that at  $25^\circ\text{C}$  the length of steel rod measured is 20 cm. Actually it is not 20 cm, it is 20 divisions on the brass scale. Now we can find the actual length of the steel rod at  $25^\circ\text{C}$  as

$$l_{25^\circ\text{C}} = (20 \text{ cm}) \times l_{\text{div}}$$

$$\text{or } l_{\text{act}} = 20[1 + \alpha_{\text{br}}(25)] \quad (\text{i})$$

The above expression is a general relation using which you can find the actual lengths of the objects of which lengths are measured by a metallic scale at some temperature other than the graduation temperature of the scale.

5. **Expansion of cavity:** Thermal expansion of an isotropic object may be imagined as a photographic enlargement. So if there is a hole  $A$  in a plate  $C$  (or cavity  $A$  inside a body



$C$ ), the area of hole (or volume of cavity) will increase when body expands on heating, just as if the hole (or cavity) were solid  $B$  of the same material. Also the expansion of area (or volume) of the body  $C$  will be independent of shape and size of hole (or cavity), i.e., will be equal to that of  $D$ .

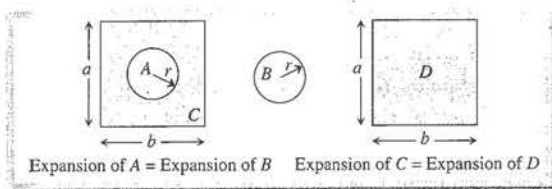


Fig. 1.20

**Note:** For a solid and hollow sphere of same radius and material, heated to the same temperature, expansion of both will be equal because thermal expansion of isotropic solids is similar to true photographic enlargement. It means the expansion of cavity is same as if it has been a solid body of the same material. But if same heat is given to the two spheres, due to lesser mass, rise in temperature of hollow sphere will be more  $\left\{ \text{As } \left( \Delta\theta = \frac{a}{mc} \right) \right\}$ . Hence its expansion will be more.

#### 6. Practical application:

- When rails are laid down on the ground, space is left between the ends of two rails to allow for expansion.
- The transmission cables are not tightly fixed to the poles.
- Pendulum of wall clock and balance wheel of wrist watch are made of invar (an alloy which has very low value of coefficient of expansion).
- Test tubes, beakers and crucibles are made of pyrex-glass or silica because they have very low value of coefficient of linear expansion.
- The iron rim to be put on a cart wheel is always of slightly smaller diameter than that of wheel to ensure tight fit.
- A glass stopper jammed in the neck of a glass bottle can be taken out by warming the neck of the bottle.

#### Concept Application Exercise 1.2

- Does the change in volume of a body when its temperature is raised depend on whether the body has cavities inside, other things being equal?
- Explain why some rubber-like substances contract with rising temperature.
- Two large holes are cut in a metal sheet. If this is heated, will their diameters increase or decrease?
- In the above question, will the distance between the holes increase or decrease on heating?

- A long metal rod is bent to form a ring with a small gap. If this is heated, will this gap increase or decrease?
- Two iron spheres of the same diameter are heated to the same temperature. One is solid, and the other is hollow. Which will expand more?
- A steel rod is 3.000 cm at 25°C. A brass ring has an interior diameter of 2.992 cm at 25°C. At what common temperature will the ring just slide on to the rod?
- A clock with a metallic pendulum gains 5 s each day at a temperature of 15°C and loses 10 s each day at a temperature of 30°C. Find the coefficient of thermal expansion of the pendulum metal.
- The design of some physical instrument requires that there be a constant difference in length of 10 cm between an iron rod and a copper cylinder laid side by side at all temperatures. Find their lengths.

$$\begin{aligned} \alpha_{\text{Fe}} &= 11 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}, \\ \alpha_{\text{Cu}} &= 17 \times 10^{-6} \text{ } ^\circ\text{C}^{-1} \end{aligned}$$

- A metal rod of 30 cm length expands by 0.075 cm when its temperature is raised from 0°C to 100°C. Another rod of a different metal of length 45 cm expands by 0.045 cm for the same rise in temperature. A composite rod C made by joining A and B end to end expands by 0.040 cm when its length is 45 cm and it is heated from 0°C to 50°C. Find the length of each portion of the composite rod.
- A brass scale is graduated at 10°C. What is the true length of a zinc rod which measures 60.00 cm on this scale at 30°C?  
Coefficient of linear expansion of brass =  $18 \times 10^{-6} \text{ K}^{-1}$ .
- A long horizontal glass capillary tube open at both ends contains a mercury thread 1 m long at 0°C. Find the length of the mercury thread, as read on this scale, at 100°C.
- A mercury-in-glass thermometer has a stem of internal diameter 0.06 cm and contains 43 g of mercury. The mercury thread expands by 10 cm when the temperature changes from 0°C to 50°C. Find the coefficient of cubical expansion of mercury. Relative density of mercury = 13.6 and  $\alpha_{\text{glass}} = 9 \times 10^{-6} \text{ K}^{-1}$ .
- A sphere of diameter 7 cm and mass 266.5 g floats in a bath of liquid. The temperature is raised, and the sphere begins to sink at 35°C. If the density of the liquid is 1.527 at 0°C, find the coefficient of cubical expansion of the liquid. Neglect the expansion of the sphere.
- A mercury thermometer is to be made with glass tubing of internal bore 0.5 mm diameter and the distance between the fixed point is to be 20 cm. Estimate the volume of the bulb below the lower fixed point, given that the coefficient of cubical expansion of mercury is 0.00018/K and the coefficient of linear expansion of glass is 0.000009/K.



16. On a Celsius thermometer the distance between the readings  $0^{\circ}\text{C}$  and  $100^{\circ}\text{C}$  is 30 cm and the area of cross section of the narrow tube containing mercury is  $15 \times 10^{-4} \text{ cm}^2$ . Find the total volume of mercury in the thermometer at  $0^{\circ}\text{C}$ .  $\alpha$  of glass =  $9 \times 10^{-6}/\text{K}$  and the coefficient of real expansion of mercury =  $18 \times 10^{-5}/\text{K}$ .
17. The height of a mercury column measured with a brass scale, which is correct and equal to  $H_0$  at  $0^{\circ}\text{C}$ , is  $H_1$  at  $t^{\circ}\text{C}$ ? The coefficient of linear expansion of brass is  $\alpha$  and the coefficient of volume expansion of mercury is  $\gamma$ . Relate  $H_0$  and  $H_1$ .
18. A glass bulb contains air and mercury. What fraction of the bulb must be occupied by mercury if the volume of air in the bulb is to remain constant at all temperatures? The coefficient of linear expansion of glass is  $9 \times 10^{-6}/\text{K}$  and the coefficient of expansion of mercury is  $1.8 \times 10^{-4}/\text{K}$ .
19. When composite rod is free, composite length increases to 2.002 m when temperature increases from  $20^{\circ}\text{C}$  to  $120^{\circ}\text{C}$ . When composite rod is fixed between the support, there is no change in component length. Find  $\gamma$  and  $\alpha$  of steel if  $\gamma_{\text{cu}} = 1.5 \times 10^{-5} \text{ N/m}^2$   $\alpha_{\text{cu}} = 1.6 \times 10^{-5}/^{\circ}\text{C}$ .

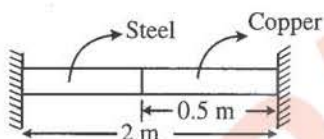


Fig. 1.21

## TRANSMISSION OF HEAT

Heat energy transfers from a body at higher temperature to a body at lower temperature. The transfer of heat from one body to another may take place by any of the following modes.

Conduction	Convection	Radiation
Heat flows from hot end to cold end. Particles of the medium simply oscillate but do not leave their place	Each particle absorbing heat is mobile	Heat flows without any intervening medium in the form of electromagnetic waves
Medium is necessary for conduction	Medium is necessary for convection	Medium is not necessary for radiation
It is a slow process	It is also a slow process	It is a very fast process
Path of heat flow may be zig-zag	Path may be zig-zag or curved	Path is a straight line

Conduction takes place in solids	Convection takes place in fluids	Radiation takes place in gaseous and transparent media
The temperature of the medium increases through which heat flows	In this process also the temperature of medium increases	There is no change in the temperature of the medium

## Conduction

The process of transmission of heat energy in which the heat is transferred from one particle to other without dislocation of the particles from their equilibrium position is called conduction.

- Conduction is a process which is possible in all states of matter.
- In solids only conduction takes place.
- In non-metallic solids and fluids the conduction takes place only due to vibrations of molecules; therefore, they are poor conductors.
- In metallic solids free electrons carry the heat energy; therefore, they are good conductors of heat.

**1. Variable and steady state:** When one end of a metallic rod is heated, heat flows by conduction from the hot end to the cold end. In the process of conduction each cross section of the rod receives heat from the adjacent cross section towards the hot end. A part of this heat is absorbed by the cross section itself whose temperature increases, another part is lost into atmosphere by convection and radiation and the rest is conducted away to the next cross section.

Because in this state temperature of every cross section of the rod goes on increasing; hence, rod is said to exist in variable state.

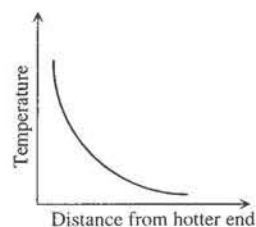


Fig. 1.22

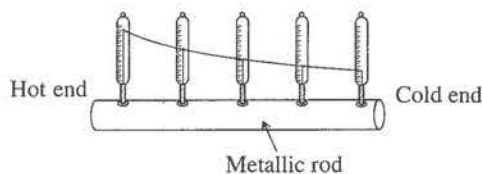


Fig. 1.23

After some time, a state is reached when the temperature of every cross section of the rod becomes constant. In this state, no heat

is absorbed by the rod. The heat that reaches any cross section is transmitted to the next except that a small part of heat is lost to surrounding from the sides by convection and radiation. This state of the rod in which no part of rod absorbs heat is called steady state.

**2. Isothermal surface:** Any surface (within a conductor) having its all points at the same temperature is called isothermal surface. The direction of flow of heat through a conductor at any point is perpendicular to the isothermal surface passing through that point.

- If the material is rectangular or cylindrical rod, the isothermal surface is a plane surface.
- If a point source of heat is situated at the centre of a sphere the isothermal surface will be spherical.
- If steam passes along the axis of the hollow cylinder, heat will flow through the walls of the cylinder so that in this condition the isothermal surface will be cylindrical.

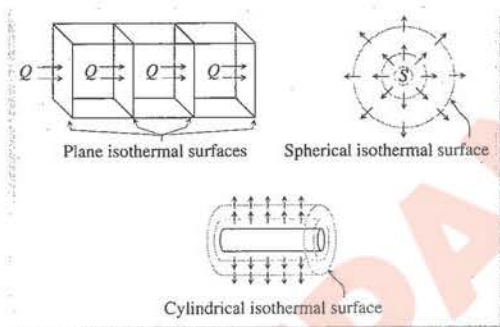


Fig. 1.24

**3. Temperature gradient:** The rate of change of temperature with distance between two isothermal surfaces is called temperature gradient.

If the temperature of two isothermal surfaces be  $\theta$  and  $(\theta - \Delta\theta)$  and the perpendicular distance between them be  $\Delta x$ , then temperature gradient =  $\frac{(\theta - \Delta\theta) - \theta}{\Delta x} = \frac{-\Delta\theta}{\Delta x}$

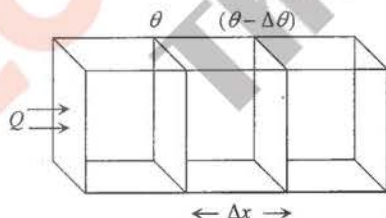


Fig. 1.25

The negative sign shows that temperature  $\theta$  decreases as the distance  $x$  increases in the direction of heat flow.

Unit: K/m (SI); dimension:  $[L^{-1} \theta]$

**4. Coefficient of thermal conductivity:** If  $L$  be the length of the rod,  $A$  the area of cross section and  $\theta_1$  and  $\theta_2$  be the temperatures of its two faces, then the amount of heat flowing from one face to the other face in time  $t$  is

$$\begin{aligned} Q &\propto A \\ &\propto (\theta_1 - \theta_2) \\ &\propto t \\ &\propto \frac{1}{l} \end{aligned}$$

In combined form

$$Q \propto \frac{A(\theta_1 - \theta_2)t}{l}$$

or

$$Q = \frac{KA(\theta_1 - \theta_2)t}{l}$$

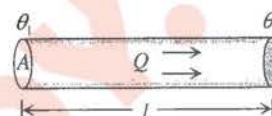


Fig. 1.26

where  $K$  is coefficient of thermal conductivity of material of rod. It is the measure of the ability of a substance to conduct heat through it.

This relation can also be expressed as

$$\frac{\Delta Q}{\Delta t} = \frac{KA(\theta_1 - \theta_2)}{l}$$

If  $A = 1 \text{ m}^2$ ,  $(\theta_1 - \theta_2) = 1^\circ\text{C}$ ,  $t = 1 \text{ s}$  and  $l = 1 \text{ m}$ , then

$$Q = K.$$

Thus, thermal conductivity of a material is the amount of heat flowing per second during steady state through its rod of length 1 m and cross section  $1 \text{ m}^2$  with a unit temperature difference between the opposite faces.

- Units: cal/cm-s°C (in CGS), kcal/m-s-K (in MKS) and W/m-K (in SI)
- Dimension:  $[MLT^{-3}\theta^{-1}]$
- The magnitude of  $K$  depends only on nature of the material.
- For perfect conductors,  $K = \infty$  and for perfect insulators,

$$K = 0$$

- Substances in which heat flows quickly and easily are known as good conductors of heat. They possess large thermal conductivity due to large number of free electrons. Example: silver, brass, etc.
- Substances which do not permit easy flow of heat are called bad conductors. They possess low thermal conductivity due to very few free electrons. Example: glass, wood, etc.
- The thermal conductivity of pure metals decreases with rise in temperature but for alloys thermal conductivity increases with increase of temperature.
- Human body is a bad conductor of heat (but it is a good conductor of electricity).

**Illustration 1.47:** A refrigerator door is 150 cm high, 80 cm wide, and 6 cm thick. If the coefficient of conductivity is  $0.0005 \text{ cal/cm s}^\circ\text{C}$ , and the inner and outer surfaces are at  $0^\circ\text{C}$  and  $30^\circ\text{C}$ , respectively, what is the heat loss per minute through the door, in calories?



**Sol.** Apply the equation of thermal conductivity

$$Q = \frac{kA(t_h - t_c)(\text{time})}{d} = \frac{0.005(150 \times 80)(30^\circ - 0^\circ)(60)}{4} = 1800 \text{ cal}$$

**Illustration 1.48** An ordinary refrigerator is thermally equivalent to a box of corkboard 90 mm thick and 5.6 m<sup>2</sup> in inner surface area. When the door is closed, the inside wall is kept, on the average, 22.2°C below the temperature of the outside wall. If the motor of the refrigerator runs 15% of the time while the door is closed, at what rate must heat be taken from the interior while the motor is running? The thermal conductivity of corkboard is  $k = 0.05 \text{ W/mK}$ .

**Sol.** Consider a time interval  $\Delta t$  during which the door is closed. As approximation, take the heat conduction to be steady over  $\Delta t$ .

Then the rate of heat into the box is

$$\frac{\Delta Q}{\Delta t} = kA \left( \frac{\Delta T}{\Delta x} \right) = (0.05)(5.6) \left( \frac{22.2}{0.090} \right) = 69.1 \text{ W}$$

To remove this heat, the motor must, since it runs only for a time (0.15)  $\Delta t$ , cause heat to leave at the rate  $69.1 / 0.15 = 460 \text{ W}$ .

**Illustration 1.49** Water is being boiled in flat bottom kettle placed on a stove. The area of the bottom is 3000 cm<sup>2</sup> and the thickness is 2 mm. If the amount of steam produced is 1 g/min, calculate the difference of temperature between the inner and outer surface of the bottom.  $K$  for the material of kettle is 0.5 cal/°C/s/cm, and the latent heat of steam is 540 cal/g.

**Sol.** Mass of steam produced  $= \frac{dm}{dt} = \frac{1}{60} \text{ g/s}$

Heat transferred per second

$$= \frac{dH}{dt} = L \frac{dm}{dt} \Rightarrow \frac{dH}{dt} = 540 \times \frac{1}{60} \text{ cal/s} = 9 \text{ cal/s}$$

Area = 3000 cm<sup>2</sup>;  $K = 0.5 \text{ cal/°C/s/cm}$

$\theta$  = temperature difference

$d$  = thickness = 2 mm = 0.2 cm

$$\begin{aligned} \frac{dH}{dt} &= \frac{KA\theta}{d} \Rightarrow L \frac{dm}{dt} = \frac{KA\theta}{d} \\ \Rightarrow 9 &= \frac{0.5 \times 3000 \times \theta}{0.2} \Rightarrow \theta = 1.2 \times 10^{-3} \text{ °C} \end{aligned}$$

**Illustration 1.50** A closed cubical box made of perfectly insulating material has walls of thickness 8 cm and the only way for the heat to enter or leave the box is through the solid, cylindrical, metallic plugs each of cross-sectional area 12 cm<sup>2</sup> and length 8 cm fixed in the opposite walls of the box as shown in Fig. 1.27. The outer surface  $A$  is kept at 100°C while the outer surface  $B$  of other plug is kept at 4°C.

$K$  of the material of the plugs is 0.5 cal/s/°C/cm. A source of energy generating 36 cal/s is enclosed inside the box. Find the equilibrium temperature of the inner surface of the box assuming that it is same at all points on the inner surface.

**Sol.** Let  $\theta$  be the temperature of inner surface of box.

Heat transfer per second through  $A$  + heat produced by source per second = Heat transfer per second through  $B$

$$\Rightarrow \left( \frac{dH}{dt} \right)_A + 36 \text{ cal/s} = \left( \frac{dH}{dt} \right)_B$$

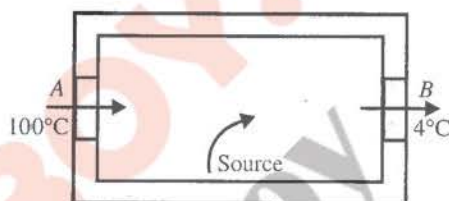


Fig. 1.27

$$\Rightarrow \frac{KA(100 - \theta)}{d} + 36 = \frac{KA(\theta - 4)}{d}$$

$$\Rightarrow KA(\theta - 4 - 100 + \theta) = 36 \times d$$

Now,  $d = 8 \text{ cm}$ ,  $A = 12 \text{ cm}^2$ ,  $K = 0.5 \text{ cal/s/°C/cm}$ .

$$\Rightarrow 2\theta - 104 = \frac{36 \times 8}{12 \times 0.5} \Rightarrow \theta = 76^\circ \text{C}$$

**Illustration 1.51** Two metal cubes  $A$  and  $B$  of same size are arranged as shown in Fig. 1.28. The extreme ends of the combination are maintained at the identical temperatures. The arrangement is thermally insulated. The coefficients of thermal conductivity of  $A$  and  $B$  are 300 W/m°C and 200 W/m°C, respectively. After steady state is reached, what will be the temperature  $T$  of the interface? (IIT-JEE-1996)

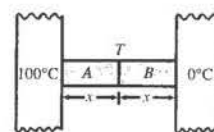


Fig. 1.28

**Sol.** In steady state, rate of flow of heat through  $A$  = Rate of flow of heat through  $B$ .

$$\text{or } K_1 A \left( \frac{100 - T}{x} \right) = K_2 A \left( \frac{T - 0}{x} \right) \quad \text{or } 300 - 3T = 2T$$

$$\therefore T = 60^\circ \text{C}$$

**5. Relation between temperature gradient and thermal conductivity:** In steady state, rate of flow of heat

$$\frac{dQ}{dt} = -KA \frac{d\theta}{dx} = -KA (\text{temperature gradient})$$

If  $\frac{dQ}{dt}$  is constant then temperature gradient  $\propto \frac{1}{K}$ .

Temperature difference between the hot end and the cold end in steady state is inversely proportional to  $K$ , i.e., in case of good conductors temperature of the cold end will be very near to hot end.

In ideal conductor where  $K = \infty$ , temperature difference in steady state will be zero.

**6. Thermometric conductivity or diffusivity:** It is a measure of rate of change of temperature (with time) when the body is not in steady state (i.e., in variable state).

The thermometric conductivity or diffusivity is defined as the ratio of the coefficient of thermal conductivity to the thermal capacity per unit volume of the material.

Thermal capacity per unit volume =  $\frac{mc}{V} = \rho c$  (as  $\rho$  is density of substance)

$$\therefore \text{Diffusivity } (D) = \frac{K}{\rho c}$$

Unit:  $\text{m}^2/\text{s}$ ; dimension:  $[L^2T^{-1}]$

**7. Thermal resistance:** The thermal resistance of a body is a measure of its opposition to the flow of heat through it.

It is defined as the ratio of temperature difference to the heat current (= rate of flow of heat).

Now, temperature difference =  $(\theta_1 - \theta_2)$  and heat current,  $H = \frac{Q}{t}$

$\therefore$  Thermal resistance

$$R = \frac{\theta_1 - \theta_2}{H} = \frac{\theta_1 - \theta_2}{Q/t} = \frac{\theta_1 - \theta_2}{KA(\theta_1 - \theta_2)/l} = \frac{l}{KA}$$

Unit:  $^\circ\text{C} \times \text{s}/\text{cal}$  or  $\text{K} \times \text{s}/\text{kcal}$ ; dimension:

$$[M^{-1}L^2T^3\theta]$$

## COMBINATION OF CONDUCTORS

**1. Series combination:** Let  $n$  slabs each of cross-sectional area  $A$ , lengths  $l_1, l_2, l_3, \dots, l_n$  and conductivities  $K_1, K_2, K_3, \dots, K_n$ , respectively, be connected in the series.

Heat current is the same in all the conductors.

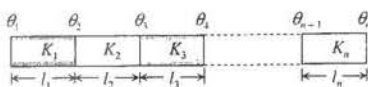


Fig. 1.29

$$\text{i.e., } \frac{Q}{t} = H_1 = H_2 = H_3 = \dots = H_n$$

$$\frac{K_1 A (\theta_1 - \theta_2)}{l_1} = \frac{K_2 A (\theta_2 - \theta_3)}{l_2} = \frac{K_3 A (\theta_3 - \theta_4)}{l_3} = \dots$$

$$= \frac{K_n A (\theta_{n-1} - \theta_n)}{l_n}$$

i. Equivalent resistance  $R = R_1 + R_2 + R_3 + \dots + R_n$

ii. If  $K_s$  is equivalent conductivity, then from relation

$$R = \frac{l}{KA}$$

$$\frac{l_1 + l_2 + l_3 + \dots + l_n}{K_s} = \frac{l_1}{K_1 A} + \frac{l_2}{K_2 A} + \frac{l_3}{K_3 A} + \dots + \frac{l_n}{K_n A}$$

$$\therefore K_s = \frac{l_1 + l_2 + l_3 + \dots + l_n}{\frac{l_1}{K_1} + \frac{l_2}{K_2} + \frac{l_3}{K_3} + \dots + \frac{l_n}{K_n}}$$

iii. Equivalent thermal conductivity for  $n$  slabs of equal length

$$K = \frac{n}{\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} + \dots + \frac{1}{K_n}}$$

For two slabs of equal length,  $K = \frac{2K_1 K_2}{K_1 + K_2}$

iv. Temperature of interface of composite bar: Let the two bars be arranged in series as shown in Fig. 1.30. Then heat current is same in the two conductors.

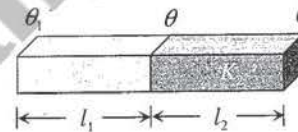


Fig. 1.30

$$\text{i.e., } \frac{Q}{t} = \frac{K_1 A (\theta_1 - \theta)}{l_1} = \frac{K_2 A (\theta - \theta_2)}{l_2}$$

$$\text{By solving we get } \theta = \frac{\frac{K_1}{l_1} \theta_1 + \frac{K_2}{l_2} \theta_2}{\frac{K_1}{l_1} + \frac{K_2}{l_2}}$$

$$\text{If } (l_1 = l_2 = l) \text{ then } \theta = \frac{K_1 \theta_1 + K_2 \theta_2}{K_1 + K_2}$$

**2. Parallel combination:** Let  $n$  slabs each of length  $l$ , areas  $A_1, A_2, A_3, \dots, A_n$  and thermal conductivities  $K_1, K_2, K_3, \dots, K_n$  be connected in parallel. Then

i. Equivalent resistance  $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n}$

ii. Temperature gradient across each slab will be same.

iii. Heat current in each slab will be different. Net heat current will be the sum of heat currents through individual slabs. i.e.,  $H = H_1 + H_2 + H_3 + \dots + H_n$

$$\begin{aligned} & \frac{K(A_1 + A_2 + A_3 + \dots + A_n)(\theta_1 - \theta_2)}{l} \\ &= \frac{K_1 A_1 (\theta_1 - \theta_2)}{l} + \frac{K_2 A_2 (\theta_1 - \theta_2)}{l} + \frac{K_3 A_3 (\theta_1 - \theta_2)}{l} \\ & \quad + \dots + \frac{K_n A_n (\theta_1 - \theta_2)}{l} \end{aligned}$$



$$K = \frac{K_1 A_1 + K_2 A_2 + K_3 A_3 + \dots + K_n A_n}{A_1 + A_2 + A_3 + \dots + A_n}$$

For  $n$  slabs of equal area  $K = \frac{K_1 + K_2 + K_3 + \dots + K_n}{n}$

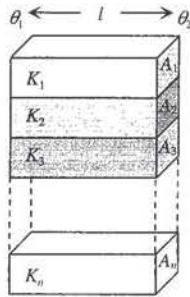


Fig. 1.31

Equivalent thermal conductivity for two slabs of equal area

$$K = \frac{K_1 + K_2}{2}$$

### Electrical Analogy for Thermal Conduction

It is an important fact to appreciate that there exists an exact similarity between thermal and electrical conductivities of a conductor.

Electrical conduction	Thermal conduction
Electric charge flows from higher potential to lower potential	Heat flows from higher temperature to lower temperature
The rate of flow of charge is called the electric current, i.e., $I = \frac{dq}{dt}$	The rate of flow of heat may be called heat current i.e., $H = \frac{dQ}{dt}$
The relation between the electric current and the potential difference is given by Ohm's law, i.e., $I = \frac{V_1 - V_2}{R}$ where $R$ is the electrical resistance of the conductor	Similarly, the heat current may be related with the temperature difference as $H = \frac{\theta_1 - \theta_2}{R}$ where $R$ is the thermal resistance of the conductor
The electrical resistance is defined as $R = \frac{\rho l}{A} = \frac{l}{\sigma A}$ , where $\rho$ = resistivity and $\sigma$ = electrical conductivity	The thermal resistance may be defined as $R = \frac{l}{KA}$ , where $K$ = Thermal conductivity of conductor

$$\begin{aligned} \frac{dq}{dt} &= I = \frac{V_1 - V_2}{R} \\ &= \frac{\sigma A}{l} (V_1 - V_2) \end{aligned}$$

$$\begin{aligned} \frac{dQ}{dt} &= H = \frac{\theta_1 - \theta_2}{R} \\ &= \frac{KA}{l} (\theta_1 - \theta_2) \end{aligned}$$

**Illustration 1.52** Three cylindrical rods  $A$ ,  $B$  and  $C$  of equal lengths and equal diameters are joined in series as shown in Fig. 1.32. Their thermal conductivities are  $2K$ ,  $K$  and  $0.5K$ , respectively. In steady state, if the free ends of rods  $A$  and  $C$  are at  $100^\circ\text{C}$  and  $0^\circ\text{C}$ , respectively, calculate the temperature at the two junction points. Assume negligible loss by radiation through the curved surface. What will be the equivalent thermal conductivity?

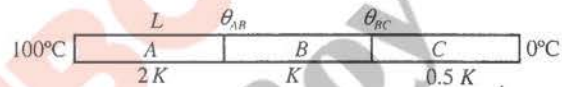


Fig. 1.32

**Sol.** As the rods are in series,  $R_{eq} = R_A + R_B + R_C$  with  $R = (L/K_A)$

$$\text{i.e., } R_{eq} = \frac{L}{2KA} + \frac{L}{KA} + \frac{L}{0.5KA} = \frac{7L}{2KA} \quad (i)$$

$$\text{And hence, } H = \frac{dQ}{dt} = \frac{\Delta\theta}{R} = \frac{(100 - 0)}{(7L/2KA)} = \frac{200KA}{7L}$$

Now in series, rate of flow of heat remains same, i.e.,  $H = H_A = H_B = H_C$ .

$$\text{So for rod } A, \quad \left[ \frac{dQ}{dt} \right]_A = \left[ \frac{dQ}{dt} \right]$$

$$\text{i.e., } \frac{(100 - \theta_{AB})2KA}{L} = \frac{200KA}{7L}$$

$$\text{or, } \theta_{AB} = 100 - (100/7) = (600/7) = 87.7^\circ\text{C}$$

$$\text{And for rod } C, \quad \left[ \frac{dQ}{dt} \right]_C = \left[ \frac{dQ}{dt} \right]$$

$$\text{i.e., } \frac{(\theta_{BC} - 0) \times 0.5KA}{L} = \frac{200KA}{7L}$$

$$\text{or, } \theta_{BC} = (400/7) = 57.1^\circ\text{C}$$

Furthermore, if  $K_{eq}$  is equivalent thermal conductivity,

$$R_{eq} = \frac{L + L + L}{K_{eq}A} = \frac{7L}{2KA} \quad [\text{from Eq. (i)}]$$

$$\text{i.e., } K_{eq} = (6/7)K$$

**Illustration 1.53** Two walls of thickness in the ratio  $1 : 3$  and thermal conductivities in the ratio  $3 : 2$  form a composite wall of a building. If the free surfaces of the wall