Physics for IIT-JEE

ELECTRICITY & MAGNETISM

B.M. Sharma

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Physics for IIT-JEE 2012-13:
Electricity & Magnetism

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Brief Contents

Contents

Preface

Since the time the IIT-JEE (Indian Institute of Technology Joint Entrance Examination) started, the examination scheme and the methodology have witnessed many a change. From the lengthy subjective problems of 1950s to the matching column type questions of the present day, the paper-setting pattern and the approach have changed. A variety of questions have been framed to test an aspirant's calibre, aptitude, and attitude for engineering field and profession. Across all these years, however, there is one thing that has not changed about the IIT-JEE, i.e., its objective of testing an aspirant's grasp and understanding of the concepts of the subjects of study and their applicability at the grass-root level.

No subject can be mastered overnight; nor can a subject be mastered just by formulae-based practice. Mastering a subject is an expedition that starts with the basics, goes through the illustrations that go on the lines of a concept, leads finally to the application domain (which aims at using the learnt concept(s) in problem-solving with accuracy) in a highly structured manner.

This series of books is an attempt at coming face-to-face with the latest IIT-JEE pattern in its own format, which is going to be highly advantageous to an aspirant for securing a good rank. A thorough knowledge of the contemporary pattern of the IIT-JEE is a must. This series of books features all types of problems asked in the examination—be it MCQs (one or more than one correct), assertion-reason type, matching column type, or paragraph-based, thought-type questions. Not discounting to need for skilled and guided practice, the material in the book has been enriched with a large number of fully solved concept-application exercises so that every step in learning is ensured for the understanding and application of the subject.

This whole series of books adopts a multi-facetted approach to mastering concepts by including a variety of exercises asked in the examination. A mix of questions helps stimulate and strengthen multi-dimensional problem-solving skills in an aspirant. Each book in the series has a sizeable portion devoted to questions and problems from previous years' IIT-JEE papers, which will help students get a feel and pattern of the questions asked in the examination. The best part about this series of books is that almost all the exercises and problem have been provided with not just answers but also solutions.

Overall the whole content of the book is an amalgamation of the theme of physics with ahead-of-time problems, which an aspirant must follow to accomplish success in IIT-JEE.

B. M. Sharma

CHAPTER

1

Coulomb's Laws and Electric Field

- Electric Charge
- Charging of a Body
- Work Function of a Body
- Properties of Electric Charge
- Coulomb's Law
- Coulomb's Law in Vector Form
- Electric Field
- Different Patterns of Electric Field Lines
- Field of Ring Charge
- Field of Uniformly Charged Disk
- Field of Two Oppositely Charged Sheets
- Electric Dipole
- Electric Field Due to a Dipole
- Electric Field Intensity Due to a Short Dipole at Some General Point
- Dipole in a Uniform Electric Field

ELECTRIC CHARGE

Electric charge, like mass, is one of the fundamental attributes of the particle of which the matter is made. Charge is the physical property of certain fundamental particles (like electron, proton) by virtue of which they interact with the other similar fundamental particles.

- Charge is an intrinsic property of some fundamental particles which accompanies these particles wherever they exist.
- Charge is that property of a body/particle which is responsible for electrical force between them.

To distinguish the nature of interaction, charges are divided into two parts:

(i) positive (ii) negative.

Fig. 1.1 shows an experiment to demonstrate that there are two types of charges.

We know that matter consists of atoms. An atom consists of a central core (called nucleus) and electrons. Electrons orbit around the nucleus. Nucleus consists of neutrons and protons. Neutrons do not contain any net charge. Protons and electrons have equal charges, but of opposite nature. Protons are positively charged while electrons are negatively charged. Protons, however, are very heavy when compared with electrons, about 1836 times. Protons are imprisoned in the nucleus along with neutrons due to the strongest binding force existing in nature called 'strong or nuclear force'. Thus, protons do not travel from atom to atom. The outermost electrons may travel from atom to atom. Hence, we say that electrons are the basis of electricity.

Charge on a proton or on an electron is of indivisible nature. We designate this charge by $+e$ and $-e$, respectively. Hence, charge in or on any object is always an integral multiple of the electronic charge.

In a normal atom:

i. Number of protons = number of electrons.
ii. Protons have the basic $+e$ charge and electrons have the basic $-e$ charge.
iii. Hence, a normal atom is electrically neutral.

Electrons can travel from one atom to another and from one body to another.

If a body loses one electron, it becomes positively charged with $+e$ charge and vice versa.

A body, however, cannot lose or gain any proton, which is heavy and remains imprisoned in the nucleus, by ordinary methods.

Note: ***Basic unit of charge = e, whose magnitude is equal to the magnitude of charge on an electron or proton, i.e., $e = 1.6 \times 10^{-19}$ C***
S.I. unit of charge: As mentioned above, $e = 1.6 \times 10^{-19}$ C. In it, e stands for one electronic charge which is the basic unit of charge. C stands for "coulomb" (note the small c in "coulomb"). "coulomb" is the S.I. unit of charge.

CHARGING OF A BODY

Ordinarily, matter contains equal number of protons and electrons. A body can be charged by the transfer of electrons or redistribution of electrons.

A body can be charged by the transfer of electrons and not due to the transfer of protons. Why?

It is because protons are inside the nucleus and it is very difficult to remove them from there. Electrons lie in the outer shells and it is easier to remove them.

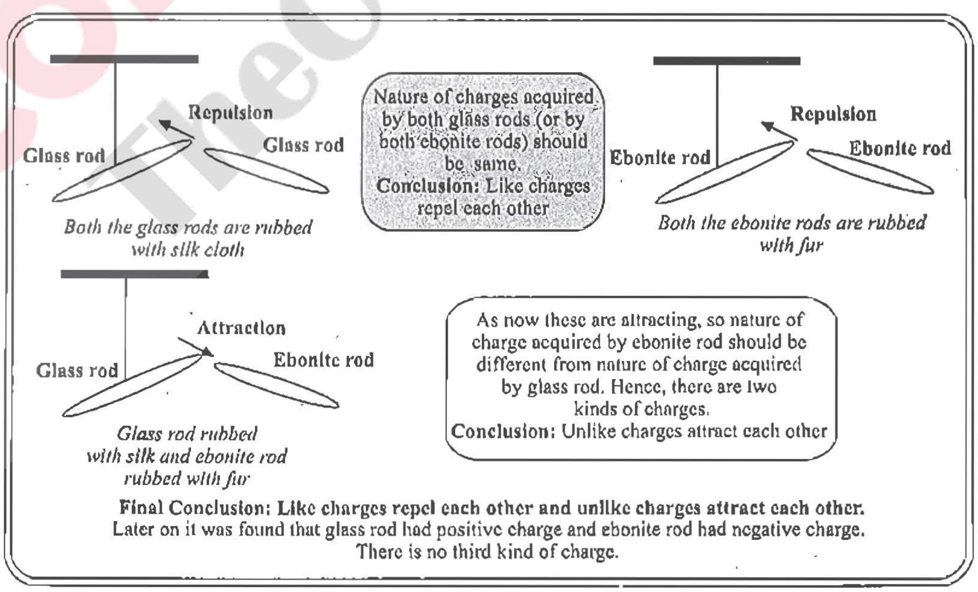


Fig. 1.1

To charge a body negatively: some electrons are given to it.
To charge a body positively: some electrons are taken from it.

WORK FUNCTION OF A BODY

The amount of work to be done on a body in order to remove electron from its surface. Obviously it is easier to remove an electron from a body whose work function is lower.

Let us see how bodies get charged due to friction:

As shown in Fig. 1.2, let $W_2 > W_1$.

Now, suppose A and B are rubbed together.

Net transfer of electrons will take place from A to B.

It is to be noted that mass is also affected during charging.

(Mass of negatively charged body increases and that of positively charged body decreases.)

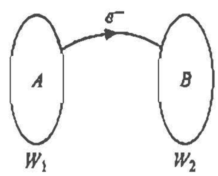


Fig. 1.2

Basically charging can be done by three methods:
1. Friction, **2.** Conduction, and **3.** Induction.

Charging by Friction

When two bodies are rubbed together, electrons are transferred from one body to the other making one body positively charged and the other negatively charged.

Example: When a glass rod is rubbed with silk, the rod becomes positively charged while silk gets negatively charged. However, ebonite on rubbing with wool becomes negatively charged making the wool positively charged.

Charging by Conduction

The process of charging from an already charged body can happen either by conduction or induction. Conduction from a charged body involves transfer of like charges. A positively charged body can cause more bodies to get positively charged but the sum of the total charge on all positively charged bodies will be the same as charge on initially considered charged body.

Charging by Induction

Induction is a process by which a charged body can be used to create other charged bodies without touching them or losing its own charge. If a charged body is brought near a neutral body, the charged body attracts opposite charge and repels similar charge present on the neutral body. If the neutral body is now earthed, the like charge is neutralized by the flow of charge from earth, leaving unlike charge on the body. Now, the earthing and the charging body is removed leaving the initially neutral body charged. The whole process is as shown in Fig. 1.3.

PROPERTIES OF ELECTRIC CHARGE

Quantization of Charge

Charge exists in discrete packets rather than in continuous amount, i.e., charge on any body is the integral multiple of the charge on an electron or proton.

$Q = \pm ne$, where $n = 0, 1, 2, \ldots$

Conservation of Charge

Charge is conserved, i.e., total charge on an isolated system is constant. By isolated system, we here mean a system through

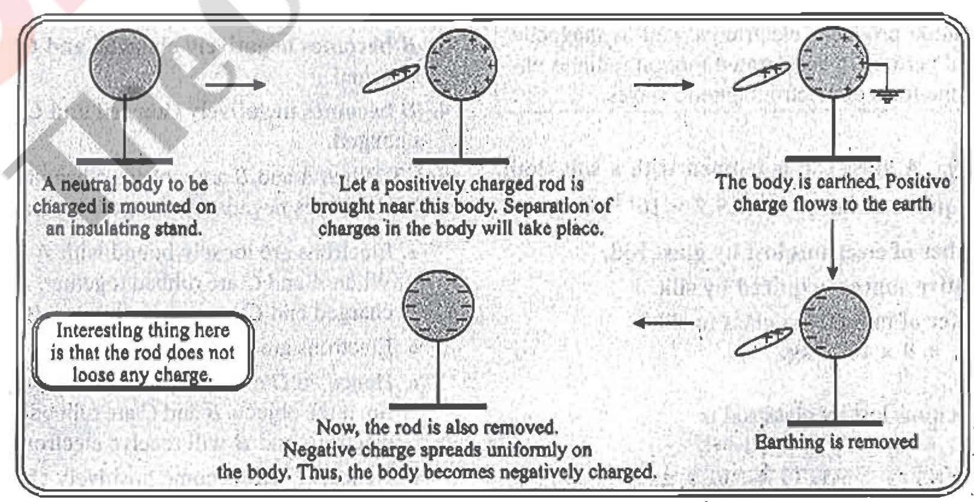


Fig. 1.3

the boundary of which no charge is allowed to escape or enter. This does not require that the amount of positive and negative charges separately be conserved.

Additivity of Charge

Total charge on a body is the algebraic sum of all the charges located anywhere on the body. While adding the charges, their sign must be taken into consideration.

For example, if a body has charges 2 C, −5 C, 4 C and 6 C (Fig. 1.4), then total charge on the body = 2 − 5 + 4 + 6 = 7 C.

Note that charges are added like real numbers. They have no direction. *So, charge is a scalar quantity.*

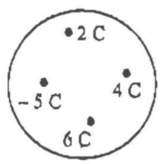


Fig. 1.4

Charge is Invariant

Charge does not depend on the speed of body.

Points to Remember

There are two types of forces which act between two charges. If the charges are stationary, there is only one type of force between them. It is called "electric" or "electrostatic" force. It is given by Coulomb's law for point charges. If the charges are moving, then two types of forces act between them. The first one is the above said electric force. The other force which emerges due to motion is called magnetic force. We shall study magnetic force in a later chapter.

Charge produces electric and magnetic fields and radiates energy: A stationary charged particle produces only electric field in the space surrounding it. A charged particle moving without acceleration produces electric as well as magnetic fields. A charged particle in accelerated motion radiates energy as well, in the form of electromagnetic waves.

Illustration 1.1 **A glass rod is rubbed with a silk cloth. The glass rod acquires a charge of $+19.2 \times 10^{-19}$ C.**

1. **Find the number of electrons lost by glass rod.**
2. **Find the negative charge acquired by silk.**
3. **Is there transfer of mass from glass to silk?**
   Given, $m_e = 9 \times 10^{-31}$kg.

Sol.

1. Number of electrons lost by glass rod is
$$n = \frac{q}{e} = \frac{19.2 \times 19^{-19}}{1.6 \times 10^{-19}} = 12$$
2. Charge on silk = -19.2×10^{-19} C
3. Since an electron has a finite mass $(m_e = 9 \times 10^{-31}\text{ kg})$, there will be transfer of mass from glass rod to silk cloth. Mass transferred $= 12 \times (9 \times 10^{-31}) = 1.08 \times 10^{-29}$ kg

Note that mass transferred is negligibly small. This is expected because the mass of an electron is extremely small.

Illustration 1.2 **Electric charges A and B attract each other. Electric charges B and C repel each other. If A and C are held close together, they will:**

1. attract 2. repel 3. not affect each other
4. more information is needed to answer.

Sol.

Case 1	Case 2
If A and B attract each other, then A: + and B: −	If A and B attract each other, then A: − and B: +
If B and C repel each other, then B: − and C: −	If B and C repel each other, then B: + and C: +

From both cases, we see that A and C will be having unlike charges. Hence, if the charges A and C are held together, they will attract each other.

Illustration 1.3 **If an object made of substance A is rubbed with an object made of substance B, then A becomes positively charged and B becomes negatively charged. If, however, an object made of substance A is rubbed against an object made of substance C, then A becomes negatively charged. What will happen if an object made of substance B is rubbed against an object made of substance C?**

1. **B becomes positively charged and C becomes positively charged.**
2. **B becomes positively charged and C becomes negatively charged.**
3. **B becomes negatively charged and C becomes positively charged.**
4. **B becomes negatively charged and C becomes negatively charged.**

Sol. 3. When A and B are rubbed, A becomes positively charged and B becomes negatively charged. It means

- Electrons are loosely bound with A in comparison to B. When A and C are rubbed together, A becomes negatively charged and C positively charged. It means
- Electrons are loosely bound with C in comparison to A.
- Hence, in C electrons are most loosely bound. So, if the objects B and C are rubbed together, C will loose electrons and B will receive electrons.
- Hence, C will become positively charged and B will become negatively charged.

Illustration 1.4 **Objects A, B and C are three identical, insulated, spherical conductors. Originally A and B both**

+3 mC, while C has a charge of −6 mC. C are allowed to touch, then they are moved ., objects B and C are allowed to touch before ıoved apart.

. jects A and B are now held near each other, they will

.attract b. repel c. have no effect on each other.

.f instead objects A and C are held near each other, they will

a. attract b. repel c. have no effect on each other.

Sol.

Initially A B C

+3 mC +3 mC − 6 mC

- When the objects A and C are allowed to touch and then moved apart:

A C $A \Longleftrightarrow C$

$[(+3\ \text{mC} + (-6\ \text{mC}) = -3\ \text{mC}]$ $-\frac{3\ \text{mC}}{2}$ $-\frac{3}{2}$ mC

- When the objects B and C are allowed to touch and then moved apart:

B C $B \Longleftrightarrow C$

$\left[(+3\ \text{mC}) + \left(-\frac{3}{2}\ \text{mC}\right) = +\frac{3}{2}\ \text{mC}\right]$ $+\frac{3}{4}$ mC $+\frac{3}{4}$ mC

Hence, if A and B are now held near each other, they will attract each other.

- If A and C are now held near each other, they will also attract each other.

Illustration 1.5 Figure 1.5 shows that a positively charged rod is brought near two uncharged metal spheres A and B attached with insulated stands and placed in contact with each other.

1. What would happen if the rod was removed before the spheres are separated?
2. Would the induced charges be equal in magnitude even if the spheres had different sizes or different conductors?
3. What will happen if the spheres are separated first and then the rod is removed far away.

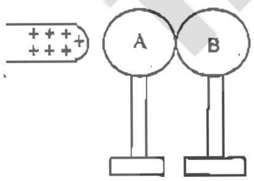


Fig. 1.5

Sol.

1. When a positively charged rod is brought near A, the free electrons in the sphere A are attracted to the rod and move in the left side of A. This movement leaves unbalanced positive charge on B. If the rod is removed before the spheres are separated, the excess electrons on sphere A would flow back to B. Both the spheres will become uncharged.

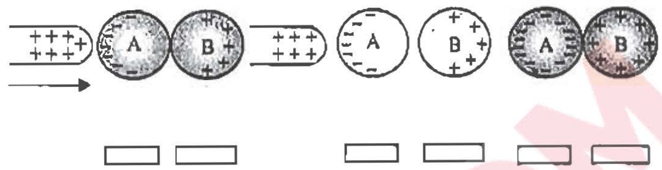


Fig. 1.6

2. Yes, net charge is conserved. Before the rod is brought near A, both A and B were neutral. They will remain so even if they have different sizes or materials.
3. If the rod is removed after the spheres are separated, the sphere A will have net negative charge and sphere B will have net positive charge of same magnitude.

Concept Application Exercise 1.1

1. a. How many electrons are in 1 coulomb of negative charge?
   b. Which is the true test of electrification, attraction or repulsion?
   c. Can a body have charge of 0.8×10^{-19} C?
2. Find the unit and dimension of permittivity of free space.
3. If only one charge is available, can it be used to obtain a charge many times greater than it in magnitude?
4. a. Can two bodies having like charges attract each other? (Yes/No)
   b. Can a charged body attract an uncharged body? (Yes/No)
   c. Two identical metallic spheres of exactly equal masses are taken, one is given a positive charge q and the other an equal negative charge. Their masses after charging are different. Comment on the statement.
5. A particle has charge of $+10^{-12}$ C.
   a. Does it contain more or less number of electrons as compared to the neutral state?
   b. Calculate the number of electrons transferred to provide this charge.
6. An ebonite rod is rubbed with fur. The ebonite rod is found to have a charge of -3.2×10^{-8} C on it.
   a. Calculate the number of electrons transferred.
   b. What is the charge on fur after rubbing?
7. The electric charge of macroscopic bodies is actually a surplus or deficiency of electrons. Why not protons?
8. A charged rod attracts bits of dry paper which after touching the rod, often jump away from it violently. Explain.
9. A person standing on an insulating stool touches a charged insulated conductor. Will the conductor get completely discharged?
10. An electron moves along a metal tube with variable cross section. How will its velocity change when it approaches the neck of the tube (Fig. 1.7)?

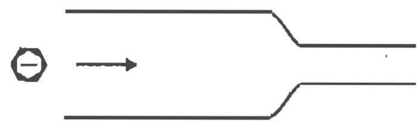


Fig. 1.7

11. Define the following statement "If there were only one electrically charged particle in the entire universe, the concept of electric charge would be meaningless".

COULOMB'S LAW

The force of interaction between two point charges is proportional to the product of magnitudes of the two charges and inversely proportional to the square of distance between them.

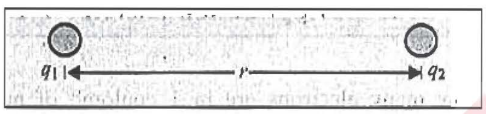


Fig. 1.8

Let two point electric charges q_1 and q_2 are at rest, separated by a distance r, then they exert a force on each other which is given by

$$F = k\frac{q_1q_2}{r^2}$$

where k is a proportionality constant known as *electrostatic force constant.*

If between the two charges there is free space (or vacuum), then $k = \dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ \text{Nm}^2\ \text{C}^{-2}$ (in SI units)

where $\varepsilon_0 = \dfrac{1}{4\pi k} = 8.85 \times 10^{-12}\ \text{C}^2\text{N}^{-1}\text{m}^{-2}$ is the absolute electric permittivity of the free space.

So, force between two charges is given as $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1q_2}{r^2}$ (i)

Equation (i) is applicable only for point charges placed in vacuum. Now, what happens if the two charges are placed in some medium?

In a medium, the force is given as: $F' = k'\dfrac{q_1q_2}{r^2}$ (ii)

where $k' = \dfrac{1}{4\pi\varepsilon}$ and in this ε is known as absolute electrical permittivity of medium.

Then, $F' = \dfrac{1}{4\pi\varepsilon}\dfrac{q_1q_2}{r^2}$ (iii)

The ratio $\dfrac{\varepsilon}{\varepsilon_0} = \varepsilon_r$ is known as relative electrical permittivity of medium.

It is also known as *dielectric constant* and denoted by K.

So, $\dfrac{\varepsilon}{\varepsilon_0} = \varepsilon_r = K$

The value of K for different materials: Vacuum = 1, air = 1.006 1, glass = 3 to 4, water = 81, conductor = ∞.

In general $K \geq 1$

Now, from (i) and (iii) : $\dfrac{F'}{F} = \dfrac{\varepsilon_0}{\varepsilon} = \dfrac{1}{K}$ =. means when the charges are placed in a medium, . creases K times.

Also, $K = \dfrac{F}{F'}$. So, *the dielectric constant of a medium, defined as the ratio of force between two charges when the; placed in vacuum to that when they are placed in that medi at same separation.*

Note:

- ***Coulomb's law is not valid for distances < 10^{-15} m.***
- ***Electrostatic forces are comparatively stronger than gravitational forces. Can you show this?***

(As an example—when we hold a book in our hand, electric force between hand and the book is sufficient to balance the gravitational force of earth on the book due to entire earth.)

Some Important Points

- Coulomb's law is applicable only for point charges.
- Coulomb's law is similar to Newton's gravitational law and both obey inverse square law.
- Coulomb's law obeys Newton's third law, i.e., the forces exerted by the two charges on each other are equal and opposite.
- This force acts along the line joining the two particles (called central force).
- Electrostatic force is a conservative force.

COULOMB'S LAW IN VECTOR FORM

Let q_1 and q_2 be two like charges placed at points A and B, respectively, in vacuum.

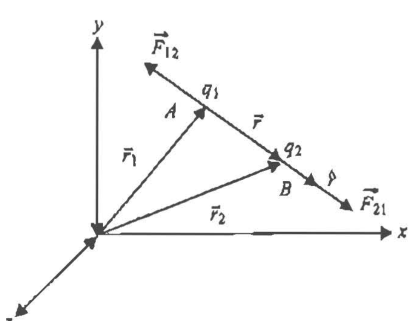


Fig. 1.9

$\vec{r}_1$ is the position vector of point A and $\vec{r}_2$ is the position vecto of point B.

Let $\vec{r}$ is vector from A to B, then $\vec{r} = \vec{r}_2 - \vec{r}_1$ and $r = |\vec{r}_2 - \vec{r}_1|$

$$\Rightarrow \qquad \hat{r} = \frac{\vec{r}}{r} = \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|}$$

Let $\vec{F}_{21}$ be the force on charge q_2 due to q_1; and

$\vec{F}_{12}$ be the force on charge q_1 due to q_2.

From Fig. 1.9, it is clear that $\vec{F}_{21}$ and $\hat{r}_{12}$ are in the same direction, so

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}\frac{\vec{r}}{r} = \frac{q_1q_2}{4\pi\varepsilon_0}\frac{\vec{r}}{r^3}$$

$$\Rightarrow \quad \vec{F}_{21} = \frac{q_1q_2}{4\pi\varepsilon_0}\frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|^3}$$

The above equations give the Coulomb's law in vector form.

As we know that charges apply equal and opposite forces on each other, so we have

$$\vec{F}_{12} = -\vec{F}_{21} \quad \Rightarrow \quad \vec{F}_{12} = \frac{q_1q_2}{4\pi\varepsilon_0}\frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|^3}$$

Also, the forces due to two point charges are parallel to the line joining the point charges; such forces are called central forces and so electrostatic forces are conservative forces.

Superposition Principle

It enables us to calculate the force acting on a charge due to more than one charge.

According to superposition principle, the total force on a given charge is vector sum of all the individual forces exerted by each of the other charge.

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_n$$

Another important point is that the force between two charges remains unaffected due to the presence of a third charge.

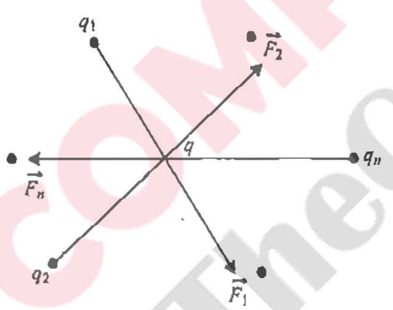


Fig. 1.10

Note:

- ***Coulomb's law and principle of superposition together can explain whole of the electrostatics.***
- ***Both Coulomb's law and Gravitational law describe inverse square law that involve a property of interacting particles—the charge in one case and mass in the other case.***

Illustration 1.6 **Two identical conducting spheres 1 and 2 carry equal amounts of charge and are fixed a certain distance apart that is large compared with their diameters. The spheres repel each other with an electrical force of 88 mN. Suppose now that a third identical sphere 3 having an insulating handle and initially uncharged, is touched first to sphere 1 then to sphere 2 and finally removed. Find the force between spheres 1 and 2 now shown in figure d.**

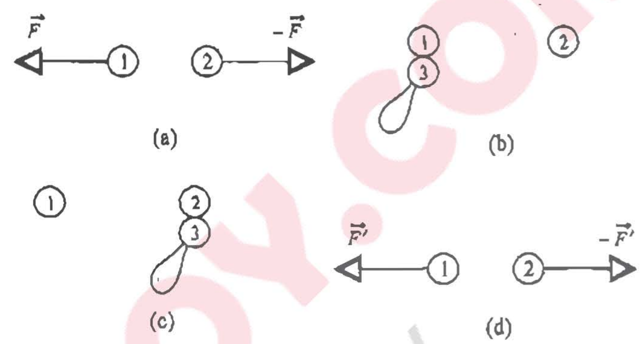


Fig. 1.11

Sol. Initial force between '1' and '2' $F = \dfrac{kq^2}{r^2} = 88$ mN

Charge on '1' after sphere '3' is touched with '1' = $q/2$. Same charge will be on sphere '3' also.

Charge on '2' after sphere '3' is touched with '2'= $\dfrac{q + q/2}{2} = \dfrac{3q}{4}$

Now, force between '1' and '2' in situation d:

$$F' = \frac{k(q/2)(3q/4)}{r^2} = \frac{3}{8}\frac{kq^2}{r^2} = \frac{3}{8} \times 88 = 33 \text{ mN}$$

Illustration 1.7 **Two identical He-filled spherical balloons each carrying a charge q are tied to a weight W with strings and float in equilibrium as shown in Fig. 1.12(a). Find:**

1. **the magnitude of q, assuming that the charge on each balloon acts as if it were concentrated at the centre.**
2. **the volume of each balloon.**

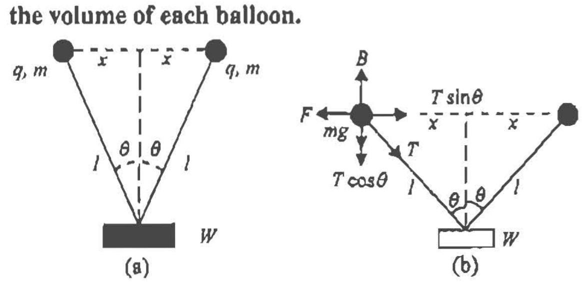


Fig. 1.12

Sol. 1. $2T\cos\theta = W$, $T\sin\theta = F$ [Fig. 1.12(b)]

$$\Rightarrow \frac{\tan\theta}{2} = \frac{F}{W} \Rightarrow F = W\frac{\tan\theta}{2}$$

$$\Rightarrow \frac{q^2}{4\pi\varepsilon_0(2x)^2} = \frac{W\tan\theta}{2} \Rightarrow q = \sqrt{8W\tan\theta\,\pi\varepsilon_0 x^2}$$

2. $T\cos\theta + mg = B \Rightarrow \dfrac{W}{2} + V\rho_{He}g = V\rho_a g$

$$\Rightarrow \qquad V = \frac{W}{2(\rho_a - \rho_{He})g}$$

Illustration 1.8 **Two particles, each having a mass of 5 g and charge 10^{-7} C, stay in limiting equilibrium on a horizontal table with a separation of 10 cm between them. Find the coefficient of friction between each particle and the table, which is same between each particle and table.**

Sol. Friction force f will balance the electrostatic repulsion,

i.e., $$f = F \Rightarrow \mu mg = \frac{q^2}{4\pi\varepsilon_0 r^2}$$

Fig. 1.13

$$\Rightarrow \mu \times \frac{5}{1000} \times 10 = \frac{9 \times 10^9 \times (10^{-7})^2}{(0.10)^2} \Rightarrow \mu = 0.18$$

Illustration 1.9 **A particle of mass m carrying a charge $-q_1$ starts moving around a fixed charge $+q_2$ along a circular path of radius r. Prove that period of revolution T of charge $-q_1$ is given by $T = \sqrt{\dfrac{16\pi^3\varepsilon_0 m r^3}{q_1 q_2}}$.**

Sol. Electrostatic force on $-q_1$ due to q_2 will provide the necessary centripetal force, hence

$$\frac{kq_1q_2}{r^2} = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{kq_1q_2}{mr}}$$

$$\text{Now, } T = \frac{2\pi r}{v} = \sqrt{\frac{16\pi^3\varepsilon_0 m r^3}{q_1 q_2}}$$

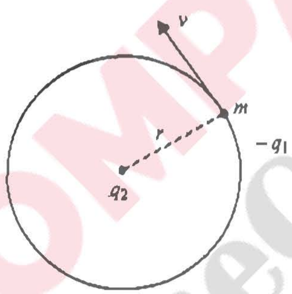


Fig. 1.14

Illustration 1.10 **Consider three charges q_1, q_2 and q_3, each equal to q, at the vertices of an equilateral triangle of side l. What is the force on a charge Q placed at the centroid of the triangle?**

1. $\dfrac{3}{4\pi\varepsilon_0}\dfrac{Qq}{l^2}$ 2. $\dfrac{\sqrt{3}}{2\pi\varepsilon_0}\dfrac{Qq}{l^2}$ 3. $\dfrac{\sqrt{3}}{4\pi\varepsilon_0}\dfrac{Qq}{l^2}$ 4. zero

Sol. Method 1. The resultant of three equal coplanar vectors acting at a point is zero if these vectors form a closed polygon (Fig. 1.15). Hence, the vector sum of the forces $\vec{F_1}$, $\vec{F_2}$ and $\vec{F_3}$ is zero.

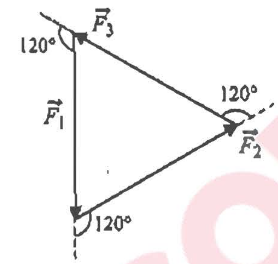


Fig. 1.15

Method 2. The forces acting on the charge Q are

$$\vec{F_1} = \text{force on } Q \text{ due to } q_1 = \frac{1}{4\pi\varepsilon_0}\frac{Qq_1}{AO^2}\widehat{AO}$$

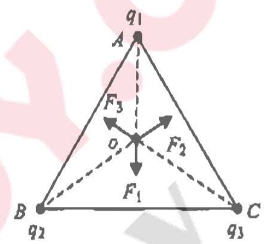


Fig. 1.16

$$\vec{F_2} = \text{force on } Q \text{ due to } q_2 = \frac{1}{4\pi\varepsilon_0}\frac{Qq_2}{BO^2}\widehat{BO}$$

$$\vec{F_3} = \text{force on } Q \text{ due to } q_3 = \frac{1}{4\pi\varepsilon_0}\frac{Qq_3}{CO^2}\widehat{CO}$$

The resultant force is $\vec{F_R} = \vec{F_1} + \vec{F_2} + \vec{F_3}$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Qq}{AO^2}(\widehat{AO} + \widehat{BO} + \widehat{CO}) = 0$$

(as $|q_1| = |q_2| = |q_3|$ and $|\vec{AO}| = |\vec{BO}| = |\vec{CO}|$)

Also, $\vec{AO} + \vec{BO} + \vec{CO} = 0$ because these are three equal vectors in a plane making angles of 120° with each other.

Method 3. The resultant force $\sum\vec{F}$ is the vector sum of individual forces

$$\sum\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3} \text{ or}$$

$$\sum F_x = F_{1x} + F_{2x} + F_{3x}$$

$$= 0 + F_2\cos 30° - F_3\cos 30° \quad \text{(i)}$$

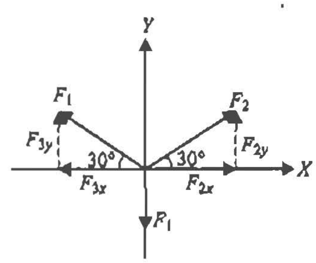


Fig. 1.17

$$\text{And } \sum F_y = F_{1y} + F_{2y} + F_{3y}$$

$$= -F_1 + F_2\sin 30° + F_3\sin 30° \quad \text{(ii)}$$

As $|F_1| = |F_2| = |F_3| = |F|$ (say), the equations (i) and (ii) become

$\sum F_x = 0$ and $\sum F_y = 0$. Hence, resultant force $\sum\vec{F} = 0$.

Illustration 1.11 **Point charges are placed at the vertices of a square of side a as shown in Fig. 1.18. What should be sign of charge q and magnitude of the ratio $\left|\frac{q}{Q}\right|$ so that:**

1. **net force on each Q is zero?**
2. **net force on each q is zero?**

Is it possible that the entire system could be in electrostatic equilibrium?

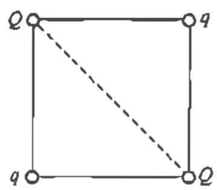
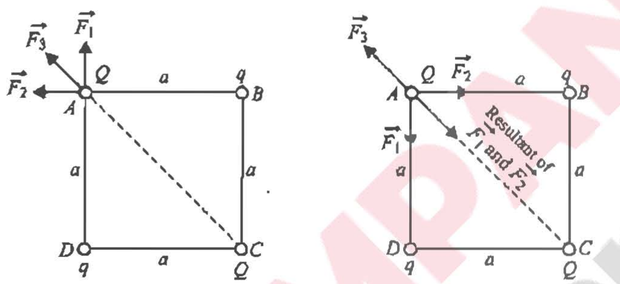


Fig. 1.18

Sol.

1. Consider the forces acting on charge Q placed at A (shown in Fig. 1.19(a) and (b))

   Case I. Let the charges q and Q are of same sign.



(a) (q and Q are of same nature) Here, net force cannot be zero.

(b) (q and Q are of opposite nature) Here, net force can be zero.

Fig. 1.19

Here, $F_1 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{qQ}{a^2}$ {force of q at D on Q at A}

$F_2 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{qQ}{a^2}$ {force of q at B on Q at A}

$F_3 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{QQ}{2a^2}$ {force of Q at C on Q at A}

In Fig. 1.19(a), resultant of forces $\vec{F_1}$ and $\vec{F_2}$ will lie along $\vec{F_3}$ so that net force on Q cannot be zero. Hence, q and Q have to be of opposite signs.

Case II. Let the charges q and Q are of opposite sign.

In this case, as shown in Fig. 1.19(b), resultant of $\vec{F_1}$ and $\vec{F_2}$ will be opposite to $\vec{F_3}$ so that it becomes possible to obtain a condition of zero net force.

Let us write $\vec{F_R} = \vec{F_1} + \vec{F_2}$

$$\therefore \quad F_R = \sqrt{F_1^2 + F_2^2} = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{a^2}\sqrt{2}$$

Direction of $\vec{F_R}$ will be along AC ($\vec{F_R}$, being resultant of forces of equal magnitude, bisects the angle between the two) $\vec{F_R}$ and $\vec{F_3}$ are in opposite directions. Net force on Q can be zero if their magnitudes are also equal, i.e.,

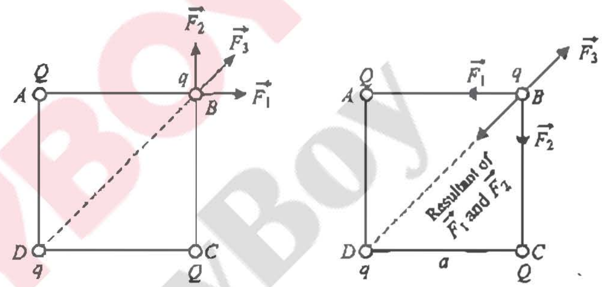
$$\frac{1}{4\pi\varepsilon_0}\frac{qQ}{a^2}\sqrt{2} = \frac{1}{4\pi\varepsilon_0}\frac{QQ}{2a^2} \text{ or } \frac{Q}{4\pi\varepsilon_0 a^2}\left(\sqrt{2}q - \frac{Q}{2}\right) = 0$$

$$\Rightarrow q = \frac{Q}{2\sqrt{2}} \Rightarrow \left|\frac{q}{Q}\right| = \frac{1}{2\sqrt{2}} \quad \{Q \neq 0\}$$

$\therefore$ The sign of q should be negative.

2. Consider now the forces acting on charge q placed at B (see Fig. 1.20(a) and (b)).

   In a similar manner, as discussed in 1, for net force on q to be zero, q and Q have to be of opposite signs. This is also shown in the given figures.



(a) (q and Q are of same sign) Here, net force cannot be zero.

(b) (q and Q are of opposite sign) Here, net force could be zero.

Fig. 1.20

Now, $F_1 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Qq}{a^2}$ {force of Q at A on q at B}

$F_2 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Qq}{a^2}$ {force of Q at C on q at B}

$F_3 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2}{2a^2}$ {force of q at D on q at B}

Referring to Fig. 1.20(b), let us write $\vec{F_R} = \vec{F_1} + \vec{F_2}$

$$\therefore \quad F_R = \sqrt{F_1^2 + F_2^2} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{a^2}\sqrt{2}$$

Resultant of $\vec{F_1}$ and $\vec{F_2}$, i.e., $\vec{F_R}$, is opposite to $\vec{F_3}$. Net force can become zero if their magnitudes are also equal, i.e.,

$$\frac{1}{4\pi\varepsilon_0}\frac{Qq}{a^2}\sqrt{2} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{2a^2} \Rightarrow \frac{q}{4\pi\varepsilon_0 a^2}\left(\sqrt{2}Q - \frac{q}{2}\right) = 0$$

$$\Rightarrow Q = \frac{q}{2\sqrt{2}} \Rightarrow \left|\frac{q}{Q}\right| = 2\sqrt{2} \quad \{q \neq 0\}$$

$\therefore$ The sign of 'q' should be negative.

In this case, we need not to repeat the calculation as the present situation is same as previous one; we can directly write $\left|\frac{q}{Q}\right| = 2\sqrt{2}$

3. The entire system cannot be in equilibrium since both conditions, i.e., $q = -\dfrac{Q}{2\sqrt{2}}$ and $Q = -\dfrac{q}{2\sqrt{2}}$ cannot be satisfied together.

Illustration 1.12 **Two identical small charged spheres, each having a mass m, hang in equilibrium as shown in**

Fig. 1.21(a). The length of each string is l and the angle made by any string with vertical is θ. Find the magnitude of the charge on each sphere.

Sol. The forces acting on the sphere are tension in the string T; force of gravity, mg; repulsive electric force, F_e, as shown in the free body diagram of the sphere (Fig. 1.21(b)). The sphere is in equilibrium. The forces in the horizontal and vertical directions must separately add up to zero..

$$\sum F_x = T\sin\theta - F_e = 0 \quad \text{(i)}$$

$$\sum F_y = T\cos\theta - mg = 0 \quad \text{(ii)}$$

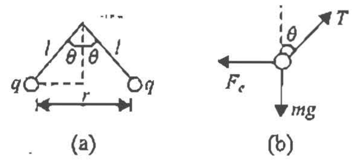


Fig. 1.21

From equation (ii), $T = \dfrac{mg}{\cos\theta}$. Thus, we can eliminate T from equation (i) to obtain

$$F_e = mg\tan\theta \text{ or } \frac{kq^2}{r^2} = mg\tan\theta \quad \text{(iii)}$$

where $k = \dfrac{1}{4\pi\varepsilon_0}$ and $r = 2l\sin\theta$.

The equation (iii) now reduces to $\dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q^2}{(2l\sin\theta)^2} = mg\tan\theta$

or $q = \sqrt{16\pi\varepsilon_0 l^2 mg\tan\theta\sin^2\theta}$

Illustration 1.13 **Two identical balls each having a density ρ are suspended from a common point by two insulating strings of equal length. Both the balls have equal mass and charge. In equilibrium, each string makes an angle θ with the vertical. Now, both the balls are immersed in a liquid. As a result, the angle θ does not change. The density of liquid is σ. Find the dielectric constant of the liquid.**

Sol. Let V is the volume of each ball, then mass of each ball:

$$m = \rho V$$

When the balls are in air, from previous problem,

$$F = mg\tan\theta = \rho Vg\tan\theta \quad \text{(i)}$$

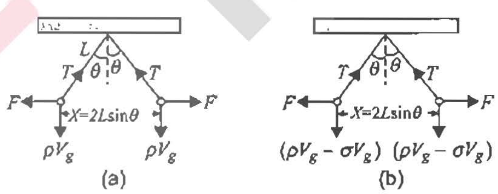


Fig. 1.22

When the balls are suspended in liquid, the Coulombic force is reduced to $F' = F/K$ and apparent weight = weight − upthrust:

$$W' = (\rho Vg - \sigma Vg).$$

According to the problem, angle θ is unchanged. So,

$$F' = W'\tan\theta = (\rho Vg - \sigma Vg)\tan\theta \quad \text{(ii)}$$

From equations (i) and (ii), we get

$$\frac{F}{F'} = K = \frac{\rho Vg}{\rho Vg - \sigma Vg} = \frac{\rho}{\rho - \sigma}$$

Illustration 1.14 **Three particles, each of mass 'm' and carrying a charge q each, are suspended from a common point by insulating massless strings, each of length 'L'. If the particles are in equilibrium and are located at the corners of an equilateral triangle of side 'a', calculate the charge q on each particle. Assume $L >> a$.**

Sol. From Fig. 1.23(b), for equilibrium of a particle along a vertical line,

$$T\cos\theta = mg \quad \text{(i)}$$

While for equilibrium in the plane of equilateral triangle,

$$T\sin\theta = 2F\cos 30° \quad \text{(ii)}$$

So, from equations (i) and (ii), we have

$$\tan\theta = \frac{\sqrt{3}F}{mg} \quad \text{(iii)}$$

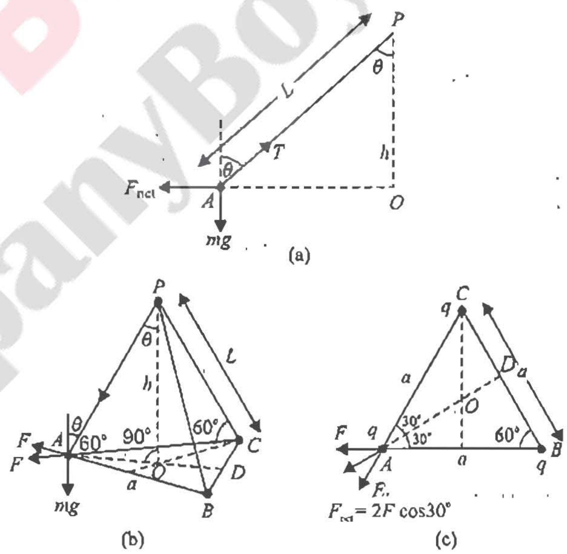


Fig. 1.23

Here, $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2}{a^2}$ and $\tan\theta = \dfrac{OA}{OP} = \dfrac{OA}{\sqrt{L^2 - OA^2}}$

Also, from Fig. 1.23(c)

$$OA = \frac{2}{3}AD = \frac{2}{3}a\sin 60° = \frac{a}{\sqrt{3}}$$

$$\text{So, } \tan\theta = \frac{(a/\sqrt{3})}{\sqrt{L^2 - (a^2/3)}} = \frac{a}{(\sqrt{3})L} \quad \{\text{as } L >> a\}$$

On substituting the above values of F and $\tan\theta$ in equation (iii), we get:

$$\frac{a}{(\sqrt{3})L} = \frac{\sqrt{3}}{mg}\frac{q^2}{4\pi\varepsilon_0 a^2}, \text{ i.e., } q = \left[\frac{4\pi\varepsilon_0 a^3 mg}{3L}\right]^{1/2}$$

Illustration 1.15 **A thin fixed ring of radius 'a' has a positive charge 'q' uniformly distributed over it. A particle of mass 'm' and having a negative charge 'Q' is placed on the**

axis at a distance of x ($x << a$) from the centre of the ring. Show that the motion of the negatively charged particle is approximately simple harmonic. Calculate the time period of oscillation.

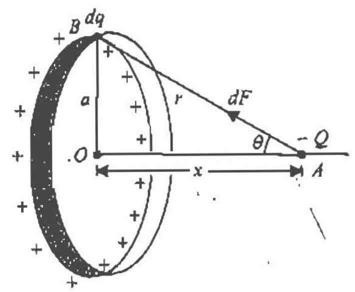


Fig. 1.24

Sol. The force on the point charge Q due to the element dq of the ring

$$dF = \frac{1}{4\pi\varepsilon_0}\frac{dq\,Q}{r^2} \text{ along } AB$$

As for every element of the ring there is symmetrically situated diametrically opposite element, the components of forces along the axis will add up while those perpendicular to it will cancel each other. Hence, net force on the charge $-Q$ is

$$F = \int dF\cos\theta = \cos\theta\int dF;$$

$$F = \frac{x}{r}\int\frac{1}{4\pi\varepsilon_0}\left[-\frac{Q\,dq}{r^2}\right]$$

So, $F = -\dfrac{1}{4\pi\varepsilon_0}\dfrac{Qx}{r^3}\displaystyle\int dq = -\dfrac{1}{4\pi\varepsilon_0}\dfrac{Qqx}{(a^2+x^2)^{3/2}}$ (i)

$$\left\{\text{as } r = (a^2+x^2)^{1/2} \text{ and } \int dq = q\right\}$$

−ve sign shows that this force will be towards the centre of ring.

As the restoring force is not linear, the motion will be oscillatory. However, if $x \ll a$ so that $x^2 \ll a^2$,

$$F = -\frac{1}{4\pi\varepsilon_0}\frac{Qq}{a^3}x = -kx \text{ with } k = \frac{Qq}{4\pi\varepsilon_0 a^3}$$

i.e., the restoring force will become linear and so the motion is simple harmonic with time period

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{4\pi\varepsilon_0 ma^3}{qQ}}$$

Illustration 1.16 **The field lines for two point charges are shown in Fig. 1.25.**

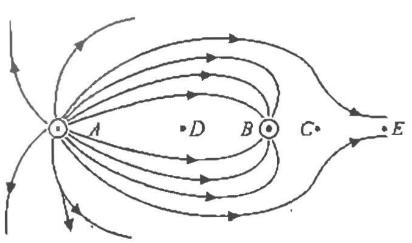


Fig. 1.25

1. **Is the field uniform?**
2. **Determine the ratio $\dfrac{q_A}{q_B}$.**
3. **What are the sign of q_A and q_B?**
4. **Apart from infinity, where is the neutral point?**
5. **If q_A and q_B are separated by a distance $10(\sqrt{2}-1)$ cm, find the position of neutral point.**
6. **Where will the lines which are not meeting at q_B meet?**
7. **Will a positive charge follow the line of force if free to move?**

Sol.

1. No.
2. Number of lines coming from or coming to a charge is proportional to magnitude of charge, so
$$\frac{q_A}{q_B} = \frac{12}{6} = 2$$
3. q_A is positive and q_B is negative.
4. C is the other neutral point.
5. For neutral point $E_A = E_B$
$$\frac{1}{4\pi\varepsilon_0}\frac{q_A}{(l+x)^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_B}{x^2}$$

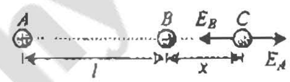


Fig. 1.26

$$\left(\frac{l+x}{x}\right)^2 = \frac{q_A}{q_B} = 2 \Rightarrow x = 10 \text{ cm}$$

6. At infinity.
7. No, as lines of force are curved, the direction of velocity and acceleration will be different. Hence, a charge cannot follow strictly the line of force.

Concept Application Exercise 1.2

1. **a.** A negatively charged particle is placed exactly midway between two fixed particles having equal positive charges. What will happen to the charge:
   i. if it is displaced at right angle to the line joining the positive charges?
   ii. if it is displaced along the line joining the positive charges?
   b. Does the Coulomb force that one charge exerts on other charges change if the other charges are brought nearby? (Yes/No)
2. **a.** Does an electric charge experience a force due to the field produced by itself? (Yes/No)
   b. Two point charges q and $-q$ are placed at a distance d apart. What are the points at which resultant electric field is parallel to line joining the two charges?
3. Two negative charges of a unit magnitude and a positive charge 'q' are placed along a straight line. At what position and value of q will the system be in equilibrium? (Negative charges are fixed.)

4. Fig. 1.27 shows three arrangements of an electron e and two protons p $(D > d)$.
   a. Rank the arrangements according to the magnitude of the net electrostatic force on the electron due to the protons, largest first.

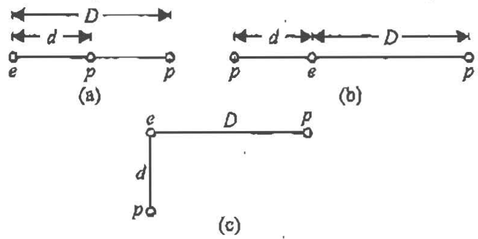


Fig. 1.27

   b. In situation c, is the angle between the net force on the electron and the line labeled horizontal less than or more than 45°?

5. Fig. 1.28 shows two charge particles on an axis. The charges are free to move. At one point, however, a third charged particle can be placed such that all three particles are in equilibrium.

Fig. 1.28

   a. Is that point to the left of the first two particles, to their right, or between them?
   b. Should the third particle be positively or negatively charged?
   c. Is the equilibrium stable or unstable?

6. In Fig. 1.29, a central particle of charge $-q$ is surrounded by two circular rings of charged particles, of radii r and R, with $R > r$. What is the magnitude and direction of the net electrostatic force on the central particle due to the other particles?

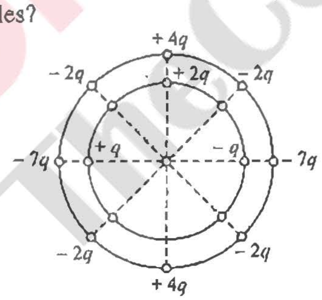


Fig. 1.29

7. Fig. 1.30 shows four situations in which particles of charge $+q$ or $-q$ are fixed in place. In each, the particles on the x-axis are equidistant from the y-axis. The particle on y-axis experiences an electrostatics force F from each of these two particles.
   a. Are the magnitudes F of those forces the same or different?
   b. Is the magnitude of the net force on the particle on y-axis equal to, greater than, or less than $2F$?
   c. Do the x components of the two forces add or cancel?
   d. Do their y components add or cancel?
   e. Is the direction of the net force on the middle particle that of the canceling components or the adding components?
   f. What is the direction of that net force on the middle particle?

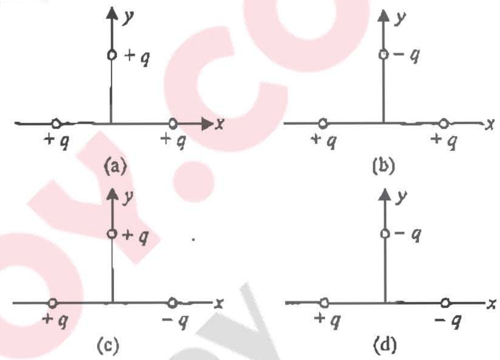


Fig. 1.30

8. Force between two point electric charges kept at a distance 'd' apart in air is F. If these charges are kept at the same distance in water, the force between the charges is F'. The ratio F'/F is equal to ______.

9. Two small balls each having charge q are suspended by two insulating threads of equal length L from a hook in an elevator. The elevator is freely falling. Calculate the angle between the two threads and tension in each thread.

10. Suppose we have a large number of identical particles, very small in size. Any two of them at 10 cm separation repel with a force of 3×10^{-10} N.
   a. If one of them is at 10 cm from a group (of very small size) of n others, how strongly do you expect it to be repelled?
   b. Suppose you measure the repulsion and find it 6×10^{-6} N. How many particles were there in the group?

ELECTRIC FIELD

If we place a single charge q at some point in space, it will experience no force. But if some other charge (say Q) is placed near it, q will start experiencing a force given by

$$F = \frac{kQq}{r^2}$$

Fig. 1.31

Now, question arises, how does Q apply a force on q or how does q know the presence of Q when there is no direct contact between them.

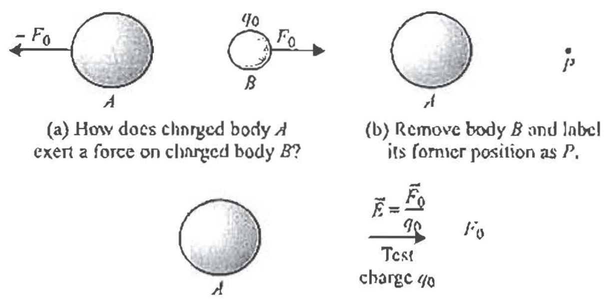
Basically, the force between two charges can be seen as a two step process:

1. Firstly, charge Q will create something around itself known as electric field.

2. Any other charge particle like q if placed at some point in that field will experience a force.

Or we can say that charges interact with each other through electric field.

So, we can define electric field as the space around a charge in which its influence can be felt by any other charged particle.



(a) How does charged body A exert a force on charged body B?

(b) Remove body B and label its former position as P.

(c) Body A sets up an electric field E at point P. E is the force per unit charge exerted.

Fig. 1.32

How to Measure Electric Field

Strength of electric field at a point in space can be measured in two measureable quantities:

1. Electric field intensity denoted by E. It is a vector quantity.
2. Electric field potential denoted by V. It is a scalar quantity.

We will first discuss them separately and then we will see what is the relation between them and how to obtain them from each other.

Electric Field Intensity E

How to find electric field intensity E at a point?

General method: Electric field intensity, E, is a vector quantity. At a point in a given space it has both magnitude and direction. Let us calculate E at some point P created due to some charges around P. Bring another small charge q_0 [test charge, generally positive] at point P. Let this charge experiences a force $\vec{F}$, then we define electric field intensity at P as force experienced per unit test charge (Fig. 1.33).

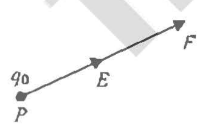


Fig. 1.33

$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}$. The direction $\vec{E}$ will be same as that of $\vec{F}$.

Note: *Q. Why the magnitude of test charge is kept small?*
Ans. *Because otherwise it may disturb the original charge distribution and then we will get electric field due to disturbed configuration and not original.*
Q. What is the minimum possible value of q_0?
Ans. 1.6×10^{-19} C

Unit of E: N/C (newton per coulomb)

Dimensional formula of E: $\frac{\text{Force}}{\text{Charge}} = \frac{MLT^{-2}}{\text{ampere} \times \text{time}}$

$$= \frac{MLT^{-2}}{AT} = [MLT^{-3}A^{-1}]$$

Note: ***If a test charge experiences no force at a point, the electric field at that point must be zero.***

Electric field due to a point charge is illustrated in Fig. 1.34.

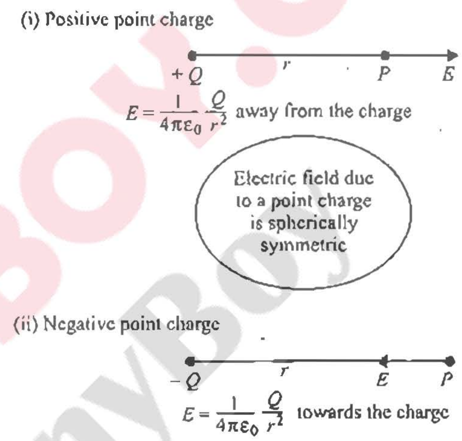


Fig. 1.34

A Point Charge in an Electric Field

What happens if a point charge q is placed at any point in an electric field which is produced by some other stationary charges. Let this electric field is $\vec{E}$. Charge q will experience a force, let this force is $\vec{F}$. Then, value of electric field at that point must be

$$\vec{E} = \frac{\vec{F}}{q} \quad \Rightarrow \quad \vec{F} = q\vec{E}. \text{ This is the force on } q \text{ by } E.$$

Direction of $\vec{F}$: The direction of $\vec{F}$ will be same as of $\vec{E}$ if q is +ve and opposite if q is −ve (Fig. 1.35).

(a) Positive charge q_0 placed in an electric field: force on q_0 is in same direction as $\vec{E}$.

(b) Negative charge q_0 placed in an electric field: force on q_0 is in opposite direction as $\vec{E}$.

Fig. 1.35

Note: ***q has no contribution in $\vec{E}$. A charge particle is not affected due to its own field. It means a charge particle can experience force due to field produced by other charge particles, but not due to field produced by itself.***

Electric Field Intensity due to a Point Charge in Position Vector Form

Electric field at P due to charge Q: $\vec{E} = \dfrac{Q(\vec{r} - \vec{r}_0)}{4\pi\varepsilon_0 |\vec{r} - \vec{r}_0|^3}$

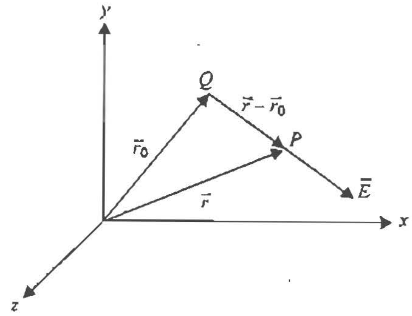


Fig. 1.36

If a charge q is placed at P, then force on this charge by Q:

$$\vec{F} = q\vec{E}$$

$$\Rightarrow \quad \vec{F} = \frac{qQ(\vec{r}-\vec{r}_0)}{4\pi\varepsilon_0 |\vec{r}-\vec{r}_0|^3}$$

Electric Field Intensity due to a Group of Charges

Using the principle of superposition, net field at point P (see Fig. 1.37)

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \cdots + \vec{E}_n$$

$$\Rightarrow \quad \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1^2}\hat{r}_1 + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2^2}\hat{r}_2 + \cdots + \frac{1}{4\pi\varepsilon_0}\frac{q_n}{r_n^2}\hat{r}_n$$

$$\Rightarrow \quad \vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^2}\hat{r}_i$$

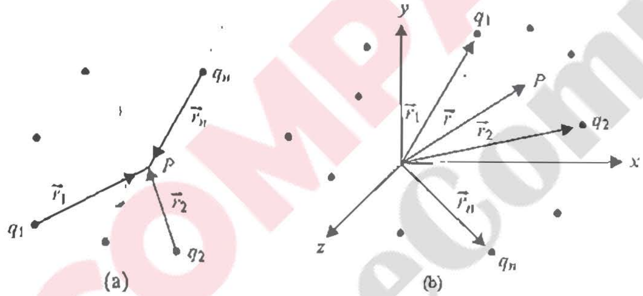


Fig. 1.37

In terms of position vectors:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i(\vec{r}-\vec{r}_i)}{|\vec{r}-\vec{r}_i|^3}$$

Illustration 1.17 **Two point-like charges a and b whose strengths are equal in absolute value are positioned at a certain distance from each other. Assuming the field strength is positive in the direction coinciding with the positive direction of the r axis, determine the signs of the charges for each distribution of the field strength between charges shown in Figs. 1.38(a), (b), (c) and (d).**

Sol.

a. As electric field tends away at a and towards at b, hence there should be + charge at a and negative charge at b, i.e., q_a is '+' and q_b is '−'.

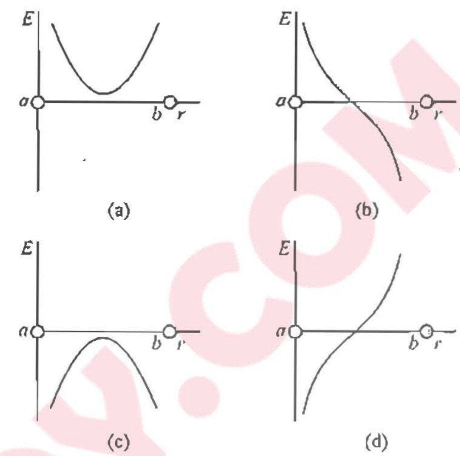


Fig. 1.38

b. The neutral point exists between a and b only when q_a and q_b both are of same sign. As direction of electric field is away from both, so both charges are positive, i.e., q_a is '+' and q_b is '+'.

Similarly, for (c) and (d) in Fig. 1.39:

c. q_a is '−' and q_b is '+'.

d. q_a is '−' and q_b is '−'.

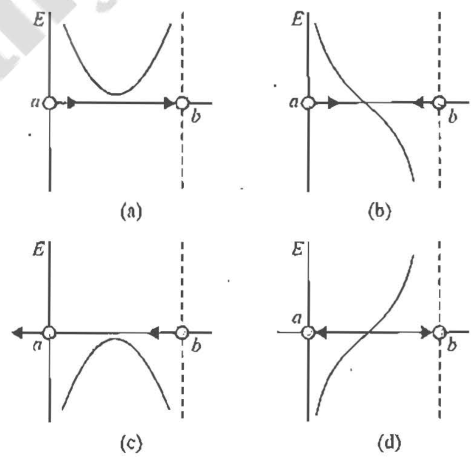


Fig. 1.39

Illustration 1.18 **Two identical positive point charges q are placed on the axis at $x = -a$ and $x = +a$, as shown in Fig. 1.40.**

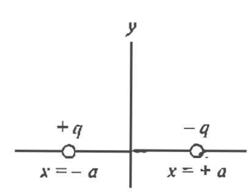


Fig. 1.40

1. Plot the variation of E along the x-axis.

2. Plot the variation of E along the y-axis

Sol.

1. Variation of E along the x-axis: 1.41(a).
2. Variation of E along the y-axis: 1.41(b)

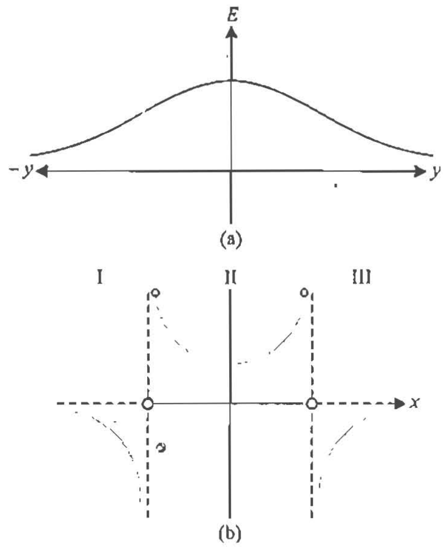


Fig. 1.41

Illustration 1.19 **In Fig. 1.42, determine the point (other than infinity) at which the electric field is zero.**

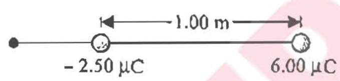


Fig. 1.42

Sol. Electric field will be zero at a point closer to the charge smaller in magnitude. Let at P electric field is zero (see Fig. 1.43). Then

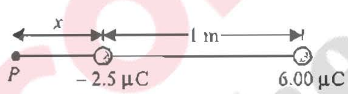


Fig. 1.43

$$\frac{k(2.5 \times 10^{-6})}{x^2} = \frac{k(6 \times 10^{-6})}{(1+x)^2}$$

$$\Rightarrow \quad x = 1.82 \text{ m}$$

Illustration 1.20 **Four charges are arranged as shown in Fig. 1.44. A point P is located at distance r from the centre of the configuration. Assuming $r >> l$, find**

1. **the magnitude of the field at point P.**
2. **the angle of its vector with x-axis.**

Sol. Electric field due to charges placed on y-axis (Fig. 1.45(a))

$$E_y = 2E_1 \sin\theta = 2\,\frac{1}{4\pi\varepsilon_0}\,\frac{q}{\left(r^2 + \left(\frac{l}{2}\right)^2\right)}\,\frac{l/2}{\left(r^2 + \frac{l^2}{4}\right)^{1/2}}$$

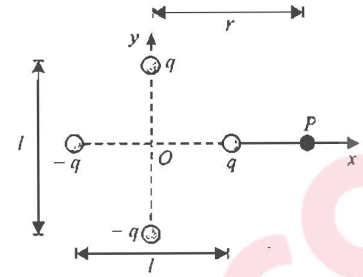


Fig. 1.44

$$E_y = \frac{1}{4\pi\varepsilon_0}\,\frac{q\,l}{\left(r^2 + \frac{l^2}{4}\right)^{3/2}} = \frac{1}{4\pi\varepsilon_0}\,\frac{ql}{r^3} \text{ (as } r >> l)$$

Electric field due to charges placed on x-axis (Fig. 1.45(b))

$$E_x = E_3 - E_4 = \frac{1}{4\pi\varepsilon_0}\,\frac{q}{\left(r - \frac{l}{2}\right)^2} - \frac{1}{4\pi\varepsilon_0}\,\frac{q}{\left(r + \frac{l}{2}\right)^2}$$

$$= \frac{1}{2\pi\varepsilon_0}\,\frac{ql}{r^3}$$

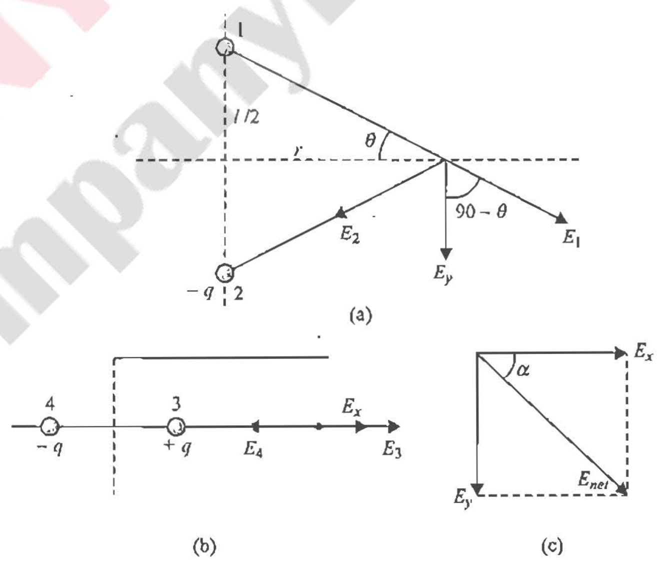


Fig. 1.45

$$\Rightarrow \quad E_{\text{net}} = \sqrt{E_x^2 + E_y^2} = \sqrt{5}\,\frac{ql}{4\pi\varepsilon_0 r^3}$$

The angle E_{net} makes with x-axis (Fig. 1.45(c))

$$\alpha = \tan^{-1}\left(\frac{E_y}{E_x}\right) = \tan^{-1}\left(\frac{1}{2}\right) \text{ below } x\text{-axis.}$$

Illustration 1.21 **A uniform electric field E exists between two metal plates. The plate length is l and the separation of the plates is d.**

1. **An electron and a proton start from the negative plate and positive plate, respectively, and go to opposite plates. Which one of them wins this race?**

2. An electron and a proton start moving parallel to the plates towards the other end from the midpoint of the separation of plates at one end of the plates. Which of the two will have greater deviation when they come out of the plates if they start with the
a. same initial velocity,
b. same initial kinetic energy, and
c. same initial momentum.

Sol.

1. $a_e = \dfrac{eE}{m_e}, a_p = \dfrac{eE}{m_p}; d = \dfrac{1}{2}at^2$

$$t = \sqrt{\frac{2d}{a}} \text{ or } t = \sqrt{\frac{2md}{eE}}; \text{ So, we have } \frac{t_e}{t_p} = \sqrt{\frac{m_e}{m_p}}.$$

As $m_e < m_p$, therefore $t_e < t_p$. Hence, electron will take less time, i.e., the electron wins the race.

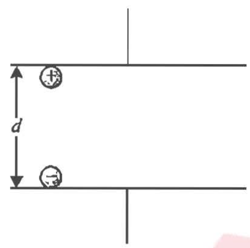


Fig. 1.46

2. Time to cross the plates $t = \dfrac{l}{u}$

Deviation: $y = \dfrac{1}{2}at^2 = \dfrac{1}{2}\dfrac{eE}{m}\left(\dfrac{l}{u}\right)^2$

$$\frac{y_e}{y_p} = \frac{m_p}{m_e}\cdot\left(\frac{u_p}{u_e}\right)^2 \quad \text{(i)}$$

a. If $u_p = u_e$, then $\dfrac{y_e}{y_p} = \dfrac{m_p}{m_e}$.

As $m_p > m_e$, therefore $y_e > y_p$.
Hence, deviation of electron will be more.

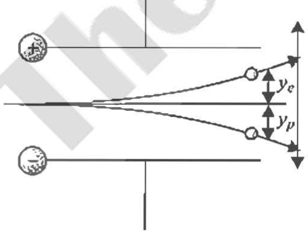


Fig. 1.47

b. From equation (i), $\dfrac{y_e}{y_p} = \dfrac{m_p u_p^2}{m_e u_e^2} = 1$ (as given)

Hence deviation of both electron and proton will be same.

c. From (i), $\dfrac{y_e}{y_p} = \left(\dfrac{m_p u_p}{m_e u_e}\right)^2 \dfrac{m_e}{m_p} = \dfrac{m_e}{m_p}$

As $m_e < m_p$, hence $y_e < y_p$.

Hence, the deviation of proton will be more.

Illustration 1.22 **A charge 10^{-9} coulomb is located at gin in free space and another charge Q at (2, 0, 0). If x-component of the electric field at (3, 1, 1) is zero, calcu the value of Q. Is the y-component zero at (3, 1, 1)?**

Sol. The electric field due to a point charge q_i at position vector from is given by

$$\vec{E}_i = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_i^3}\vec{r}_i$$

Here: $\vec{r}_1 = (3-0)\hat{i} + (1-0)\hat{j} + (1-0)\hat{k} = 3\hat{i} + \hat{j} +$

with $r_1 = \sqrt{(3^2+1^2+1^2)} = \sqrt{11}$ m

$$\vec{r}_2 = (3-2)\hat{i} + (1-0)\hat{j} + (1-0)\hat{k} = \hat{i} + \hat{j} + \hat{k}$$

with $r_2 = \sqrt{(1^2+1^2+1^2)} = \sqrt{3}$ m

So,

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{10^{-9}}{(11)^{3/2}}[3\hat{i} + \hat{j} + \hat{k}] \text{ and}$$

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(3)^{3/2}}[\hat{i} + \hat{j} + \hat{k}]$$

Hence, net field:

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\left[\left(\frac{3\times10^{-9}}{11\sqrt{11}} + \frac{Q}{3\sqrt{3}}\right)\hat{i} + \left(\frac{10^{-9}}{11\sqrt{11}} + \frac{Q}{3\sqrt{3}}\right)\hat{j} + \left(\frac{10^{-9}}{11\sqrt{11}} + \frac{Q}{3\sqrt{3}}\right)\hat{k}\right]$$

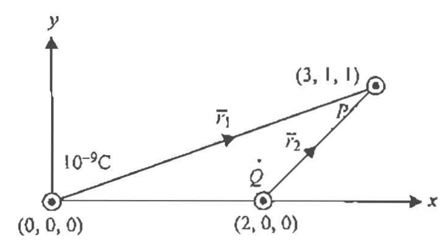


Fig. 1.48

According to given problem:

$$E_x = 0, \text{ i.e., } \frac{1}{4\pi\varepsilon_0}\left[\frac{3\times10^{-9}}{11\sqrt{11}} + \frac{Q}{3\sqrt{3}}\right] = 0$$

$$\text{So, } Q = -\left[\frac{3}{11}\right]^{3/2} \times 3\times10^{-9} \text{ coulomb}$$

And for this value of Q

$$E_y = \frac{1}{4\pi\varepsilon_0}\left[\frac{10^{-9}}{11\sqrt{11}} - \frac{(3/11)^{3/2}\times3\times10^{-9}}{3\sqrt{3}}\right]$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{2\times10^{-9}}{11\sqrt{11}} \neq 0, \text{ i.e., } E_y \text{ is not zero.}$$

Lines of Force

This idea was given by Michael Faraday. The lines of provide a nice idea to visualise the pattern of electric fie a given space. We assume that space around a charged bo filled with some lines known as electric lines of force. 7

lines of force are drawn in space in such a way that tangent to the line at any point gives the direction of electric field at that point. It has been found quite convenient to visualize the electric field in terms of lines of force.

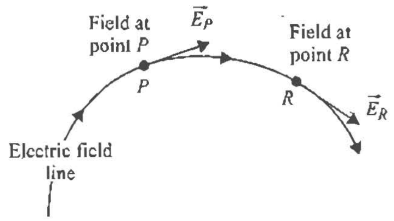


Fig. 1.49

Properties of Electric Lines of Force

- Electric lines of force start (or diverge out) from a positive charge and end (or converge) on a negative charge.
- The tangent drawn at any point on the line of force gives the direction of force acting on a positive charge at that point (see Fig. 1.50).
- In S.I. system of units, the number of electric lines of force originating or terminating on a charge of q coulomb is equal to $\frac{q}{\varepsilon_0}$, i.e., $\left(\frac{q}{\varepsilon_0}\right)$ electric lines are associated with unit charge.

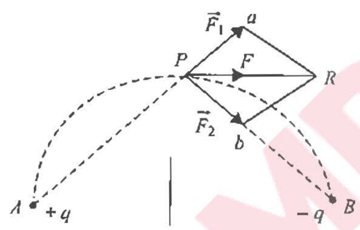


Fig. 1.50

- Two electric lines of force never cross each other, because if they do so then at the point of intersection, intensity will have two directions which is absurd.
- Electric lines of force can never be closed loops, as a line can never start and end on the same charge.
- The electric lines of force do not pass through a conductor as electric field inside a conductor is always zero.
- Lines of force have a tendency to contract longitudinally like a stretched elastic string producing attraction between opposite charges and repel each other laterally resulting in repulsion between similar charges and edge effect (curving of lines of force near the edges of a charged conductor).
- Electric lines of force end or start normally on the surface of a conductor.
- Tangent to the line of force at a point in an electric field gives the direction of intensity or force or acceleration which a positive charge will experience there but not the direction of motion always, so a positive point charge free to move in an electric field may or may not follow the line of force. It will follow the line of force if it is a straight line (as direction of velocity and acceleration will be same) and will not follow the line if it is curved as the direction of motion will be different from that of acceleration and the particle will move neither in the direction of motion nor acceleration (line of force).

The use of the electric lines of force is that we can compare the intensities at two points just by looking at the distribution of lines of force. Where the field lines are close together, E is large and where they are far apart, E is small.

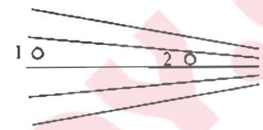
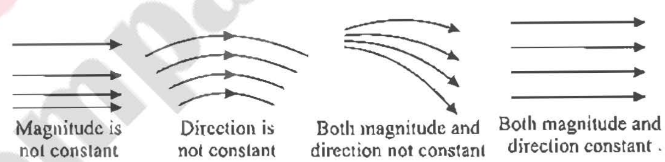


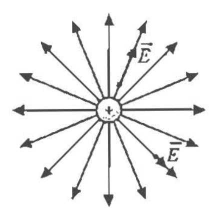
Fig. 1.51

As an example in the figure electric lines of forces are shown. At point 2 the electric field intensity will be greater in comparison to that at point 1.

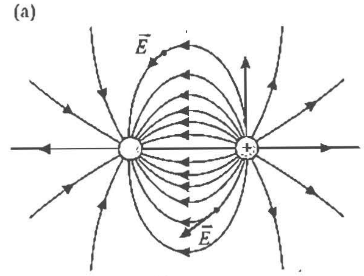
DIFFERENT PATTERNS OF ELECTRIC FIELD LINES



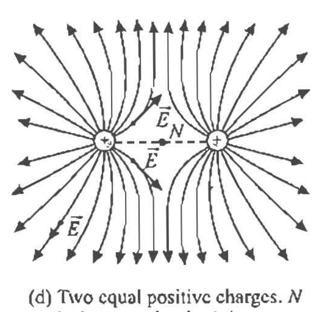
(a)



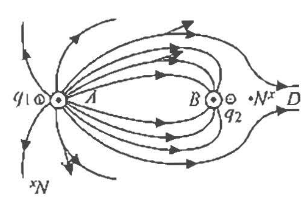
(b) A single positive charge



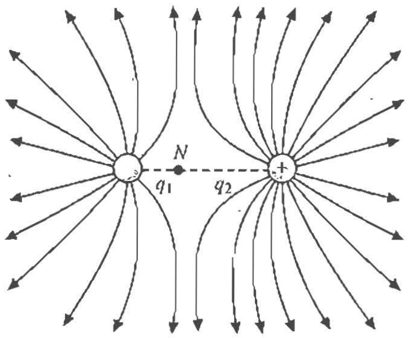
(c) A positive charge and a negative charge of equal magnitude (an electric dipole)



(d) Two equal positive charges. N is the neutral point lying at the middle of the charges.



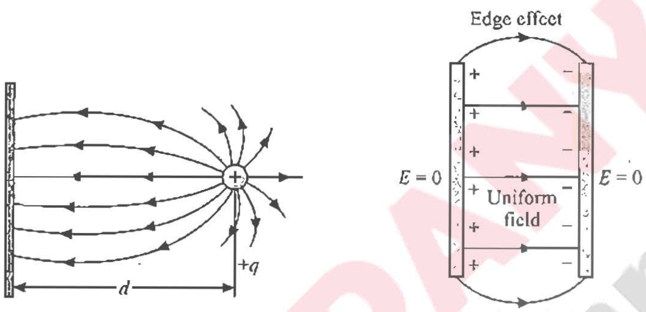
(e) A is a positive charge and B a negative charge of different magnitudes $(|q_2| < |q_1|)$



(f) Two positive charges of different magnitudes ($q_1 < q_2$)

Fig. 1.52

Note: ***Neutral point (N) is the location where the net electric field due to charges is zero. It lies near the charge of smaller magnitude.***



Parallel metal plates having dissimilar charges

Fig. 1.53

Illustration 1.23 **Fig. 1.54 shows the sketch of field lines for two point charges $2Q$ and $-Q$. The pattern of field lines can be deduced by considering the following points:**

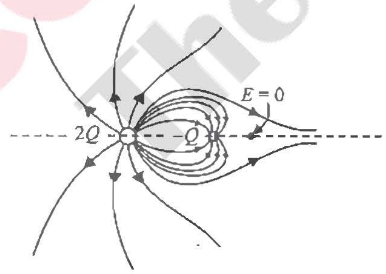


Fig. 1.54

Sol.

1. Symmetry: For every point above the line joining the two charges, there is an equivalent point below it. Therefore, the pattern must be symmetrical about the line joining the two charges.
2. Near field: Very close to a charge, its field predominates. Therefore, the lines are radial and spherically symmetric.
3. Far field: Far from the system of charges, the pattern should look like that of a single point charge of value $(2Q - -Q) = +Q$, i.e., the lines should be radially outward.
4. Null point or neutral point: There is one point at which $E = 0$. No lines should pass through this point.
   - Neutral point lies near the position of charge of smaller magnitude.
5. Number of lines: Twice as many lines leave $+2Q$ as enter $-Q$.

Note: ***Excess lines from $2Q$ charge will meet at infinity.***

Illustration 1.24 **Charges $+q$ and $-2q$ are fixed a distance d apart as shown in figure.**

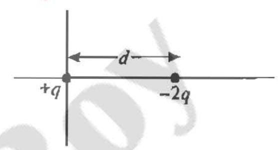


Fig. 1.55

1. **Sketch roughly the pattern of electric field lines, showing position of neutral point.**
2. **Where should a charge particle q be placed so that it experiences no force?**

Sol. Let net force on q at P is zero, then

$$\frac{kq^2}{x^2} = \frac{kq\,2q}{(d+x)^2} \Rightarrow x = \frac{d}{\sqrt{2}-1}$$

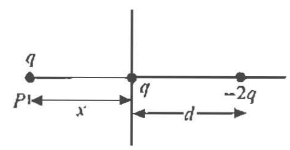


Fig. 1.56

P is the neutral potential where electric field will be zero.

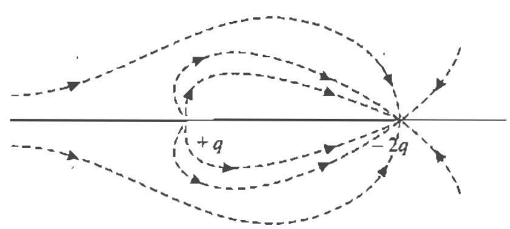


Fig. 1.57

FIELD OF RING CHARGE

A ring-shaped conductor with radius a carries a total charge Q uniformly distributed around it. Let us calculate the electric field at a point P that lies on the axis of the ring at a distance x from its center.

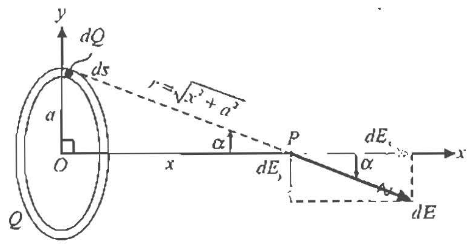


Fig. 1.58

As shown in the figure, we imagine the ring divided into infinitesimal segments each of length ds. Each segment has charge dQ and acts as a point charge source of electric field. Let $d\vec{E}$ be the electric field from one such segment; the net electric field at P is then the sum of all contributions $d\vec{E}$ from all the segments that make up the ring. (This same technique works for any situation in which charge is distributed along a line or a curve.) The calculation of $\vec{E}$ is greatly simplified because the field point P is on the symmetry axis of the ring. If we consider two ring segments at the top and bottom of the ring, we see that the contributions $d\vec{E}$ to the field at P from these segments have the same x-component but opposite y-components. Hence, the total y-component of field due to this pair of segments is zero. When we add up the contributions from all such pairs of segments, the total field $\vec{E}$ will have only a component along the ring's symmetry axis (the x-axis), with no component perpendicular to that axis (that is, no y-component or z-component). So, the field at P is described completely by its x-component E_x.

To calculate E_x note that the square of the distance r from a ring segment to the point P is $r^2 = x^2 + a^2$. Hence, the magnitude of this segment's contribution to the electric field at P is

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{x^2+a^2}$$

Using $\cos\alpha = \frac{x}{r} = \frac{x}{(x^2+a^2)^{1/2}}$, the component dE_x of this field along the x-axis is

$$dE_x = dE\cos\alpha = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{x^2+a^2}\frac{x}{\sqrt{x^2+a^2}} = \frac{1}{4\pi\varepsilon_0}\frac{x\,dQ}{(x^2+a^2)^{3/2}}$$

To find the total x-component E_x of the field at P, we integrate this expression over all segments of the ring:

$$E_x = \int \frac{1}{4\pi\varepsilon_0}\frac{x\,dQ}{(x^2+a^2)^{3/2}}.$$

Since x does not vary as we move from point to point around the ring, all the factors on the right side except dQ are constant and can be taken outside the integral. The integral of dQ is just the total charge Q and we finally get

$$\vec{E} = E_x\hat{i} = \frac{1}{4\pi\varepsilon_0}\frac{xQ}{(x^2+a^2)^{3/2}}\hat{i} \qquad \text{(i)}$$

- Electric field is directed away from positively charged ring.
- For $x = 0$, $E = 0$. This conclusion may be arrived at by the symmetry consideration.
- At a large distance from the ring, the electric field will be zero. Hence, it should have certain maximum value between $x = 0$ and $x = \infty$ (or $x = -\infty$).
- If we maximize equation (i), we can get the value of x_m as well as E_{max}.

  For maximum value of E_x:

$$\frac{d}{dx}\left\{\frac{1}{4\pi\varepsilon_0}Q\frac{x}{(x^2+a^2)^{3/2}}\right\} = 0$$

$$\frac{(x^2+a^2)^{3/2}.1 - x\frac{3}{2}.(x^2+a^2)^{1/2}.2x}{(x^2+a^2)^3} = 0$$

$$(x^2+a^2) - 3x^2 = 0 \quad\Rightarrow\quad x = \pm\frac{a}{\sqrt{2}}$$

and the maximum value of the electric field is

$$E_{a(\max)} = \frac{1}{4\pi\varepsilon_0}\left(\frac{2Q}{3\sqrt{3}R^2}\right)$$

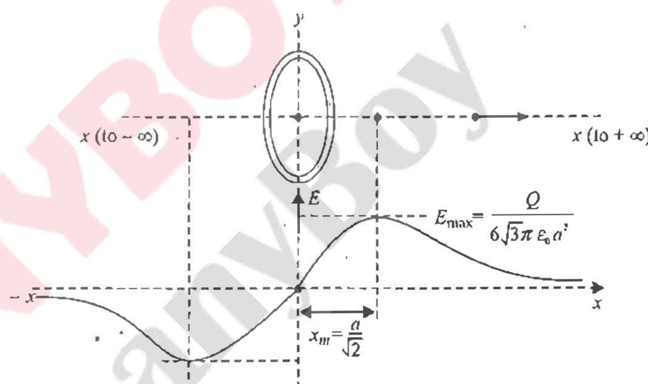


Fig. 1.59

Illustration 1.25 **If we place a negative charge (of magnitude $-q$ and mass m) at the center of a charged ring and slightly displace it along the axis of ring and release. Examine whether it will perform simple harmonic motion. If yes, then find the time period of oscillation of the particle.**

Sol. $E = \frac{kQx}{(a^2+x^2)^{3/2}}$

Force on charge $F = -qE = -\frac{kqQx}{(a^2+x^2)^{3/2}}$

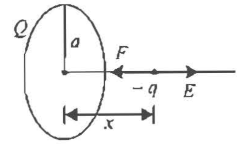


Fig. 1.60

$$a = \frac{F}{m} = \frac{-kqQx}{m(a^2+x^2)^{3/2}}$$

Hence, acceleration is opposite to displacement, so motion will be oscillatory.

But a is not directly proportional to x so motion is not SHM.

If $x << a$, then $a = -\frac{kqQx}{ma^3}$

Here $a \propto x$, so the motion will be SHM. Comparing with $a = -\omega^2 x$

We get $\omega = \sqrt{\frac{kqQ}{ma^3}}$

Illustration 1.26 **Two identical point charges having magnitude q each are placed as shown in the figure.**

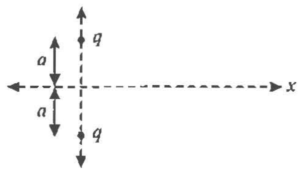


Fig. 1.61

1. **Plot the variation of electric field on x-axis.**
2. **Where will the magnitude of electric field be maximum on x-axis? Find the maximum value of electric field on x-axis.**
3. **If we place a negative charge (of magnitude $-q$ and mass m) at the mid point of charges and displaced along x-axis, examine whether it will perform simple harmonic motion. If yes, then find the time period of oscillation of the particle.**

Sol. 1.

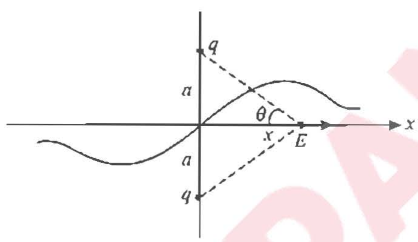


Fig. 1.62

2. Field at $x = x$: $E = 2\left[\dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{(a^2+x^2)}\right]\cos\theta$

$$\Rightarrow \quad E = \frac{q}{2\pi\varepsilon_0}\frac{x}{(a^2+x^2)^{3/2}}$$

For E to be maximum, $\dfrac{dE}{dx} = 0$

Solve to get $x = \pm\dfrac{a}{\sqrt{2}} \Rightarrow E_{\max} = \dfrac{q}{3\sqrt{3}\pi\varepsilon_0 a^2}$

3. Force on particle: $F = -qE = \dfrac{-q^2}{2\pi\varepsilon_0}\dfrac{x}{(a^2+x^2)^{3/2}}$

For $x << a$, particle will execute SHM with time period

$$T = 2\pi\sqrt{\frac{2\pi\varepsilon_0 m a^3}{q^2}}$$

Positive electric charge Q is distributed uniformly along a line, lying along the y-axis. Let us find the electric field at point D on the x-axis at a distance r_0 from the origin.

We divide the line charge into infinitesimal segments, each of which acts as a point charge; let the length of a typical segment at height l be dl. If the charge is distributed uniformly with the linear charge density λ, then the charge dQ in a segment of length dl is $dQ = \lambda\, dl$. At point D, the differential electric field dE created by this element,

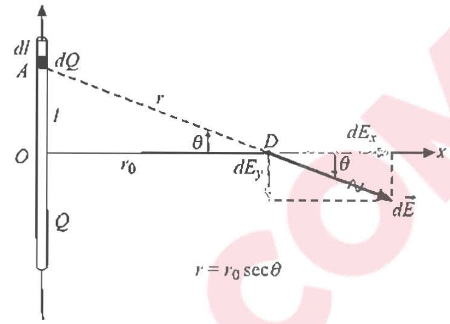


Fig. 1.63

$$dE = \frac{dQ}{4\pi\varepsilon_0 r^2} = \frac{\lambda\, dl}{4\pi\varepsilon_0 r^2} = \frac{\lambda\, dl}{4\pi\varepsilon_0 r_0^2\sec^2\theta} \qquad \text{(i)}$$

In triangle AOD; $OA = OD\tan\theta$, i.e.,

$l = r_0\tan\theta$; Differentiating this equation with respect to θ; $dl = r_0\sec^2\theta\, d\theta$

Substituting the value of dl in equation (i);

$$dE = \frac{\lambda\, d\theta}{4\pi\varepsilon_0 r_0}$$

Field dE has components dE_x, dE_y given by

$$dE_x = \frac{\lambda\cos\theta\, d\theta}{4\pi\varepsilon_0 r_0} \text{ and } dE_y = \frac{\lambda\sin\theta\, d\theta}{4\pi\varepsilon_0 r_0}$$

On integrating expression for dE_x and dE_y in limits $\theta = -\dfrac{\pi}{2}$ to $\theta = +\dfrac{\pi}{2}$, we obtain E_x and E_y. Note that as the length of wire increases, the angle θ increases; for a very long wire (infinitely long wire), it approaches $\pi/2$.

$$E_x = \int_{-\pi/2}^{+\pi/2}\frac{\lambda\cos\theta\, d\theta}{4\pi\varepsilon_0 r_0} = \frac{\lambda}{2\pi\varepsilon_0 r_0} \text{ and}$$

$$E_y = \int_{-\pi/2}^{+\pi/2}\frac{\lambda\sin\theta\, d\theta}{4\pi\varepsilon_0 r_0} = 0$$

Thus, $E = E_x = \dfrac{\lambda}{2\pi\varepsilon_0 r_0}$

Note: ***Using a symmetry argument, we could have guessed that E_y would be zero; if we place a positive test charge at D, the upper half of the line of charge pushes downward on it, and the lower half pushes up with equal magnitude.***

- If the wire has finite length and the angle subtended by ends of wire at a point are θ_1 and θ_2, the limits of integration would change.

$$E_x = \int_{-\theta_1}^{+\theta_2}\frac{\lambda\cos\theta\, d\theta}{4\pi\varepsilon_0 r_0}$$

$$= \frac{\lambda}{4\pi\varepsilon_0 r_0}(\sin\theta_1 + \sin\theta_2)$$

$$E_y = \int_{-\theta_1}^{+\theta_2}\frac{\lambda\sin\theta\, d\theta}{4\pi\varepsilon_0 r_0}$$

$$= \frac{\lambda}{4\pi \varepsilon_0 r_0}(\cos\theta_1 - \cos\theta_2)$$

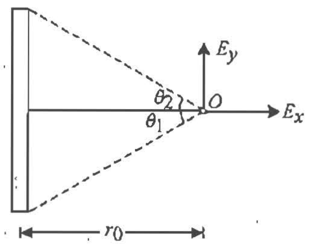


Fig. 1.64

- If we wish to determine field at the end of a long wire, we may substitute $\theta_1 = 0$ and $\theta_2 = \pi/2$ in the expressions for E_x and E_y.

$$E_x = \frac{\lambda}{4\pi\varepsilon_0 r_0}\left[\sin(0) + \sin\left(\frac{\pi}{2}\right)\right] = \frac{\lambda}{4\pi\varepsilon_0 r_0} \text{ and}$$

$$E_y = \frac{\lambda}{4\pi\varepsilon_0 r_0}\left[\cos(0) - \cos\left(\frac{\pi}{2}\right)\right] = \frac{\lambda}{4\pi\varepsilon_0 r_0}$$

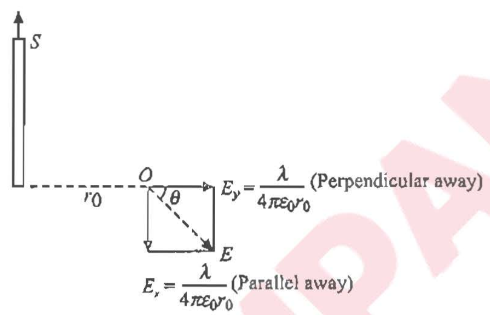


Fig. 1.65

Magnitude of resultant field $\vec{E}$:

$$|\vec{E}| = \sqrt{E_x^2 + E_y^2} = \frac{\sqrt{2}\lambda}{4\pi\varepsilon_0 r_0}$$

$\vec{E}$ makes an angle θ with the x-axis, where $\tan\theta = \frac{|E_y|}{|E_x|} = 1$; $\theta = 45°$

FIELD OF UNIFORMLY CHARGED DISK

Let us find the electric field caused by a disk of radius R with a uniform positive surface charge density (charge per unit area) σ, at a point along the axis of the disk a distance x from its center.

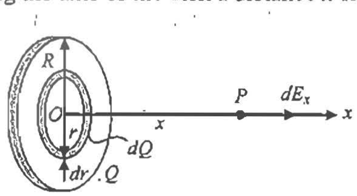


Fig. 1.66

The situation is shown in Fig. 1.66. We can represent this charge distribution as a collection of concentric rings of charge. We already know how to find the field of a single ring on its axis of symmetry, so all we have to do is to add the contribution of all the rings. As shown in the figure, a typical ring has charge dQ, inner radius r and outer radius $r + dr$. Its area dA is approximately equal to its width dr times its circumference $2\pi r$, or $dA = 2\pi r\, dr$. The charge per unit area is $\sigma = \frac{dQ}{dA}$, so the charge of ring is $dQ = \sigma(2\pi r\, dr)$, or $dQ = 2\pi\sigma r\, dr$. The field component dE_x at point P due to charge dQ of a ring of radius r

$$dE_x = \frac{1}{4\pi\varepsilon_0}\frac{(2\pi\sigma r dr)x}{(x^2 + r^2)^{3/2}}$$

To find the total field due to all the rings, we integrate dE_x over r. To include the whole disk, we must integrate from 0 to R (not from $-R$ to R) :

$$E_x = \int dE_x = \int_0^R dE_x = \int_0^R \frac{1}{4\pi\varepsilon_0}\frac{(2\pi\sigma r dr)x}{(x^2 + r^2)^{3/2}}$$

Remember that x is a constant during the integration and that the integration variable is r. The integral can be evaluated by use of the substitution $z = x^2 + r^2$. We will let you work out the details; the result is

$$E_x = \frac{\sigma x}{2\varepsilon_0}\left[-\frac{1}{\sqrt{x^2 + R^2}} + \frac{1}{x}\right] = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{(x^2 + R^2)}}\right] \quad \text{(i)}$$

In this figure, the charge is assumed to be positive. At a point on the symmetry axis of a uniformly charged ring, the electric field due to the ring has no components perpendicular to the axis. Hence, at point P in the figure, $dE_y = dE_z = 0$ for each ring, and thus the total field has $E_y = E_z = 0$.

Again, we can ask what happens if the charge distribution gets very large. Suppose we keep increasing the radius R of the disk, simultaneously adding charge so that the surface charge density σ (charge per unit area) is constant. In the limit that R is much larger than the distance x of the field point from the disk ($R >> x$), i.e., the situation becomes the electric field near infinite plane sheet of charge.

From (i)

$$E_x = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{x\sqrt{1 + \frac{R^2}{x^2}}}\right] = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{1}{\sqrt{1 + \frac{R^2}{x^2}}}\right];$$

As $R >> x$, then the term $\frac{1}{\sqrt{1 + \frac{R^2}{x^2}}} \to 0$

And we get $E_x = \frac{\sigma}{2\varepsilon_0}$

Our final result does not contain the distance x from the plane. This is correct but rather surprising result.

It means:

- That the electric field produced by an infinite plane sheet of charge is independent of the distance from the sheet.
- Thus, the field is uniform; its direction is everywhere perpendicular to the sheet and away from it.

- Infinite plane sheet of charge is a hypothetical case. In real practice, there is no such infinite plane sheet of charge. Again, there is no such thing as an infinite sheet of charge, but if the dimensions of the sheet are much larger than the distance x of the observation point P from the sheet, the field is very nearly the same as for an infinite sheet.

FIELD OF TWO OPPOSITELY CHARGED SHEETS

Two infinite plane sheets are placed parallel to each other, separated by a distance d (as shown in figure). The lower sheet has a uniform positive surface charge density σ, and the upper sheet has a uniform negative surface charge density $-\sigma$ with the same magnitude. Let us find the electric field between the two sheets, above the upper sheet and below the lower sheet.

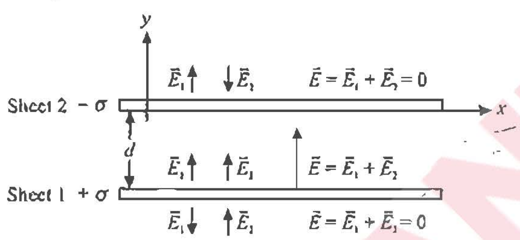


Fig. 1.67

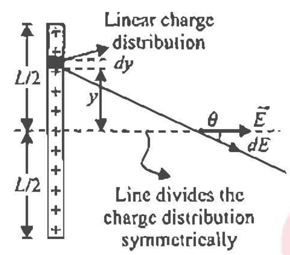
The situation described in this example in an idealization of two finite, oppositely charged sheets, like the plates shown in the figures. If the dimensions of the sheets are large in comparison to the separation d, then we can to good approximation consider the sheets to be infinite in extent. We know the field due to a single infinite plane sheet of charge. We can then find the total field by using the principle of superposition of electric fields. Let sheet 1 be the lower sheet of positive charge, and let sheet 2 be the upper sheet of negative charge; the fields due to each sheet are $\vec{E}_1$ and $\vec{E}_2$, respectively, and both have the same magnitude at all points, no matter how far from either sheet, i.e., $E_1 = E_2 = \dfrac{\sigma}{2\varepsilon_0}$.

At all points, the direction of $\vec{E}_1$ is away from the positive charge of sheet 1, and the direction of $\vec{E}_2$ is towards the negative charge of sheet 2. These fields, as well as the x- and y-axes, are shown in figure. At points between the sheets, the fields at each other and at points above the upper sheet or below the lower sheet cancel each other. Thus, the total field is

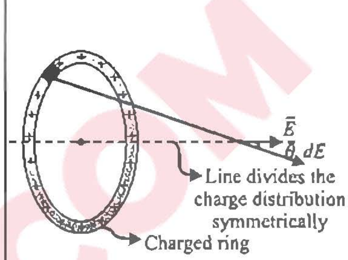
$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \begin{cases} 0 & \text{above the upper sheet} \\ \dfrac{\sigma}{\varepsilon_0}\hat{j} & \text{between the sheets} \\ 0 & \text{above the upper sheet} \end{cases}$$

Because we considered the sheets to be infinite, our result does not depend on the separation d.

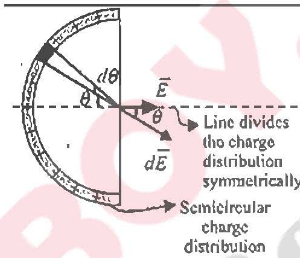
Symmetry plays very important role in problem solving. Electric field is in the direction along the line which divides the charge distribution symmetrically.



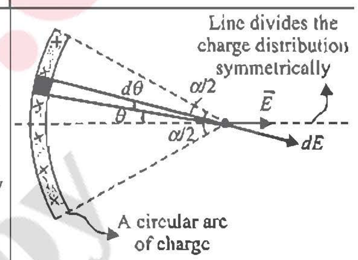
$E_{net} = \int dE \cos\theta$



$E_{net} = \int dE \cos\theta$

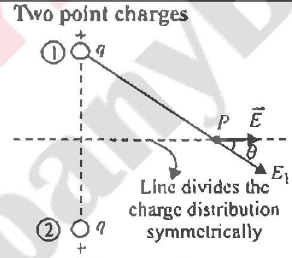


$E_{net} = \int dE \cos\theta$



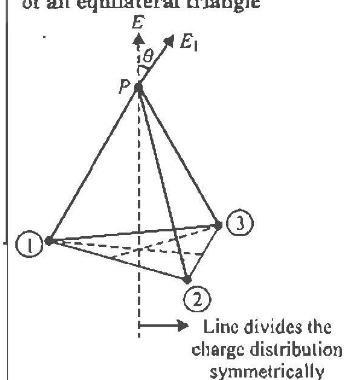
$E_{net} = \int dE \cos\theta$

Two point charges



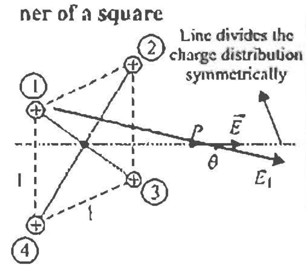
Here, $|\vec{E}_1| = |\vec{E}_2|$
$E_{net} = 2|E_1|\cos\theta$

Three point charges at the corner of an equilateral triangle



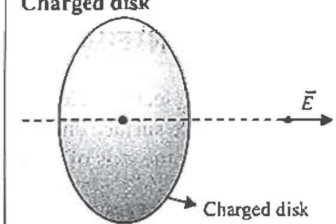
Here, electric field at P due to charges (1), (2) and (3) are equal, i.e., $|\vec{E}_1| = |\vec{E}_2| = |\vec{E}_3|$.
Hence, $E_{net} = 3\,|\vec{E}_1|\cos\theta$

Four point charges at the corner of a square

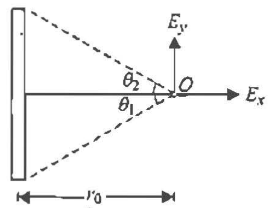
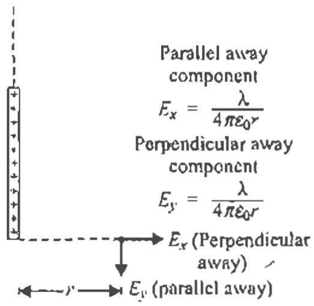
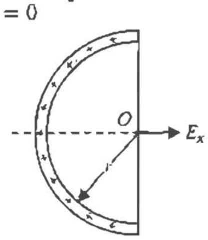
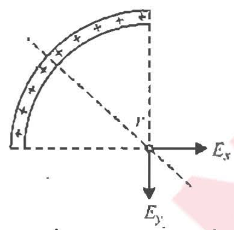
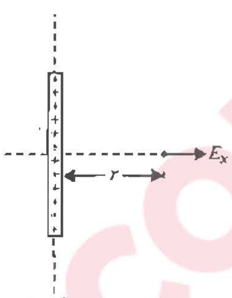
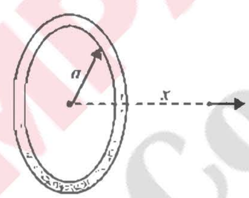
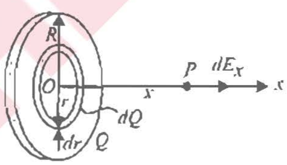
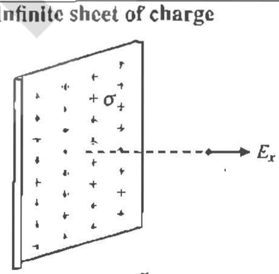


The electric field at point P due to charges (1), (2), (3) and (4), $|\vec{E}_1| = |\vec{E}_2| = |\vec{E}_3| = |\vec{E}_4|$
Hence net electric field at P
$|\vec{E}_{net}| = 4\,|\vec{E}_1|\cos\theta$

Charged disk



Some Useful Results

A charged rod of fixed length having charge density λ  $E_x = \frac{\lambda}{4\pi\varepsilon_0 r_0}(\sin\theta_1 + \sin\theta_2)$ $E_y = \frac{\lambda}{4\pi\varepsilon_0 r_0}(\cos\theta_1 - \cos\theta_2)$	**Semi-infinite rod having charge density λ**  Parallel away component $E_x = \frac{\lambda}{4\pi\varepsilon_0 r}$ Perpendicular away component $E_y = \frac{\lambda}{4\pi\varepsilon_0 r}$ E_x (Perpendicular away) E_y (parallel away)
Semicircular ring having charge density λ $E_x = \frac{\lambda}{2\pi\varepsilon_0 r}$ $E_y = 0$ 	**Quarter circular ring having charge density λ**  $E_x = \frac{\lambda}{4\pi\varepsilon_0 r} \Rightarrow E_y = \frac{\lambda}{4\pi\varepsilon_0 r}$
Infinite line charge  $E_x = \frac{\lambda}{2\pi\varepsilon_0 r}$ $E_y = 0$	**Charged ring**  $E_x = \frac{kQx}{(x^2 + a^2)^{3/2}}$ $\Rightarrow \quad E_y = 0$
Charged disk  $E_x = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{(x^2 + R^2)}}\right]$; $E_y = 0$	**Infinite sheet of charge**  $E_x = \frac{\sigma}{2\varepsilon_0}$ $E_y = 0$

Concept Application Exercise 1.3

1. A particle with positive charge Q is held fixed at the origin. A second particle with positive charge q is fired at the first particle, and follows the trajectory as shown in the figure. Is the angular momentum of second particle constant about some axis? Why or why not? Give reason to support your answer.

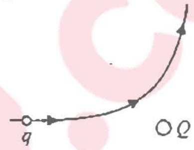


Fig. 1.68

2. Figure shows some of the electric field lines due to three point charges arranged along the vertical axis. All three charges have the same magnitude.
   a. What are the signs of each of the three charges? Explain your reasoning.
   b. At what point(s) is the magnitude of the electric field the smallest ? Explain your reasoning. Explain how the fields produced by each individual point charge combine to give a small net field at this point or points.

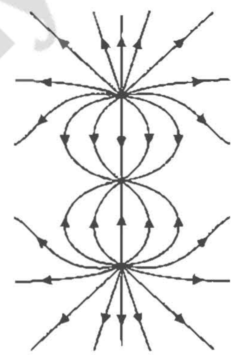


Fig. 1.69

3. Two point charges Q and $4Q$ are fixed at a distance of 12 cm from each other. Sketch lines of force and locate the neutral point, if any.
4. Is an electric field of the type shown by the electric lines in the Fig. 1.70 below physically possible?

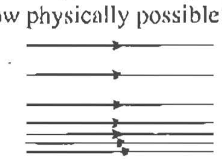


Fig. 1.70

5. Figure 1.71 shows three electric field lines. What is the direction of the electrostatic force on a positive test charge placed at
   a. points A and B?
   b. At which point, A or B, will the acceleration of the test charge be greater if the charge is released?

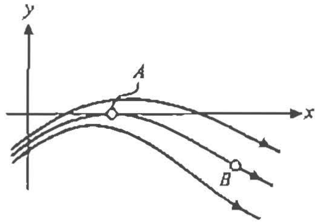


Fig. 1.71

6. A thin metallic spherical shell contains a charge Q on it. A point charge q is placed at the center of the shell and another charge q_1 is placed outside it as shown in figure. All the three charges are positive. Find the force on the charge

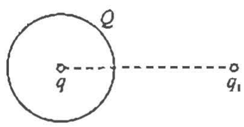


Fig. 1.72

a. at center due to all charges.
b. at center due to shell.

7. In Fig. 1.73, two particles each of charge $-q$, are arranged symmetrically about the y-axis; each producing an electric field at point P on y-axis.

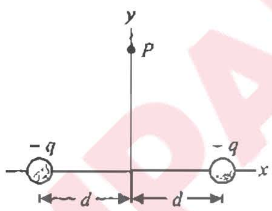


Fig. 1.73

a. Are the magnitude of the fields at P equal?
b. Is each electric field directed toward or away from the charge producing it?
c. Is the magnitude of the net electric field at P equal to the sum of the magnitudes E of the two field vectors (is it equal to $2E$)?
d. Do the x-components of those two field vectors add or cancel?
e. Do their y-components add or cancel?
f. Is the direction of the net field at P that of the canceling components or the adding components?
g. What is the direction of the net field?

8. In Fig. 1.74(a), a plastic rod in the form of circular arc with charge $+Q$ uniformly distributed on it produces an electric field of magnitude E at the center of curvature (at the origin). In figures (b), (c), and (d) more circular rods with identical uniform charges $+Q$ are added until the circle is complete. A fifth arrangement (which would be labeled e) is like that in d except that the rod in the fourth quadrant has charge $-Q$. Rank all the five arrangements according to the magnitude of the electric field at the center of curvature, greatest first.

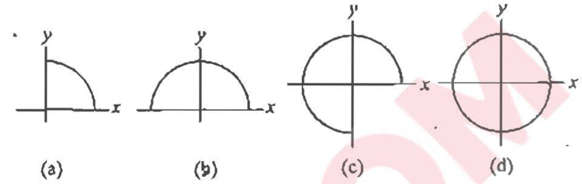


Fig. 1.74

9. Figure shows that E has the same value for all points in front of an infinitely charged sheet. Is this reasonable? One might think that the field should be stronger near the sheet because the charges are so much closer.

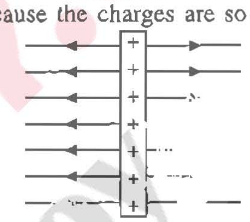


Fig. 1.75

10. Figure shows the tracks of three charged particles in a uniform electrostatic field projected parallel to plate with same velocity. Give the signs of the three charges. Which of the three particles has the highest charge to mass ratio?

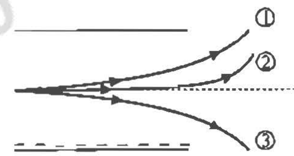


Fig. 1.76

11. Three small spheres x, y and z carry charges of equal magnitudes and with signs shown in figure. They are placed at the vertices of an isosceles triangle with the distance between x and y equal to the distance between x and z. Spheres y and z are held in place but sphere x is free to move on a frictionless surface.

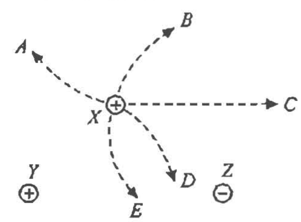


Fig. 1.77

a. What is the direction of the electric force on sphere x at the point shown in the figure?
b. Which path is sphere X likely to take when released?

12. Two identical positive charges are fixed on the y-axis, at equal distances from the origin O. A particle with a negative charge starts on the x-axis at a large distance from O, moves along the x-axis, passes through O and moves far away from O on the other side. Its acceleration a is taken as positive along its direction of motion. Plot the particle's acceleration a against its x-coordinate.

13. Electric field is defined in terms of q_0, a small positive charge. If instead the definition were in terms of a small negative charge of the same magnitude, then compared to the original field, the newly defined electric field
   a. would point in the same direction and have the same magnitude.
   b. would point in the opposite direction and have the same magnitude.
   c. would point in the same direction and have a different magnitude.
   d. would point in the opposite direction and have a different magnitude.
14. Three identical positive charges Q are arranged at the vertices of an equilateral triangle. The side of the triangle is a. Find the intensity of the field at the vertex of a regular tetrahedran of which the triangle is the base.
15. Two point charges of $+5\times10^{-19}$ C and $+20\times10^{-19}$ C are separated by a distance of 2 m. The electric field intensity will be zero at a distance d = ______ from 5×10^{-19} C charge.
16. An electron (mass m_e) falls through a distance 'd' in a uniform electric field of magnitude E.

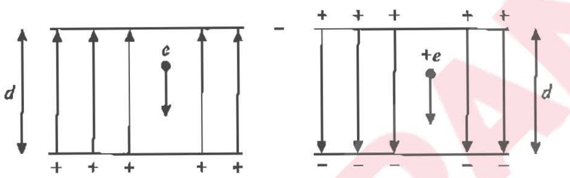


Fig. 1.78

The direction of the field is reversed keeping its magnitude unchanged and a proton (mass m_p) falls through the same distance. If the times taken by electron and proton to fall the distance d is 't_{electron}' and 't_{proton}', respectively, then the ratio $\frac{t_{\text{electron}}}{t_{\text{proton}}}$ = ______

17. Two charged metal plates in vacuum are 10 cm apart. A uniform electric field of intensity $(45/16)\times10^3$ NC^{-1} is applied between the plates. An electron is released between the plates from rest at a point just outside the negative plate. Calculate
   a. how long (t) will electron take to reach the other plate?
   b. At what velocity (v) will it be going just before it hits the other plate?
18. A polythene piece rubbed with wool is found to have a negative charge of 3.2×10^{-7} C.
   a. The number of electrons transferred is ______
   b. Is there a transfer of mass from wool to polythene? (Yes/No) ______
19. Two identical point charges 'Q' are kept at a distance 'r' from each other. A third point charge is placed on the line joining the above two charges such that all the three charges are in equilibrium. The third charge
   a. should be of magnitude q = ...
   b. should be of sign ...
   c. should be placed ...
20. If we introduce a large thin metal plate between two point charges, what will happen to the force between the charges?
21. Two point electric charges of unknown magnitude and sign are placed a certain distance apart. The electric field intensity is zero at a point not between the charges but on the line joining them. Write two essential conditions for this to happen.
22. A ball of charge q is placed in a hollow conducting uncharged sphere. After this, the sphere is connected with earth for a short time and the ball is then removed from the sphere. The ball has not been brought into contact with the sphere.
   a. What charge will the sphere have after these operations? Where and how will this charge be distributed?
   b. What will be the electric field inside as well on outside of sphere?
23. Two pieces of plastic, a full ring and a half ring, have the same radius and charge density. Which electric field at the center has the greater magnitude?
   Define your answer.

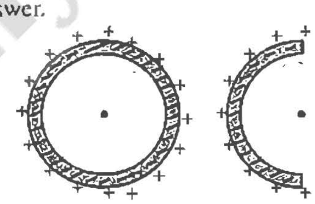


Fig. 1.79

24. A droplet of ink in an industrial ink-jet printer carries a charge of 1.6×10^{-10} C and is deflected onto paper by a force of 3.2×10^{-4} N. Find the strength of the electric field to produce this force.
25. An electric dipole of length 4 cm, when placed with its axis making an angle of 60° with a uniform electric field experiences a torque of $4\sqrt{3}$ N m. Calculate the (a) magnitude of the electric field and (b) potential energy of the dipole, if the dipole has charges of $\pm$ 8 nC.
26. An electric dipole consists of two opposite charges each of 1 μC separated by 2 cm. The dipole is placed in an external uniform field of 10^5 NC^{-1} intensity. Find
   a. maximum torque exerted by the field on the dipole and
   b. the work done in rotating the dipole through 180° starting from the position $\theta = 0°$.

ELECTRIC DIPOLE

- An electric dipole is a system of two equal and opposite point charges separated by a very small and finite distance.
- Fig. 1.80 shows an electric dipole consisting of two equal and opposite point charges $-q$ and $+q$ separated by a small

distance $2l$. The strength of an electric dipole is measured by a vector quantity known as *electric dipole moment.* Its magnitude is equal to the product of the magnitude of either charge and the distance between the two charges.

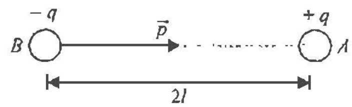


Fig. 1.80

$$\therefore \quad p = q2l$$

The direction of p is from negative charge to positive charge.

- In S.I. system of units, p is measured in coulomb-metre.

ELECTRIC FIELD DUE TO A DIPOLE

Electric Field Intensity due to an Electric Dipole at a Point on the Axial Line

- A line passing through the negative and positive charges of the electric dipole is called the axial line of the electric dipole.

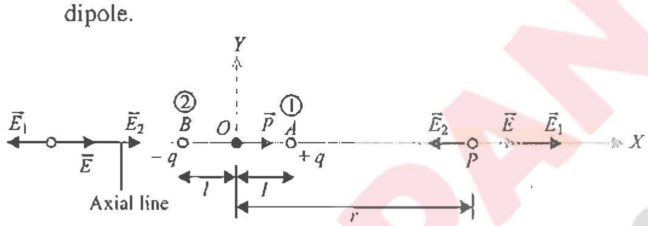


Fig. 1.81

- Suppose an electric dipole AB is located in a medium of dielectric constant K (as shown in Fig. 1.81). Let the dipole consists of two point charges of $-q$ and $+q$ coulomb separated by a short distance $2l$ meter. Let P be an observation point on the axial line such that its distance from the mid point O of the electric dipole is r. We are interested to calculate the intensity of electric field at P.

- $E_1 = \dfrac{1}{4\pi\varepsilon_0 K}\dfrac{q}{(r-l)^2}$ due to q at P {along the direction OX}

and $E_2 = \dfrac{1}{4\pi\varepsilon_0 K}\dfrac{q}{(r+l)^2}$ due to $-q$ at P {along the direction OB}

The intensities E_1 and E_2 are along the same line but in opposite directions. Since $E_1 > E_2$, hence resultant intensity E at the point P will be equal to their differences and in the direction $\overline{AP}$. Thus,

$$E = E_1 - E_2 = \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r-l)^2} - \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r+l)^2}$$

$$E = \frac{q}{4\pi\varepsilon_0 K}\left[\frac{4lr}{(r^2-l^2)^2}\right] = \frac{1}{4\pi\varepsilon_0 K}\left[\frac{2(2ql)r}{(r^2-l^2)^2}\right]$$

But $2ql = p$ =electric dipole moment;

$$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0 K}\frac{2pr}{(r^2-l^2)^2}$$

- If l is very small compared to r ($l \ll r$), then l^2 can be neglected in comparison to r^2. Then, the electric field intensity at the point P due to a short dipole is given by

$$E = \frac{1}{4\pi\varepsilon_0 K}\frac{2pr}{r^4} = \frac{1}{4\pi\varepsilon_0 K}\frac{2p}{r^3}$$

$$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0 K}\frac{2p}{r^3}$$

- If dipole is placed in air or vacuum, then $K = 1$ and $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2p}{r^3}$

Note: ***The direction of electric field E is in the direction of $\vec{p}$, i.e., parallel to the axis of dipole from the negative charge towards the positive charge.***

In vector form, we can write:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}\hat{i} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{r^3}$$

Electric Field Intensity due to an Electric Dipole at a Point on the Equatorial Line

An equatorial line of the electric dipole is a line perpendicular to the axial line and passing through a point mid way between charges.

- Let us now suppose that the observation point P is situated on the equatorial line of dipole such that its distance from mid-point O of the electric dipole is r (as shown in Fig. 1.82). Let us assume again that the medium between the electric dipole and the observation point has dielectric constant K.

- $E_1 = \dfrac{1}{4\pi\varepsilon_0 K}\dfrac{q}{(r^2+l^2)}$ {along the direction PD}

and $E_2 = \dfrac{1}{4\pi\varepsilon_0 K}\dfrac{q}{(r^2+l^2)}$ {along the direction PC}

The magnitude of E_1 and E_2 are equal but directions are different.

Net intensity: $E = E_1\cos\theta + E_2\cos\theta$

[sine components cancel out]

$$E = \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r^2+l^2)}\cos\theta + \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r^2+l^2)}\cos\theta$$

$$= \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r^2+l^2)} \times 2\cos\theta \text{ along } PR$$

But from the figure,

$$\cos\theta = \frac{OA}{PA} = \frac{OA}{(OP^2+OA^2)^{1/2}} = \frac{l}{(r^2+l^2)^{1/2}}$$

$$\therefore \quad E = \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r^2+l^2)} \times \frac{2l}{(r^2+l^2)^{1/2}} = \frac{1}{4\pi\varepsilon_0 K} \times \frac{2ql}{(r^2+l^2)^{3/2}}$$

But $2ql = p$ = electric dipole moment

$$\therefore \quad E = \frac{1}{4\pi\varepsilon_0 K} \times \frac{p}{(r^2+l^2)^{3/2}}$$

- If l is very small as compared to r ($l \ll r$), then l^2 can be neglected in comparison to r^2. Then, the electric field

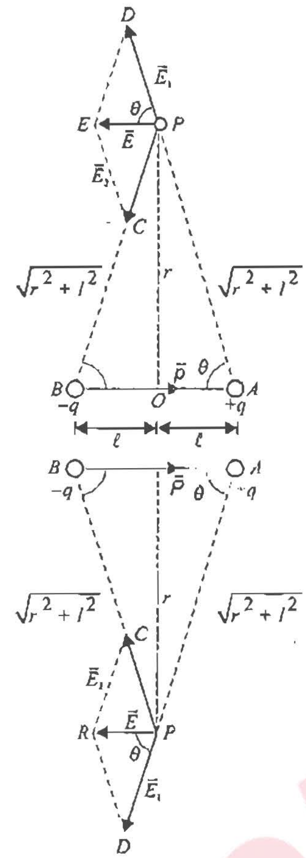


Fig. 1.82

intensity at the point P due to a short dipole is given by

$$E = \frac{1}{4\pi\varepsilon_0 K}\frac{p}{(r^2)^{3/2}} = \frac{1}{4\pi\varepsilon_0 K}\frac{p}{r^3}$$

- If dipole is placed in air or vacuum, then $K = 1$ and $E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$

As direction of resultant electric field is along the negative x-axis, hence in vector form we can write

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \times \frac{p}{r^3}(-\hat{i}) = -\frac{1}{4\pi\varepsilon_0} \times \frac{\vec{p}}{r^3}$$

Note: ***The direction of electric field E is opposite to the direction of $\vec{p}$, i.e., antiparallel to the axis of dipole from the positive charge towards the negative charge.***

ELECTRIC FIELD INTENSITY DUE TO A SHORT DIPOLE AT SOME GENERAL POINT

- Let AB be a short electric dipole of dipole moment $\vec{p}$ (directed from B to A). We are interested to find the electric field at some general point P. The distance of observation point P w.r.t. mid point O of the dipole is r and the angle made by the line OP w.r.t. axis of dipole is θ.
- We know that dipole moment of a dipole is a vector quantity. It can be resolved into two rectangular components $\vec{p}_1$ and $\vec{p}_2$ as shown in figure, so that $\vec{p} = \vec{p}_1 + \vec{p}_2$.
  The magnitude of $\vec{p}_1$ and $\vec{p}_2$ are $p_1 = p\cos\theta$ and $p_2 = p\sin\theta$.
- It is clear from figure that point P lies on the axial line of dipole with moment $\vec{p}_1$. Hence, magnitude of the electric field intensity $\vec{E}_1$ at P due to $\vec{p}_1$ is

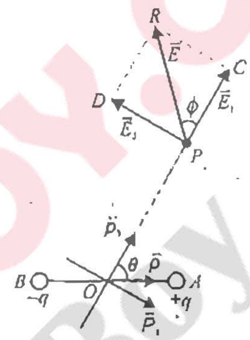


Fig. 1.83

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{2p\cos\theta}{r^3}\ \{\text{along } OC\} \qquad \text{(i)}$$

Similarly, P lies on the equatorial line of dipole with moment $\vec{p}_2$. Hence, magnitude of electric field intensity $\vec{E}_2$ at P due to $\vec{p}_2$ is

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{p\sin\theta}{r^3}\ \{\text{opposite to } p_2\} \qquad \text{(ii)}$$

Hence, resultant intensity at P is $\vec{E} = \vec{E}_1 + \vec{E}_2$

Magnitude of $\vec{E}$ is: $E = \sqrt{(E_1^2 + E_2^2)}$ (as $\vec{E}_1$ and $\vec{E}_2$ are mutually perpendicular).

or $$E = \sqrt{\left(\frac{2p\cos\theta}{4\pi\varepsilon_0 r^3}\right)^2 + \left(\frac{p\sin\theta}{4\pi\varepsilon_0 r^3}\right)^2}$$

$$= \frac{p}{4\pi\varepsilon_0 r^3}\sqrt{4\cos^2\theta + \sin^2\theta} = \frac{p}{4\pi\varepsilon_0 r^3}\sqrt{1 + 3\cos^2\theta}$$

- If the resultant field intensity vector $\vec{E}$ makes an angle ϕ with the direction of $\vec{E}_1$, then

$$\tan\phi = \frac{E_2}{E_1} = \frac{(p\sin\theta/4\pi\varepsilon_0 r^3)}{(2p\cos\theta/4\pi\varepsilon_0 r^3)} = \frac{1}{2}\tan\theta$$

Illustration 1.27 **Three charges $-q$, $+2q$ and $-q$ are arranged on a line as shown in the Fig. 1.84. Calculate the field at a distance $r >> a$ on the line.**

Sol. The field at point P is superposition of fields $\vec{E}_1$, $\vec{E}_2$, $\vec{E}_3$ due to each charge.

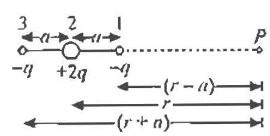


Fig. 1.84

$$\vec{E}_1 = -\frac{q}{4\pi\varepsilon_0(r-a)^2}\hat{i};\quad \vec{E}_2 = +\frac{2q}{4\pi\varepsilon_0 r^2}\hat{i};$$

$$\vec{E}_3 = -\frac{q}{4\pi\varepsilon_0(r+a)^2}\hat{i};\ \text{Now}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = \frac{q}{4\pi\varepsilon_0}\left[-\frac{1}{(r-a)^2} + \frac{2}{r^2} - \frac{1}{(r+a)^2}\right]\hat{i}$$

$$E = \frac{q}{4\pi\varepsilon_0 r^2}\left[-\left\{1-\left(\frac{a}{r}\right)^{-2}\right\} + 2 - \left\{1+\left(\frac{a}{r}\right)\right\}^{-2}\right]$$

If $r >> a$, we can use binomial approximation:

$$(1+\alpha)^n \simeq 1 + n\alpha + \frac{n(n-1)}{\underline{|2}}\alpha^2 + \cdots \text{ for } \alpha << 1$$

Therefore,

$$E = \frac{q}{4\pi\varepsilon_0 r^2}\left[-\left\{\left(1-2(-\frac{a}{r})\right) + \frac{-2(-2-1)}{2}\left(\frac{-a}{r}\right)^2\right\}\right.$$

$$\left. + 2 - \left\{1 - 2\frac{a}{r} + \frac{-2(-2-1)}{2}\left(\frac{a}{r}\right)^2\right\}\right] = \frac{6a^2q}{4\pi\varepsilon_0 r^4}$$

The charge in this problem may be considered as two dipoles placed close together. Such an arrangement of charge is called an electric quadrupole.

Illustration 1.28 **What is the force on a dipole of dipole moment p placed as shown in the Fig. 1.85.**

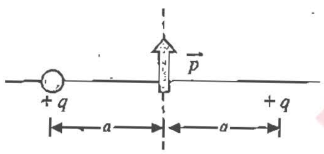


Fig. 1.85

Sol. Force on any q by dipole:

$$F = qE_{\text{dipole}} = \frac{q}{4\pi\varepsilon_0}\frac{p}{a^3} \text{ downward}$$

So from third law, force on dipole due to both charges

$$= 2F = \frac{qp}{2\pi\varepsilon_0 a^3} \text{ upward}$$

Net Force on a Dipole in a Non-Uniform Field

Suppose an electric dipole with dipole moment $\vec{P}$ is placed in a non-uniform electric field $\vec{E} = E\hat{i}$ that points along x-axis (Fig. 1.86). Let E depends only on x. The electric field at the position of negative charge is E and at the position of positive charge $(E + \Delta E)$. Net force acting on the dipole is then

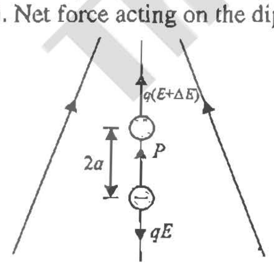


Fig. 1.86

$$F = q(E + \Delta E) - qE = q\,\Delta E = q\left[\frac{\Delta E}{\Delta x}2a\right]$$

$$\left[\text{as } \frac{\Delta E}{\Delta x} = \frac{dE}{dx}\right]$$

$$\therefore \quad F = 2aq\frac{dE}{dx} = p\frac{dE}{dx}$$

$$|\vec{F}| = \left|p\frac{d\vec{E}}{dx}\right|$$

where $\dfrac{dE}{dx}$ is the gradient of the field in the x-direction.

Illustration 1.29 **Find the force on a small electric dipole of dipole moment $\vec{p}$ due to a point charge Q placed at a distance r.**

Fig. 1.87

Sol. Electric field of a point charge is a non-uniform electric field. Electric field at a distance x from the point charge is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{x^2} \Rightarrow \frac{dE}{dx} = -\frac{1}{4\pi\varepsilon_0}\frac{2Q}{x^3}$$

magnitude of force on the dipole:

$$F = \left|p\frac{dE}{dx}\right|_{x=r} = \frac{1}{4\pi\varepsilon_0}\frac{2pQ}{r^3}$$

Alternatively: Same can be calculated as force on the point charge due to dipole which is same as the force on dipole due to point charge (Newton's 3rd law). The electric field of small dipole at a distance r is

$E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2p}{r^3}$. Hence, force on the point charge Q is

$$F = \frac{1}{4\pi\varepsilon_0}\frac{2pQ}{r^3}$$

DIPOLE IN A UNIFORM ELECTRIC FIELD

Torque: When a dipole is placed in a uniform field as shown in Fig. 1.88, the net force on it: $F_R = \left[q\vec{E} + (-q)\vec{E}\right] = 0$

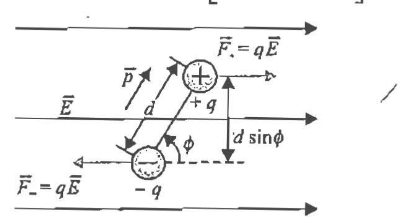


Fig. 1.88

Hence, net force on a dipole is zero in a uniform electric field. While the torque $\tau = qE \times d\sin\phi$

i.e., $\quad \tau = pE\sin\phi$ {as $p = qd$}

or $\quad \vec{\tau} = \vec{p} \times \vec{E}$ (by electric field)

and $\quad \vec{\tau} = \vec{E} \times \vec{p}$ (by us if the dipole is in equilibrium)

From the expression, it is clear that couple acting on a dipole is maximum ($= pE$) when dipole is perpendicular ($\phi = 90°$) to the field and minimum ($= 0$) when dipole is parallel ($\phi = 0°$) or antiparallel ($\phi = 180°$) to the field.

By applying a torque, electric field tends to align a dipole in its own direction.

Illustration 1.30 **An electric dipole consists of two charges of 0.1 μC separated by a distance of 2.0 cm. The dipole is placed in an external field of 10^5 NC^{-1}. What maximum torque does the field exert on the dipole?**

Sol. $\tau = pE\sin\theta = q \times 2a \times E\sin\theta$. Max. value of τ will be when $\sin\theta = 1$

$\therefore \tau_{max} = 10^{-7} \times 2 \times 10^{-2} \times 10^{5} \times 1 = 2 \times 10^{-4}$ N-m

Concept Application Exercise 1.4

1. State the following statements as true / false:
   a. An electric dipole is kept in a uniform electric field at some angle with it. It experiences a force but no torque.
   b. An electric dipole may experience a net force when it is placed in a non-uniform electric field.
   c. An electric dipole is kept in a non-uniform electric field. It can experience a force and a torque.
2. Electric intensity due to an electric dipole varies with distance as $E \propto r^n$, where n is ______.
3. An electric dipole of moment $\vec{p}$ is placed at the origin along the x-axis. The electric field E at a point P, whose position vector makes an angle θ with the x-axis, will make an angle with x-axis is ______.

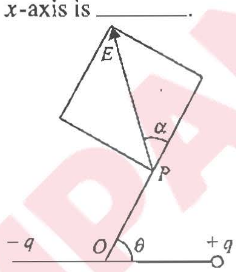


Fig. 1.89

4. Two point charges of 1 μC and −1μC are separated by a distance of 100 Å. A point P is at a distance of 10 cm from the mid point and on the perpendicular bisector of the line joining the two charges. Find the electric field at P.
5. An electric dipole consists of two opposite charges of magnitude 2×10^{-6} C each and separated by a distance of 3 cm. It is placed in an electric field of 2×10^5 NC^{-1}. Determine the maximum torque on the dipole.
6. Three charges are arranged on the vertices of an equilateral triangle as shown in Fig. 1.90. Find the dipole moment of the combination.

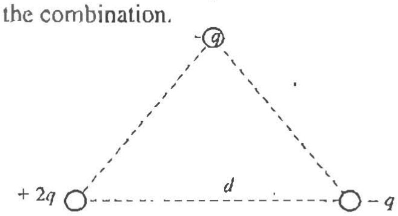


Fig. 1.90

7. The electric field at A due to dipole p is perpendicular to p. the angle θ is______

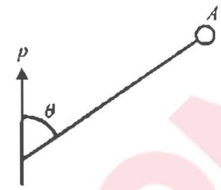


Fig. 1.91

8. A dipole lies on the x-axis, with the positive charge $+q$ at $x = +\frac{d}{2}$ and the negative charge at $-\frac{d}{2}$. Find the electric flux ϕ_E through the yz plane midway between the charges.
9. An electric dipole is formed by two particles fixed at the end of a light rod of length l. The mass of each particle is m and the charges are $-q$ and $+q$. The system is placed in such a way that the dipole axis is parallel to a uniform electric field E that exists in region. The dipole is slightly rotated about its center and released. Show that for small angular displacement motion is SHM. Evaluate its time period.

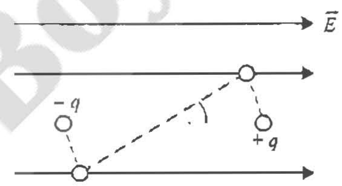


Fig. 1.92

10. A dipole consists of two particles carrying charges +2 and −2μC and masses 1 and 2 kg, respectively, separated by a distance of 6 m. It is placed in a uniform electric field of 8×10^4 Vm^{-1}. For small oscillations about its equilibrium position, find the angular frequency.
11. A small electric dipole of dipole moment P is placed near a point charge $+Q$ as shown. Then, the net force on the dipole is towards ______.

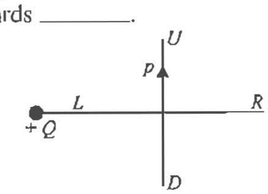


Fig. 1.93

Solved Examples

Example 1.1 **A uniformly charged wire with linear charge density λ is laid in the form of a semicircle of radius R. Find the electric field generated by the semicircle at the center.**

Sol. We consider a differential element dl on the ring, that subtends an angle $d\theta$ at the center of the ring,

$dl = R\,d\theta$. Charge on this element $= dQ = \lambda R\,d\theta$.

This element creates a field dE which makes an angle θ at the center as shown in Fig. 1.94. For each differential element

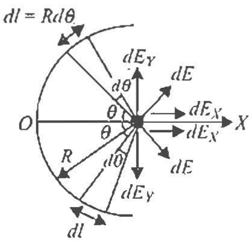


Fig. 1.94

in the upper half of the ring, there corresponds a symmetrically placed charge element in the lower half plane. The y-components of field due to these symmetric elements cancel out and x-components remain.

$$dE_x = dE\cos\theta = \frac{dQ}{4\pi\varepsilon_0 R^2}\cos\theta = \frac{\lambda(R\,d\theta)\cos\theta}{4\pi\varepsilon_0 R^2}$$

On integrating the expression for dE_x, w.r.t. angle θ, in limits $\theta = -\pi/2$ to $\theta = +\pi/2$, we obtain

$$E = \int_{-\pi/2}^{+\pi/2} \frac{\lambda R}{4\pi\varepsilon_0 R^2}\cos\theta\, d\theta = \frac{\lambda}{2\pi\varepsilon_0 R}$$

In terms of total charge, say Q, on the ring, $\lambda = \dfrac{Q}{\pi R}$ and we get $E = \dfrac{Q}{2\pi^2\varepsilon_0 R^2}$.

If we consider the wire in the form of an arc as shown in the figure, the symmetry consideration is not useful in canceling out x- and y-components of the fields, if θ_1 and θ_2 are different. We will integrate dE_x as well as dE_y in limits $\theta = -\theta_1$ to $\theta = +\theta_2$.

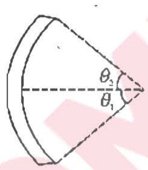


Fig. 1.95

$$E_x = \int_{-\theta_1}^{+\theta_2} \frac{\lambda R}{4\pi\varepsilon_0 R^2}\cos\theta\, d\theta = \frac{\lambda}{4\pi\varepsilon_0 R}(\sin\theta_1 + \sin\theta_2)$$

$$E_y = -\int_{-\theta_1}^{+\theta_2} \frac{\lambda R}{4\pi\varepsilon_0 R^2}\sin\theta\, d\theta = \frac{\lambda}{4\pi\varepsilon_0 R}(\cos\theta_2 - \cos\theta_1)$$

For a symmetrical arc, $\theta_1 = \theta_2$. Thus, E_y vanishes and

$$E_x = \frac{\lambda\sin\theta}{2\pi\varepsilon_0 R}$$

Example 1.2 **A long wire with a uniform charge density λ is bent in two configurations shown in figure (a) and (b). Determine the electric field intensity at point O.**

Sol. Consideration of Fig. 1.96(a)

Field due to segment (1):

$$\vec{E}_1 = \left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)\hat{i} + \left(-\frac{\lambda}{4\pi\varepsilon_0 R}\right)\hat{j}$$

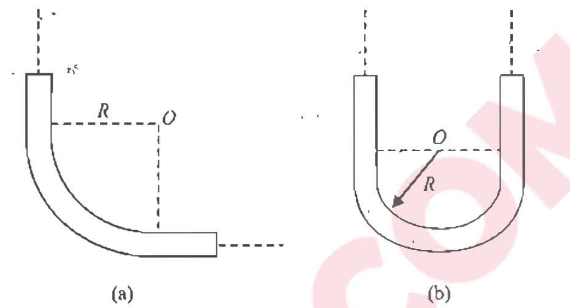


Fig. 1.96

Field due to segment (2):

$$\vec{E}_2 = \left(-\frac{\lambda}{4\pi\varepsilon_0 R}\right)\hat{i} + \left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)\hat{j}$$

Field due to quarter shape wire segment (3):

$$\vec{E}_3 = \left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)\hat{i} + \left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)\hat{j} \quad (\because \theta_1 = 90^\circ\ \theta_2 = 0^\circ)$$

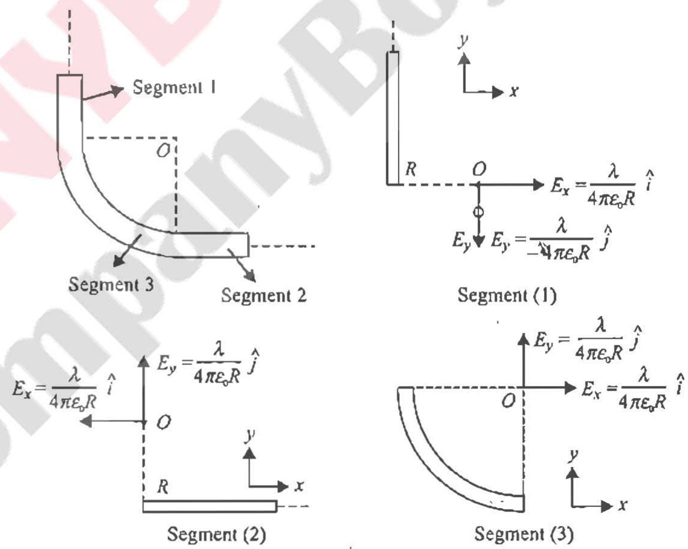


Fig. 1.97

Resultant field is superposition of fields due to each part.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 \qquad \text{(i)}$$

Substituting the values of $\bar{E}_1$, $\bar{E}_2$ and $\bar{E}_3$ in (i),

$$\vec{E} = \left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)\hat{i} + \left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)\hat{j}$$

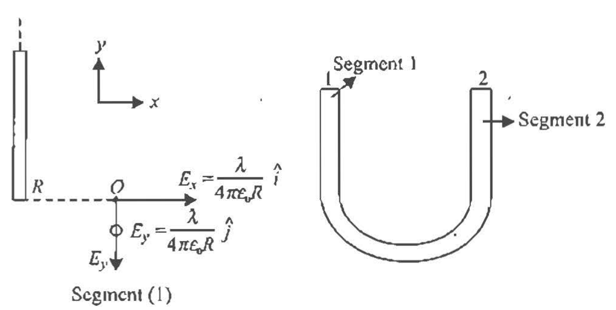


Fig. 1.98

$$|\vec{E}| = \left[\left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)^2 + \left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)^2\right]^{1/2} = \frac{\sqrt{2}\lambda}{4\pi\varepsilon_0 R}$$

Here, $E_x = E_y = \dfrac{\lambda}{4\pi\varepsilon_0 R}$. Hence, the resultant field will make an angle of 45° with the axis.

b. Field due to segment 1,

$$\vec{E}_{x_1} = \frac{\lambda}{4\pi\varepsilon_0 R}\hat{i}$$

$$\vec{E}_{y_1} = -\frac{\lambda}{4\pi\varepsilon_0 R}\hat{j}$$

$$\vec{E}_1 = \frac{\lambda}{4\pi\varepsilon_0 R}[\hat{i} - \hat{j}]$$

Field due to segment 2, $\vec{E}_{x_2} = -\dfrac{\lambda}{4\pi\varepsilon_0 R}\hat{i}$

$$\vec{E}_{y2} = -\frac{\lambda}{4\pi\varepsilon_0 R}\hat{j}$$

$$\vec{E}_2 = -\frac{\lambda}{4\pi\varepsilon_0 R}[\hat{i} + \hat{j}]$$

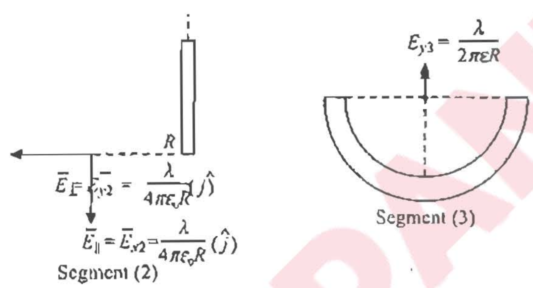


Fig. 1.99

Field due to segment 3, $\vec{E}_{x_3} = 0$, $\vec{E}_{y_3} = \dfrac{\lambda}{2\pi\varepsilon_0 R}\hat{j}$

$$\Rightarrow \quad \vec{E}_3 = \frac{\lambda}{2\pi\varepsilon_0 R}\hat{j}$$

From principle of superposition of electric fields,

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = \frac{\lambda}{4\pi\varepsilon_0 R}[\hat{i} - \hat{j}] - \frac{\lambda}{4\pi\varepsilon_0 R}[\hat{i} + \hat{j}] + \frac{\lambda}{2\pi\varepsilon_0 R}\hat{j} = 0$$

Hence, net field is zero.

Example 1.3 **A particle having charge that of an electron and mass 1.6×10^{-30} kg is projected with an initial speed u at an angle 45° to the horizontal from the lower plate of a parallel plate capacitor as shown in figure. The plates are sufficiently long and have separation 2 cm. Find the maximum value of velocity of particle for it not to hit the upper plate. Take electric field between the plates = 10^3 Vm^{-1} directed upward.**

Sol. Resolving the velocity of particle parallel and perpendicular to the plate.

$$u_{\parallel} = u\cos 45^\circ = \frac{u}{\sqrt{2}} \text{ and } u_{\perp} = u\sin 45^\circ = \frac{u}{\sqrt{2}}$$

Force on the charged particle in downward direction normal to the plate = eE

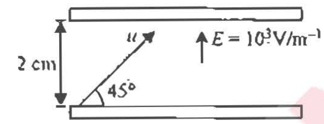


Fig. 1.100

$\therefore$ Acceleration $a = \dfrac{eE}{m}$, where m is the mass of charged particle.

The particle will not hit the upper plate, if the velocity component normal to plate becomes zero before reaching it, i.e.,

$0 = u_{\perp}^2 - 2ay$ with $y \le d$, where d is the distance between the plates.

$\therefore$ Maximum velocity for the particle not to hit the upper plate, (for this $y = d = 2$ cm)

$$u_{\perp} = \sqrt{2ay} = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 10^3 \times 2 \times 10^{-2}}{1.6 \times 10^{-30}}}$$

$$= 2 \times 10^6 \text{ ms}^{-1}$$

$$\Rightarrow \quad u_{max} = u_{\perp}/\cos 45^\circ = 2\sqrt{2} \times 10^6 \text{ ms}^{-1}$$

Example 1.4 **A particle of mass m and charge q is released at rest in a uniform field of magnitude E. The uniform field is created between two parallel plates of charge densities $+\sigma$ and $-\sigma$, respectively. The particle accelerates towards the other plate a distance d away. Determine the speed at which it strikes the opposite plate.**

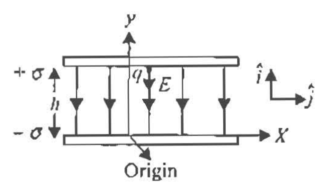


Fig. 1.101

Sol. The applied electric field is $\vec{E} = -E_0\hat{j}$

The force experienced by the charge q, $\vec{F} = q\vec{E} = -qE_0\hat{j}$

The force is constant, and so the acceleration is constant as well

$$\therefore \quad \vec{a} = \frac{\vec{F}}{m} = -\frac{qE_0}{m}\hat{j}$$

Due to constant acceleration, the particle moves in −ve y-direction; the problem is analogous to motion of a mass released from rest in a gravitational field.

From equations of motion,

$$v_y = v_{y0} + a_y t = 0 - \frac{qE_0}{m}t \qquad \text{(i)}$$

$$\text{And } y = y_0 + v_0 t + \frac{1}{2}a_y t^2\ ;\ 0 = d + 0 - \frac{1}{2}\frac{qE_0}{m}t^2 \qquad \text{(ii)}$$

Particle starts at $y_0 = d$ and impact occurs at $y = 0$

From equation (ii), $t = \left(\dfrac{2dm}{qE_0}\right)^{1/2}$,

From equation (i), $v_y = -\dfrac{qE_0}{m}\left(\dfrac{2dm}{qE_0}\right)^{1/2} = -\sqrt{\dfrac{2qE_0 d}{m}}$

Example 1.5 **Two balls of charges q_1 and q_2 initially have a velocity of the same magnitude and direction. After**

a uniform electric field has been applied for a certain time interval, the direction of first ball changes by 60° and the velocity magnitude is reduced by half. The direction of velocity of the second ball changes thereby 90°. In what ratio will the velocity of the second ball change? Determine the magnitude of the charge-to-mass ratio of the second ball if it is equal to α_1 for the first ball. Ignore the electrostatic interaction between the balls.

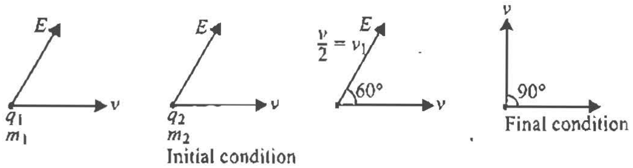


Fig. 1.102

Sol. Let the electric field on each ball be given by

$$E = E_x\hat{i} + E_y\hat{j}$$

From impulse–momentum equation, we have

Impulse = Change in momentum

Let the final velocities of the balls be v_1 and v_2. Nothing that $v_1 = v/2$, we have

$$q_1(E_x\hat{i}+E_y\hat{j})\Delta t = m_1\left(\frac{v}{2}\cos 60°\hat{i}+\frac{v}{2}\sin 60°\hat{j}\right) - m_1 v\hat{i} \quad \text{(i)}$$

$$q_2(E_x\hat{i}+E_y\hat{j})\Delta t = m_2\left(v_2\cos 90°\hat{i} + v_2\sin 90°\hat{j}\right) - m_2 v\hat{i} \quad \text{(ii)}$$

On comparing the x- and y-components on both sides of equation (i), we get

$$\frac{q_1}{m_1}E_x\,\Delta t = -\frac{3}{4}v \text{ and } \frac{q_1}{m_1}E_y\,\Delta t = \frac{\sqrt{3}}{4}v \quad \text{(iii)}$$

Similarly, for equation (ii), we get

$$\frac{q_2}{m_2}E_x\,\Delta t = -v \quad \text{and} \quad \frac{q_2}{m_2}E_y\,\Delta t = v_2 \quad \text{(iv)}$$

From equations (iii) and (iv), by dividing the equations expression for x-components, we get

$$\frac{q_1/m_1}{q_2/m_2} = \frac{3}{4} \quad \text{(v)}$$

or $$\frac{q_2}{m_2} = \frac{4}{3}\frac{q_1}{m_1} = \frac{4}{3}\alpha_1$$

Also, $$\frac{q_1/m_1}{q_2/m_2} = \frac{\sqrt{3}\,v}{4v_2} \Rightarrow \frac{\sqrt{3}\,v}{4v_2} = \frac{3}{4} \Rightarrow v_2 = \frac{v}{\sqrt{3}}$$

Example 1.6 **A rigid insulated wire frame, in the form of right triangle ABC is set in a vertical plane. Two beads of equal masses m each carrying charges q_1 and q_2 are connected by a chord of length l and can slide without friction on the wires. Considering the case when the beads are stationary, determine (IIT-JEE, 1978)**

1. **the angle α.**
2. **the tension in the chord, and**
3. **the normal reactions on the beads if the chord is not cut. What are the values of the charges for which the beads continue to remain stationary?**

Sol. Because of equilibrium of charge q_1

$$N_1 = mg\sin 60° + (T - F)\sin\alpha \ldots \quad \text{(i)}$$

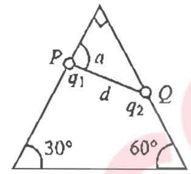


Fig. 1.103

and $(T - F)\cos\alpha = mg\cos 60° \ldots$ (ii)

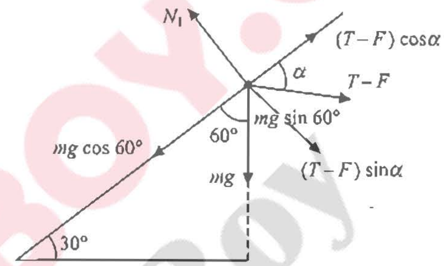


Fig. 1.104

Because of equilibrium of charge q_2

$(T - F)\sin\alpha = mg\cos 30°$ (iii)

From (i) and (iii), $N_1 = mg\sin 60° + mg\cos 30°$ (iv)

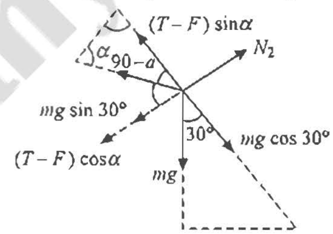


Fig. 1.105

$$\Rightarrow \quad N_1 = mg\left(\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\right) = \sqrt{3}\,\text{mg}$$

From (ii) and (iv),

$$N_2 = mg\cos 60° + mg\sin 30° = mg\left(\frac{1}{2}+\frac{1}{2}\right) = mg$$

Also, $F = k\frac{q_1q_2}{l^2}$

Now, from equations (ii) and (iii), we get

$$(T - F)^2\cos^2\alpha + (T - F)^2\sin^2\alpha = m^2g^2\cos^2 60° + m^2g^2\cos^2 30°$$

$$\Rightarrow (T - F)^2 = m^2g^2\left[\frac{1}{4}+\frac{3}{4}\right] = m^2g^2$$

$$\Rightarrow \quad T - F = \pm mg \quad \text{(v)}$$

$$\Rightarrow T = mg + F = mg + k\frac{q_1q_2}{l^2} \quad \text{(vi)}$$

[Taking positive sign]

From (ii) and (v),

$$mg\cos\alpha = mg\cos 60° \quad \Rightarrow \cos\alpha = \cos 60°$$

When the string is cut, $T = 0$

$\therefore$ From (vi), $\text{mg} = \pm k\frac{q_1q_2}{l^2} \Rightarrow q_1q_2 = \pm\frac{mgl^2}{k}$

EXERCISES

Subjective Type

Solutions on page 1.45

1. Calculate the number of electrons in a small, electrically neutral silver pin that has a mass of 10.0 g. Silver has 47 electrons per atom, and its molar mass is 107.87 $gmol^{-1}$.
2. A charged particle of radius 5×10^{-7} m is located in a horizontal electric field of intensity 6.28×10^5 Vm^{-1}. The surrounding medium has coefficient of viscosity $\eta = 1.6 \times 10^5$ Nsm^{-2}. The particle starts moving under the effect of electric field and finally attains a uniform horizontal speed of 0.02 ms^{-1}. Find the number of electrons on it. Assume gravity free space.
3. Suppose that 1.00 g of hydrogen is separated into electrons and protons. Suppose also that the protons are placed at the Earth's north pole and the electrons are placed at the south pole. What is the resulting compression force on the Earth? (Given: Radius of earth is 6400 km).
4. Two identical conducting small spheres are placed with their centers 0.300 m apart. One is given a charge of 12.0 nC and the other a charge of -18.0 nC.
   a. Find the electric force exerted by one sphere on the other?
   b. If the spheres are connected by a conducting wire, find the electric force between the two after they have come to equilibrium.
5. Four equal point charges each of magnitude $+Q$ are to be placed in equilibrium at the corners of a square. What should be the magnitude and sign of the point charge that should be placed at the center of square to do this job?
6. Two point electric charges of values q and $2q$ are kept at a distance d apart from each other in air. A third charge Q is to be kept along the same line in such a way that the net force acting on q and $2q$ is zero. Find the location of the third charge from charge 'q'.
7. Two fixed point charges $+4e$ and $+e$ unit are separated by a distance 'a'. Where the third point charge should be placed from $+4e$ charge for it to be in equilibrium.
8. Two identical particles are charged and held at a distance of 1 m from each other. They are found to be attracting each other with a force of 0.027 N. Now, they are connected by a conducting wire, so that charge flows between them. When the charge flow stops, they are found to be repelling each other with a force of 0.009 N. Find the initial charge on each particle.
9. Two similarly and equally charged identical metal spheres A and B repel each other with a force of 2×10^{-5} N. A third identical uncharged sphere C is touched with A and then placed at the mid-point between A and B. Find the net electric force on C.
10. Three point charges of $+2$ μC, -3 μC and -3 μC are kept at the vertices A, B and C respectively, of an equilateral triangle of side 20 cm as shown in the figure. What should be the sign and magnitude of the charge (q) to be placed at the mid point (M) of side BC so that the charge at A remains in equilibrium?

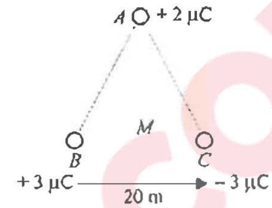


Fig. 1.106

11. Two small beads having positive charges $3q$ and q are fixed at the opposite ends of a horizontal, insulating rod, extending from the origin to the point $x = d$. As shown in figure, a third small charged bead is free to slide on the rod. At what position is the third bead in equilibrium? Can it be in stable equilibrium?

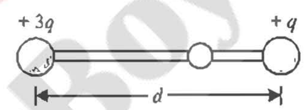


Fig. 1.107

12. A copper atom consists of copper nucleus surrounded by 29 electrons. The atomic weight of copper is 63.5 $gmol^{-1}$. Let us now take two pieces of copper each weighing 10 g. Let us consider one electron from one piece is transferred to another for every 1000 atoms in a piece.
    a. Find the magnitude of charge appearing on each piece.
    b. What will be the Coulomb force between the two pieces after the transfer of electrons if they are 10 cm apart?

    [Avogadro's number $= 6 \times 10^{23}$ mol^{-1}]
13. A flat square sheet of charge of side 50 cm carries a uniform surface charge density. An electron 0.5 cm from a point near the center of the sheet experiences a force of 1.8×10^{-12} N directed away from the sheet. Determine the total charge on the sheet.
14. Particle of mass 9×10^{-31} kg and a negative charge of 1.6×10^{-19} C is projected horizontally with a velocity of 10^6 ms^{-1} into a region between two infinite horizontal parallel plates of metal. The distance between the plates is d = 0.3 cm and the particle enters 0.1 cm below the top plate. The top and bottom plates are connected, respectively, to the positive and negative terminals of a 30 V battery. Find the components of the velocity of the particle just before it hits one of the plates.

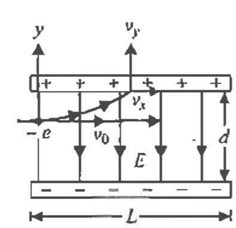


Fig. 1.108

15. A solid spherical region having a spherical cavity whose diameter 'R' is equal to the radius of the spherical region, has a total charge 'Q'. Find the electric field at a point P as shown.

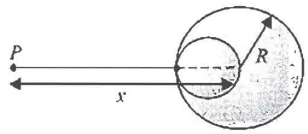


Fig. 1.109

16. A sphere of radius R has a uniform volume density ρ. A spherical cavity of radius b whose center lies at $\vec{r} = \vec{a}$ is removed from the sphere.
   a. Find the electric field at any point inside the spherical cavity.
   b. Find the electric field outside the cavity.

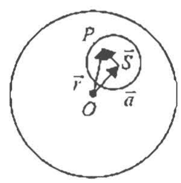
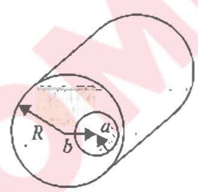


Fig. 1.110

17. A very long, solid insulating cylinder with radius R has a cylindrical hole with radius a bored along its entire length. The axis of the hole is a distance b from the axis of the cylinder, where $a < b < R$ (as shown in figure). The solid material of the cylinder has a uniform volume charge density ρ. Find the magnitude and direction of the electric field inside the hole, and show that this is uniform over the entire hole.



Charge density ρ

Fig. 1.111

18. Point charges q and $-q$ are located at the vertices of a square with diagonals $2l$ as shown in figure. Evaluate the magnitude of the electric field strength at a point located symmetrically with respect to the vertices of the square at a distance x from the center.

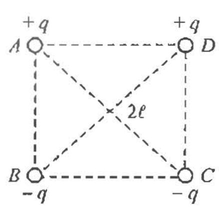


Fig. 1.112

19. Two mutually perpendicular long straight conductors carrying uniformly distributed charges of linear charge densities λ_1 and λ_2 are positioned at a distance a from each other. How does the interaction between the rods depend on a?

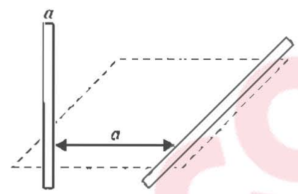


Fig. 1.113

20. A ring of radius 0.1 m is made out of a thin metallic wire of area of cross section 10^{-6} m^2. The ring has a uniform charge of π coulombs. Find the change in the radius of the ring when a charge of 10^{-8} coulomb is placed at the center of the ring. Young's modulus of the metal is 2×10^{11} Nm^{-2}.

21. A charged cork ball of mass m is suspended on a light string in the presence of a uniform electric field as shown in figure. When $E = (A\hat{i} + B\hat{j})$NC^{-1}, where A and B are positive numbers, the ball is in equilibrium at the angle θ. Find **a.** the charge on the ball and **b.** the tension in the string.

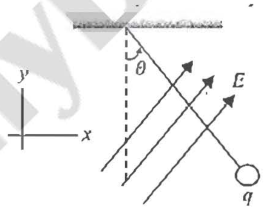


Fig. 1.114

22. A ring of radius R has charge $-Q$ distributed uniformly over it. Calculate the charge that should be placed at the center of the ring such that the electric field becomes zero at a point on the axis of the ring distant 'R' from the center of the ring.

23. Two identical small equally charged conducting balls are suspended from long threads secured at one point. The charges and masses of the balls are such that they are in equilibrium when the distance between them is a (the length of thread $L \gg a$). One of the balls is then discharged. What will be the distance b ($b \ll l$) between the balls when equilibrium is restored?

24. Two point charges Q_a and Q_b are positioned at points A and B. The field strength to the right of charge Q_b on the line that passes through the two charges varies according to a law that is represented schematically in the figure accompanying the problem (without employing a definite scale). The field strength is assumed to be positive if its direction coincides with the positive direction of x-axis. The distance between the charges is l = 21 cm (Fig. 1.115). Find
   a. the signs of the charges.
   b. the ratio of the absolute values of charges Q_a and Q_b.
   c. the coordinate x of the point where the field strength is maximum.

25. Two semicircular wires ABC and ADC each of radius 'R' are lying on x–y and x–z plane, respectively, as shown in the Fig. 1.116. If the linear charge density of the semicircular

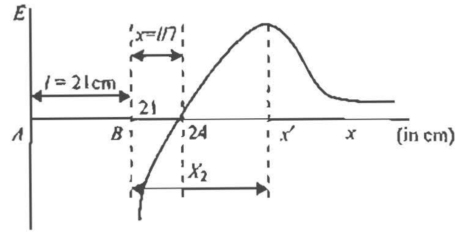


Fig. 1.115

parts and straight parts is λ, find the electric field intensity $\vec{E}$ at the origin.

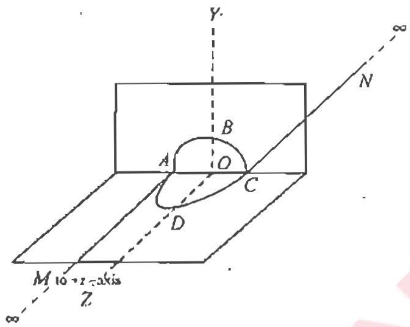


Fig. 1.116

26. An infinite wire having linear charge density λ is arranged as shown in the Fig. 1.117. A charge particle of mass m and charge q is released from point P. Find the initial acceleration of the particle (at $t = 0$) just after the particle is released.

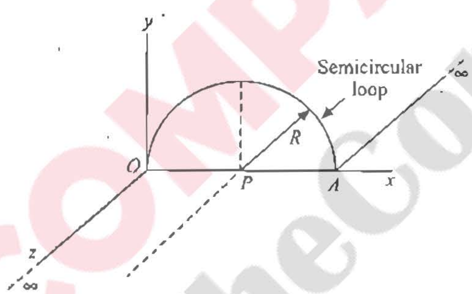


Fig. 1.117

27. Three small balls, each of mass m are suspended separately from a common point by three silk threads, each of length l. The balls are identically charged and hang at the corners of an equilateral triangle of side x. What is the charge on each ball?

28. Two similar balls, each of mass m and charge q, are hung from a common point by two silk threads, each of length l (Fig. 1.118). Prove that separation between the balls is

$$x = \left[\frac{q^2 l}{2\pi\varepsilon_0 mg}\right]^{1/3}, \text{ if } \theta \text{ is small.}$$

Find the rate $\frac{dq}{dt}$ with which the charge should leak off each sphere if their velocity of approach varies as $v = a/\sqrt{x}$, where a is a constant.

29. Three equal negative charges, $-q_1$ each, form the vertices of an equilateral triangle. A particle of mass m and a positive

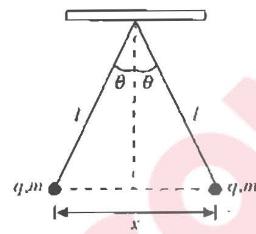


Fig. 1.118

charge q_2 is constrained to move along a line perpendicular to the plane of triangle and through its center which is at a distance r from each of the negative charges as shown in figure. The whole system is kept in gravity free space.

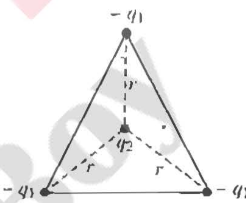


Fig. 1.119

Find the time period of vibration of the particle for small displacement from equilibrium position.

30. A ball of radius R carries a positive charge whose volume density at a point is given as $\rho = \rho_0(1 - r/R)$, where ρ_0 is a constant and r is the distance of the point from the center. Assuming the permittivities of the ball and the environment to be equal to unity, find
   a. the magnitude of the electric field strength as a function of the distance r both inside and outside the ball
   b. the maximum intensity E_{max} and the corresponding distance r_m.

31. The Fig. 1.120 shows two dipole moments parallel to each other and placed at a distance x apart. What is the magnitude of force of interaction? What is the nature of force, attractive or repulsive?

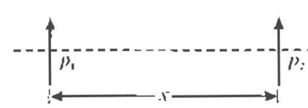


Fig. 1.120

32. Two dipoles p_1 and p_2 are placed along the same axis at a distance x apart, as shown in Fig. 1.121. What is magnitude of force of interaction? What is the nature of force, attractive or repulsive?

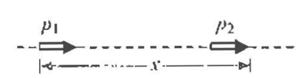


Fig. 1.121

33. A short dipole is placed along x-axis at $x = x$ (Fig. 1.122).
   a. Find the force acting on the dipole due to a point charge q placed at origin.
   b. Find the force on dipole if the dipole is rotated by 180° about z-axis.

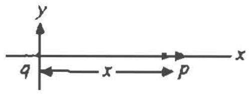


Fig. 1.122

c. Find the force on dipole if the dipole is rotated by 90° anti-clockwise about z-axis, i.e., it becomes parallel to y-axis.

Objective Type

Solutions on page 1.51

1. If a body is charged by rubbing it, its weight
   a. always decreases slightly
   b. always increases slightly
   c. may increase slightly or may decrease slightly
   d. remains precisely the same
2. In S.I. system, the value of ε_0 is
   a. $1\ C^2N^{-1}m^{-2}$
   b. $9\times10^9\ C^2N^{-1}m^{-2}$
   c. $\frac{1}{9\times10^9}\ C^2N^{-1}m^{-2}$
   d. $\frac{1}{4\pi\times9\times10^9}C^2N^{-1}m^{-2}$
3. Dimensions of ε_0 are
   a. $M^{-1}L^{-3}T^4A^2$ b. $M^0L^{-3}T^3A^3$
   c. $M^{-1}L^{-3}T^3A$ d. $M^{-1}L^{-3}TA^2$
4. The dimensional formula of electric intensity is
   a. $MLT^{-2}A^{-1}$ b. $MLT^{-3}A^{-1}$
   c. $ML^2T^{-3}A^{-1}$ d. $ML^2T^{-3}A^{-2}$
5. The dielectric constant K of an insulator can be
   a. -1 b. 0 c. 0.5 d. 5
6. Choose the correct statement:
   a. The total charge of the universe is constant.
   b. The total number of the charged particles is constant.
   c. The total positive charge of the universe remains constant.
   d. The total negative charge of the universe remains constant.
7. Two neutrons are placed at some distance apart from each other. They will
   a. attract each other
   b. repel each other
   c. neither attract nor repel each other
   d. cannot say
8. When a soap bubble is charged, its size
   a. increases
   b. decreases
   c. remains the same
   d. increases if it is given positive charge and decreases if it is given negative charge
9. Two point charges certain distance apart in air repel each other with a force F. A glass plate is introduced between the charges. The force becomes F_1, where
   a. $F_1 < F$ b. $F_1 = F$
   c. $F_1 > F$ d. data is insufficient
10. There are two charges $+1\ \mu C$ and $+5\ \mu C$. The ratio of the forces (force on one due to other) acting on them will be
    a. 1 : 1 b. 1 : 2 c. 1 : 3 d. 1 : 4
11. Two point charges Q_1 and Q_2 are 3 m apart, and their sum of charges is 10 μC. If force of attraction between them is 0.075 N, then the values of Q_1 and Q_2 respectively, are
    a. 5 μC, 5 μC b. 15 μC, −5 μC
    c. 5 μC, 15 μC d. −15 μC, 5 μC
12. A certain charge 'Q' is to be divided into two parts q and $Q-q$. What is the relationship of 'Q' to 'q' if the two parts, placed at a given distance 'r' apart are to have maximum Coulomb repulsion?
    a. $q=\frac{Q}{2}$ b. $q=\frac{Q}{3}$
    c. $q=\frac{2Q}{2}$ d. $q=\frac{Q}{4}$
13. Three charged particles are placed on a straight line as shown in figure. q_1 and q_2 are fixed but q_3 can be moved. Under the action of the forces from q_1 and q_2, q_3 is in equilibrium. What is the relation between q_1 and q_2?

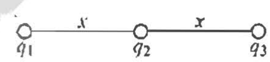


Fig. 1.123

    a. $q_1=4q_2$ b. $q_1=-q_2$
    c. $q_1=-4q_2$ d. $q_1=q_2$
14. Two particles A and B (B is right of A) having charges 8×10^{-6} C and -2×10^{-6} C, respectively, are held fixed with separation of 20 cm. Where should a third charged particle be placed so that it does not experience a net electric force.
    a. 5 cm right of B b. 5 cm left of A
    c. 20 cm left of A d. 20 cm right of B
15. Five balls numbered 1, 2, 3, 4, 5 are suspended using separate threads. The balls (1, 2), (2, 4) and (4, 1) show electrostatic attraction, while balls (2, 3) and (4, 5) show repulsion. Therefore, ball 1 must be
    a. negatively charged b. positively charge
    c. neutral d. made of metal
16. Electric charges A and B repel each other. Electric charges B and C also repel other. If A and C are held close together, they will
    a. attract b. repel
    c. not affect each other d. none of these
17. Two point charges repel each other with a force of 100 N. One of the charges is increased by 10% and the other is reduced by 10%. The new force of repulsion at the same distance would be
    a. 100 N b. 121 N
    c. 99 N d. none of these

18. Three charges $+Q_1$, $+Q_2$ and q are placed on a straight line such that q is somewhere in between $+Q_1$ and $+Q_2$. If this system of charges is in equilibrium, what should be the magnitude and sign of charge q?
    a. $\dfrac{Q_1Q_2}{(\sqrt{Q_1}+\sqrt{Q_2})^2}$, + ve
    b. $\dfrac{Q_1+Q_2}{2}$, + ve
    c. $\dfrac{Q_1Q_2}{(\sqrt{Q_1}+\sqrt{Q_2})^2}$, – ve
    d. $\dfrac{Q_1+Q_2}{2}$, – ve

19. Two positive and equal charges are fixed at a certain distance. A third small charge is placed in between the two charges and it experiences zero net force due to the other two.
    a. The equilibrium is stable if small charge is positive
    b. The equilibrium is stable if small charge is negative
    c. The equilibrium is always stable
    d. The equilibrium is not stable

20. An isolated charge q_1 of mass m is suspended freely by a thread of length l. Another charge q_2 is brought near it ($r >> l$). When q_1 is in equilibrium, tension in thread will be

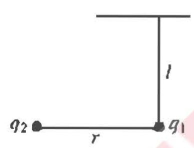


Fig. 1.124

    a. mg b. $>mg$
    c. $<mg$ d. none of these

21. Three equal charges, each $+q$, are placed on the corners of an equilateral triangle of side a. Then, the coulomb force experienced by one charge due to the rest of the two is
    a. kq^2/a^2 b. $2kq^2/a^2$
    c. $\sqrt{3}kq^2/a^2$ d. zero

22. A positively charged ball hangs from a long silk thread. Electric field at a certain point (at the same horizontal level of ball) due to this charge is E. Let us put a positive test charge q_0 at this point and measure F/q_0 on this charge. Then, E
    a. $>F/q_0$ b. $<F/q_0$
    c. $=F/q_0$ d. none of these

23. Electric field near a straight wire carrying a steady current is
    a. proportional to the distance from the wire
    b. proportional to inverse square of the distance from the wire
    c. inversely proportional to the distance from the wire
    d. zero

24. A force of 2.25 N acts on a charge of 15×10^{-4} C. Calculate the intensity of electric field at the point.
    a. 1500 NC^{-1} b. 150 NC^{-1}
    c. 15000 NC^{-1} d. none of these

25. An α particle is situated in an electric field of strength 15 $\times$ 10^4 NC^{-1}. Force acting on it is
    a. 4.8×10^{-12} N b. 4.8×10^{-14} N
    c. 48×10^{-14} N d. none of these

26. Two particles of masses in the ratio 1 : 2, with charges in the ratio 1 : 1, are placed at rest in a uniform electric field. They are released and allowed to move for the same time. The ratio of their kinetic energies will be finally
    a. 2 : 1 b. 8 : 1 c. 4 : 1 d. 1 : 4

27. Three equal charges, each $+q$, are placed on the corners of an equilateral triangle. The electric field intensity at the centroid of the triangle is
    a. kq/r^2 b. $3kq/r^2$
    c. $\sqrt{3}kq/r^2$ d. zero

28. A point charge of 100 μC is placed at $3\hat{i}+4\hat{j}$ m. Find the electric field intensity due to this charge at a point located at $9\hat{i}+12\hat{j}$ m.
    a. 8000 Vm^{-1} b. 9000 Vm^{-1}
    c. 2250 Vm^{-1} d. 4500 Vm^{-1}

29. Electric lines of force
    a. exist everywhere
    b. exist only in the immediate vicinity of electric charges
    c. exist only when both positive and negative charges are near one another
    d. are imaginary

30. Two charges Q_1 = 18 μC and Q_2 = – 2 μC are separated by a distance R and Q_1 is to the left of Q_2. The distance of the point where the net electric field is zero is
    a. between Q_1 and Q_2 b. left of Q_1 at $R/2$
    c. right of Q_2 at R d. right of Q_2 at $R/2$

31. Determine the electric field intensity at point P due to quadruple distribution shown in figure for $r >> a$.

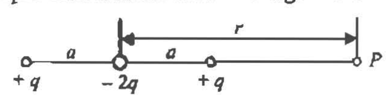


Fig. 1.125

    a. 0 b. kqa^2/r^4
    c. $6kqa^2/r^4$ d. $6kqa^2/r^2$

32. An oil drop, carrying six electronic charges and having a mass of 1.6×10^{-12} g, falls with some terminal velocity in a medium. What magnitude of vertical electric field is required to make the drop move upward with the same speed as it was formerly moving downward with? Ignore buoyancy.
    a. 10^5 NC^{-1} b. 10^4 NC^{-1}
    c. 3.3×10^4 NC^{-1} d. 3.3×10^5 NC^{-1}

33. What is the largest charge a metal ball of 1 mm radius can hold? Dielectric strength of air is 3×10^6 Vm^{-1}.

a. 3 nC b. 1/3 nC
c. 2 nC d. 1/2 nC

34. Five point charges, $+q$ each, are placed at the five vertices of a regular hexagon. The distance of center of hexagon from any of the vertices is a. The electric field at the center of the hexagon is

a. $\dfrac{q}{4\pi\varepsilon_0 a^2}$ b. $\dfrac{q}{8\pi\varepsilon_0 a^2}$
c. $\dfrac{q}{16\pi\varepsilon_0 a^2}$ d. zero

35. A ring of charge with radius 0.5 m has 0.002π m gap. If the ring carries a charge of +31 C, the electric field at the center is

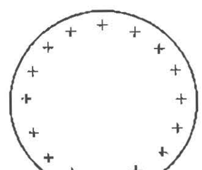


Fig. 1.126

a. 7.5×10^7 NC^{-1} b. 7.2×10^7 NC^{-1}
c. 6.2×10^7 NC^{-1} d. 6.5×10^7 NC^{-1}

36. A block of mass m containing a net negative charge $-q$ is placed on a frictionless horizontal table and is connected to a wall through an unstretched spring of spring constant k as shown. If horizontal electric field E parallel to the spring is switched on, then the maximum compression of the spring is

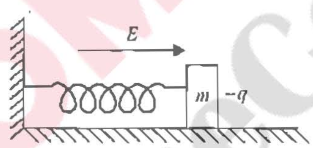


Fig. 1.127

a. $\sqrt{qE/k}$ b. $2qE/k$
c. qE/k d. zero

37. Figure shows the electric lines of force emerging from a charged body. If the electric fields at A and B are E_A and E_B, respectively, and if the distance between A and B is r, then

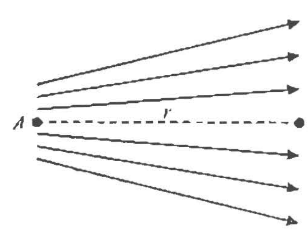


Fig. 1.128

a. $E_A > E_B$ b. $E_A < E_B$
c. $E_A = E_B/r$ d. $E_A = E_B/r^2$

38. If an electron has an initial velocity in a direction different from that of a uniform electric field, the path of the electron is

a. a straight line b. a circle
c. an ellipse d. a parabola

39. An electron is taken from a point A to point B along the path AB in a uniform electric field of intensity $E = 10$ Vm^{-1}. Side $AB = 5$ m and side $BC = 3$ m. Then, the amount of work done is

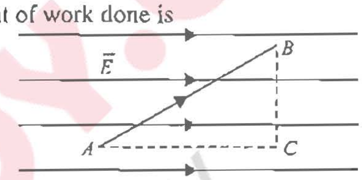


Fig. 1.129

a. 50 eV b. 40 eV
c. −50 eV d. −40 eV

40. A point charge q_1 is moved along a circular path of radius r in the electric field of another point charge q_2 at the center of the path. The work done by the electric field on the charge q_1 in half revolution is

a. zero b. positive
c. negative d. none of these

41. A spherical conducting ball is suspended by a grounded conducting thread. A positive point charge is moved near the ball. The ball will

a. be attracted to the point charge and swing toward it.
b. be repelled from the point charge and swing away from it.
c. not be affected by the point charge
d. none of these

42. Two point charges are located on the positive x-axis of a coordinate system (as shown in figure). Charge $q_1 = 1.0$ nC is 2.0 cm from the origin, and charge $q_2 = -3.0$ nC is 4.0 cm from the origin. What is the total force exerted by these two charges on a charge $q_3 = 5.0$ nC located at the origin? Gravitational forces are negligible.

a. 28 μN directed to the left
b. 28 μN directed to the right
c. 196 μN directed to the left
d. 196 μN directed to the right

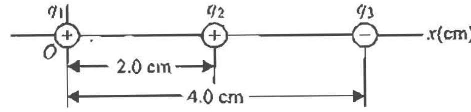


Fig. 1.130

43. Three +ve charges of equal magnitude 'q' are placed at the vertices of an equilateral triangle of side 'l'. How can the system of charges be placed in equilibrium?

a. By placing a charge $Q = \left(-\dfrac{q}{\sqrt{3}}\right)$ at the centroid of the triangle

b. By placing a charge $Q = \left(\frac{q}{\sqrt{3}}\right)$ at the centroid of the triangle
c. By placing a charge $Q = q$ at a distance l from all the three charges
d. By placing a charge $Q = -q$ above the plane of the triangle at a distance l from all the three charges

44. In figure, two equal positive point charges $q_1 = q_2 = 2.0\ \mu C$ interact with a third point charge $Q = 4.0\ \mu C$. Find the magnitude and direction of the net force on Q.

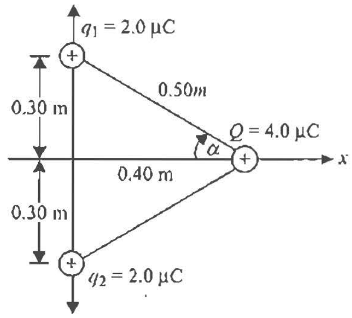


Fig. 1.131

a. 0.23 N in $+x$ direction
b. 0.46 N in $+x$ direction
c. 0.23 N in $-x$ direction
d. 0.46 N in $-x$ direction

45. Three identical spheres, each having a charge q and radius R, are kept in such a way that each touches the other two. Find the magnitude of the electric force on any sphere due to other two.

a. $\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{R}\right)^2$ b. $\frac{\sqrt{3}}{4\pi\varepsilon_0}\left(\frac{q}{R}\right)^2$
c. $\frac{\sqrt{3}}{16\pi\varepsilon_0}\left(\frac{q}{R}\right)^2$ d. $\frac{\sqrt{5}}{16\pi\varepsilon_0}\left(\frac{q}{R}\right)^2$

46. Five point charges, each of value $+q$, are placed on five vertices of a regular hexagon of side L. What is the magnitude of the force on a point charge of value $-q$ coulomb placed at the center of the hexagon?

a. $\frac{1}{\pi\varepsilon_0}\left(\frac{q}{L}\right)^2$ b. $\frac{2}{\pi\varepsilon_0}\left(\frac{q}{L}\right)^2$
c. $\frac{1}{2\pi\varepsilon_0}\left(\frac{q}{L}\right)^2$ d. $\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{L}\right)^2$

47. It is required to hold equal charges, q, in equilibrium at the corners of a square. What charge when placed at the center of the square will do this?

a. $-\frac{q}{2}\left(1+2\sqrt{2}\right)$ b. $\frac{q}{2}\left(1+2\sqrt{2}\right)$
c. $\frac{q}{4}\left(1+2\sqrt{2}\right)$ d. $-\frac{q}{4}\left(1+2\sqrt{2}\right)$

48. A point charge $q = -8.0$ nC is located at the origin. Find the electric field (in NC^{-1}) vector at the point $x = 1.2$ m, $y = -1.6$ m (as shown in Fig. 1.132).

a. $-14.4\hat{i} + 10.8\hat{j}$ b. $-14.4\hat{i} - 10.8\hat{j}$
c. $-10.8\hat{i} + 14.4\hat{j}$ d. $-10.8\hat{i} - 14.4\hat{j}$

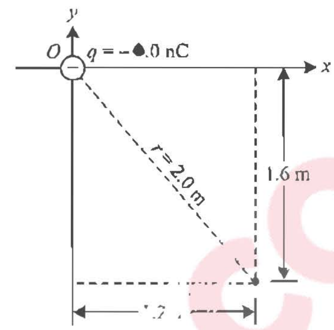


Fig. 1.132

49. A positive point charge 50 μC is located in the plane xy at a point with radius vector $\vec{r}_0 = 2\hat{i} + 3\hat{j}$. Evaluate the electric field vector $\vec{E}$ at a point with radius vector $\vec{r} = 8\hat{i} - 5\hat{j}$, where r_0 and r are expressed in meters.

a. $(1.4\hat{i} - 2.6\hat{j})\ kNC^{-1}$ b. $(1.4\hat{i} + 2.6\hat{j})\ kNC^{-1}$
c. $(2.7\hat{i} - 3.6\hat{j})\ kNC^{-1}$ d. $(2.7\hat{i} + 3.6\hat{j})\ kNC^{-1}$

50. A charge $q = 1\ \mu C$ is placed at point (3 m, 2 m, 5 m). Find the electric field vector at point P (0 m, −4 m, 3 m).

a. $-\frac{9}{343}(3\hat{i} + 6\hat{j} + 2\hat{k})\ kNC^{-1}$
b. $\frac{9}{343}(3\hat{i} - 6\hat{j} + \hat{k})\ kNC^{-1}$
c. $\frac{3}{343}(3\hat{i} + 6\hat{j} + 2\hat{k})\ kNC^{-1}$
d. $\frac{9}{343}(3\hat{i} + 6\hat{j} + 2\hat{k})\ kNC^{-1}$

51. Four identical charges Q are fixed at the four corners of a square of side a. Find the electric field at a point P located symmetrically at a distance $\frac{a}{\sqrt{2}}$ from the center of the square.

a. $\frac{Q}{2\sqrt{2}\pi\varepsilon_0 a^2}$ b. $\frac{Q}{\sqrt{2}\pi\varepsilon_0 a^2}$
c. $\frac{2\sqrt{2}Q}{\pi\varepsilon_0 a^2}$ d. $\frac{\sqrt{2}Q}{\pi\varepsilon_0 a^2}$

52. A thin glass rod is bent into a semicircle of radius r. A charge $+Q$ is uniformly distributed along the upper half and a charge $-Q$ is uniformly distributed along the lower half, as shown in Fig. 1.133. Calculate electric field E at P, the center of semicircle.

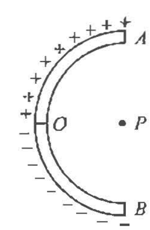


Fig. 1.133

a. $\frac{Q}{\pi^2\varepsilon_0 r^2}$ b. $\frac{2Q}{\pi^2\varepsilon_0 r^2}$