PHYSICS

For Joint Entrance Examination (JEE)

Electrostatics
and
Current Electricity

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CENGAGE
Learning·

Brief Contents

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Preface

While the paper-setting pattern and assessment methodology have been revised many times over and newer criteria devised to help develop more aspirant-friendly engineering entrance tests, the need to standardize the selection processes and their outcomes at the national level has always been felt. A combined national-level engineering entrance examination has finally been proposed by the Ministry of Human Resource Development, Government of India. The Joint Entrance Examination (JEE) to India's prestigious engineering institutions (IITs, IIITs, NITs, ISM, IISERs, and other engineering colleges) aims to serve as a common national-level engineering entrance test, thereby eliminating the need for aspiring engineers to sit through multiple entrance tests.

While the methodology and scope of an engineering entrance test are prone to change, there are two basic objectives that any test needs to serve:

1. The objective to test an aspirant's caliber, aptitude, and attitude for the engineering field and profession.

2. The need to test an aspirant's grasp and understanding of the concepts of the subjects of study and their applicability at the grassroot level.

Students appearing for various engineering entrance examinations cannot bank solely on conventional shortcut measures to crack the entrance examination. Conventional techniques alone are not enough as most of the questions asked in the examination are based on concepts rather than on just formulae. Hence, it is necessary for students appearing for joint entrance examination to not only gain a thorough knowledge and understanding of the concepts but also develop problem-solving skills to be able to relate their understanding of the subject to real-life applications based on these concepts.

This series of books is designed to help students to get an all-round grasp of the subject so as to be able to make its useful application in all its contexts. It uses a right mix of fundamental principles and concepts, illustrations which highlight the application of these concepts, and exercises for practice. The objective of each book in this series is to help students develop their problem-solving skills/accuracy, the ability to reach the crux of the matter, and the speed to get answers in limited time. These books feature all types of problems asked in the examination—be it MCQs (one or more than one correct), assertion-reason type, matching column type, comprehension type, or integer type questions. These problems have skillfully been set to help students develop a sound problem-solving methodology.

Not discounting the need for skilled and guided practice, the material in the books has been enriched with a number of fully solved concept application exercises so that every step in learning is ensured for the understanding and application of the subject. This whole series of books adopts a multi-faceted approach to mastering concepts by including a variety of exercises asked in the examination. A mix of questions helps stimulate and strengthen multi-dimensional problem-solving skills in an aspirant.

It is imperative to note that this book would be as profound and useful as you want it to be. Therefore, in order to get maximum benefit from this book, we recommend the following study plan for each chapter.

Step 1: Go through the entire opening discussion about the fundamentals and concepts.

Step 2: After learning the theory/concept, follow the illustrative examples to get an understanding of the theory/concept.

Overall the whole content of the book is an amalgamation of the theme of physics with ahead-of-time problems, which equips the students with the knowledge of the field and paves a confident path for them to accomplish success in the JEE.

With best wishes!

B.M. Sharma

Coulomb's Laws and Electric Field

ELECTRIC CHARGE

Electric charge, like mass, is one of the fundamental attributes of the particle of which the matter is made of. Charge is the physical property of certain fundamental particles (such as, electron, proton) by virtue of which they interact with the other similar fundamental particles.

- Charge is an intrinsic property of some fundamental particles which accompanies these particles wherever they exist.
- Charge is that property of a body/particle which is responsible for 'electrical force' between them.

To distinguish the nature of interaction, charges are divided into two parts: (i) positive (ii) negative

Figure 1.1 shows an experiment to demonstrate that there are two types of charges.

We know that matter consists of atoms. An atom consists of a central core (called nucleus) and electrons. Electrons orbit around the nucleus. Nucleus consists of neutrons and protons. Neutrons do not contain any net charge. Protons and electrons have equal charges, but of opposite nature. Protons are positively charged, whereas electrons are negatively charged. Protons, however, are very heavy when compared with electrons, about 1836 times. Protons are imprisoned in the nucleus along with neutrons due to the strongest binding force existing in nature called 'strong or nuclear force'. Thus, protons do not travel from atom to atom. The outermost electrons may travel from atom to atom. Hence, we say that electrons are the basis of electricity.

Charge on a proton or on an electron is of indivisible nature. We designate this charge as $+e$ and $-e$, respectively. Hence, charge in or on any object is always an integral multiple of the electronic charge.

In a normal atom:

i. Number of protons are equal to number of electrons.
ii. Protons have the basic $+e$ charge and electrons have the basic $-e$ charge.
iii. Hence, a normal atom is electrically neutral.

Electrons can travel from one atom to another and from one body to another. If a body loses one electron, it becomes positively charged with $+e$ charge and vice versa.

A body, however, cannot lose or gain any proton, which is heavy and remains imprisoned in the nucleus, by ordinary methods.

Note: *Basic unit of charge = e, whose magnitude is equal to the magnitude of charge on an electron or proton, i.e., $e=1.6\times10^{-19}$ C.*

S.I. unit of charge: As mentioned, $e=1.6\times10^{-19}$ C. Here, e stands for one electronic charge which is the basic unit of charge; C stands for coulomb (note the small c in coulomb). coulomb is the S.I. unit of charge.

CHARGING OF A BODY

Ordinarily, matter contains equal number of protons and electrons. A body can be charged by the transfer of electrons or redistribution of electrons.

A body can be charged by the transfer of electrons and by the transfer of protons. Why?

It is because protons are inside the nucleus and it is very difficult to remove them from there. Electrons lie in the outer shells and it is easier to remove them.

Glass rod — **Repulsion** — **Glass rod**
Both the glass rods are rubbed with silk cloth

Nature of charges acquired by both glass rods (or by both ebonite rods) should be the same.
Conclusion: Like charges repel each other

Ebonite rod — **Repulsion** — **Ebonite rod**
Both the ebonite rods are rubbed with fur

Glass rod — **Attraction** — **Ebonite rod**
Glass rod rubbed with silk and ebonite rod rubbed with fur

As now these are attracting, so nature of charge acquired by ebonite rod should be different from nature of charge acquired by glass rod. Hence, there are two kinds of charges.
Conclusion: Unlike charges attract each other

Final conclusion: Like charges repel each other and unlike charges attract each other.
Later on, it was found that glass rod had positive charge and ebonite rod had negative charge. There is no third kind of charge.

Fig. 1.1

To charge a body negatively: some electrons are given to it.

To charge a body positively: some electrons are taken from it.

WORK FUNCTION OF A BODY

It is the amount of work to be done on a body in order to remove an electron from its surface. Obviously, it is easier to remove an electron from a body whose work function is lower.

Let us see how bodies get charged due to friction:

As shown in Fig. 1.2, let $W_2 > W_1$. Now, suppose A and B are rubbed together.

Net transfer of electrons will take place from A to B.

It is because electrons in A are loosely bound as work function of A is less than B.

It is to be noted that mass is also affected during charging.

(Mass of negatively charged body increases and that of positively charged body decreases.)
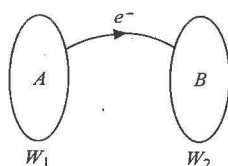
Fig. 1.2

Basically charging can be done by three methods:

1. Friction, **2.** Conduction, and **3.** Induction

Charging by Friction

When two bodies are rubbed together, electrons are transferred from one body to the other making one body positively charged and the other negatively charged.

Example: When a glass rod is rubbed with silk, the rod becomes positively charged, whereas silk gets negatively charged. However, ebonite on rubbing with wool becomes negatively charged making the wool positively charged.

Charging by Conduction

The process of charging from an already charged body can happen by either conduction or induction. Conduction from a charged body involves transfer of like charges. A positively charged body can cause more bodies to get positively charged, but the sum of the total charge on all positively charged bodies will be the same as charge on initially considered charged body.

Charging by Induction

Induction is a process by which a charged body can be used to charge neutral bodies without touching them or losing its own charge. If a charged body is brought near a neutral body, the charged body attracts opposite charge and repels similar charge present on the neutral body. If the neutral body is now earthed, the like charge is neutralized by the flow of charge from earth, leaving unlike charge on the body. Now, the earthing and the charging body is removed leaving the initially neutral body charged. The whole process is as shown in Fig. 1.3.

PROPERTIES OF ELECTRIC CHARGE

Quantization of Charge

Charge exists in discrete packets rather than in continuous amount, i.e., charge on any body is the integral multiple of the charge on an electron or proton.

$$Q = \pm ne, \text{ where } n = 0, 1, 2, \ldots$$

Conservation of Charge

Charge is conserved, i.e., total charge on an isolated system is constant. By isolated system, we here mean a system through the boundary of which no charge is allowed to escape or enter. This

A neutral body to be charged is mounted on an insulating stand.

Let a positively charged rod is brought near this body. Separation of charges in the body will take place.

The body is earthed. Positive charge flows to the earth.

Interesting thing here is that the rod does not loose any charge.

Now, the rod is also removed. Negative charge spreads uniformly on the body. Thus, the body becomes negatively charged.

Earthing is removed

Fig. 1.3

does not require that the amounts of positive and negative charges are separately conserved.

Additivity of Charge

Total charge on a body is the algebraic sum of all the charges located anywhere on the body. While adding the charges, their sign must be taken into consideration.

For example, if a body has charges 2 C, −5 C, 4 C and 6 C (Fig. 1.4), then the total charge on the body is $2 − 5 + 4 + 6 = 7$ C.

Note that charges are added like real numbers. They have no direction. *So, charge is a scalar quantity.*
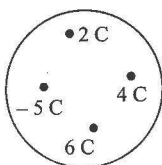
Fig. 1.4

Charge is Invariant

Charge does not depend on the speed of body.

Points to Remember

- There are two types of forces which act between two charges. If the charges are stationary, there is only one type of force between them. It is called electric or electrostatic force. It is given by Coulomb's law for point charges. If the charges are moving, then two types of forces act between them. The first one is electric force. The other force which emerges due to motion is called magnetic force. We shall study magnetic force the following chapter.
- **Charge produces electric and magnetic fields and radiates energy:** A stationary charged particle produces only electric field in the space surrounding it. A charged particle moving without acceleration produces electric as well as magnetic field. A charged particle in accelerated motion radiates energy as well, in the form of electromagnetic waves.

Illustration 1.1 A glass rod is rubbed with a silk cloth. The glass rod acquires a charge of $+19.2 \times 10^{-19}$ C.

1. Find the number of electrons lost by glass rod.
2. Find the negative charge acquired by silk.
3. Is there transfer of mass from glass to silk?
   Given, $m_e = 9 \times 10^{-31}$ kg.

Sol.

1. Number of electrons lost by glass rod is
$$n = \frac{q}{e} = \frac{19.2 \times 10^{-19}}{1.6 \times 10^{-19}} = 12$$

2. Charge on silk = $−19.2 \times 10^{-19}$ C
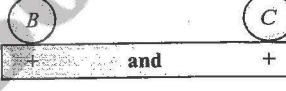
3. Since an electron has a finite mass ($m_e = 9 \times 10^{-31}$ kg), there will be transfer of mass from glass rod to silk cloth. Mass transferred = $12 \times (9 \times 10^{-31}) = 1.08 \times 10^{-29}$ kg

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Note that mass transferred is negligibly small. This is expected because the mass of an electron is extremely small.

Illustration 1.2 Electric charges A and B attract each other. Electric charges B and C repel each other. If A and C are held close together, they will

a. attract 　 b. repel. 　 c. not affect each other
d. more information is needed to answer.

Sol. a.

Case 1	Case 2
If A and B attract each other, then	If A and B repel each other, then
A　　　　　B　$+$　and　$−$	A　　　　　B　　　and　$+$
If B and C attract each other, then	If B and C repel each other, then
B　　　　　C　　　and　$−$	B　　　　　C　$+$　and　$+$

From both cases, we see that A and C will be having unlike charges. Hence, if the charges A and C are held together, they will attract each other.

Illustration 1.3 If an object made of substance A is rubbed with an object made of substance B, then A becomes positively charged and B becomes negatively charged. If, however, an object made of substance A is rubbed against an object made of substance C, then A becomes negatively charged. What will happen if an object made of substance B is rubbed against an object made of substance C?

a. B becomes positively charged and C becomes positively charged.

b. B becomes positively charged and C becomes negatively charged.

c. B becomes negatively charged and C becomes positively charged.

d. B becomes negatively charged and C becomes negatively charged.

Sol. c. When A and B are rubbed, A becomes positively charged and B becomes negatively charged. It means Electrons are loosely bound with A in comparison to B. When A and C are rubbed together, A becomes negatively charged and C positively charged. It means Electrons are loosely bound with C in comparison to A. Hence, in C, electrons are most loosely bound. So, if B and C are rubbed together, C will loose electrons and B will receive electrons. Hence, C will become positively charged and B will become negatively charged.

Illustration 1.4 Objects A, B and C are three identical, insulated, spherical conductors. Originally, A and B have

charges of +3 mC, whereas *C* has a charge of −6 mC. Objects *A* and *C* are allowed to touch, then they are moved apart. Objects *B* and *C* are allowed to touch before they are moved apart.

  i. If objects *A* and *B* are now held near each other, they will

    a. attract   b. repel   c. have no effect on each other.

  ii. If instead objects *A* and *C* are held near each other, they will

    a. attract   b. repel   c. have no effect on each other.

Sol.

Initially,   (*A*) +3 mC   (*B*) +3 mC   (*C*) −6 mC

- When the objects *A* and *C* are allowed to touch and then moved apart:

(*A*)   (*C*)   (*A*) ⟷ (*C*)

$[(+3 \text{ mC} + (-6 \text{ mC}) = -3 \text{ mC})]$ $-\frac{3}{2}$ mC $-\frac{3}{2}$ mC

- When the objects *B* and *C* are allowed to touch and then moved apart:

(*B*)   (*C*)   (*B*) ⟷ (*C*)

$\left[(+3 \text{ mC}) + \left(-\frac{3}{2} \text{ mC}\right) = +\frac{3}{2} \text{ mC}\right]$ $+\frac{3}{4}$ mC $+\frac{3}{4}$ mC

i.a. Hence, if *A* and *B* are now held near each other, they will attract each other.

ii.a. If *A* and *C* are now held near each other, they will also attract each other.

Illustration 1.5 Figure 1.5 shows that a positively charged rod is brought near two uncharged metal spheres *A* and *B* clam red on insulated stands and placed in contact with each other.

  i. What would happen if the rod is removed before the spheres are separated?

  ii. Would the induced charges be equal in magnitude even if the spheres had different sizes or different conductors?

  iii. What will happen if the spheres are separated first and then the rod is removed far away?
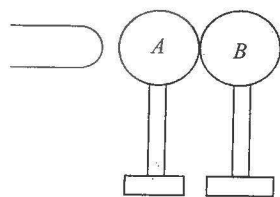
Fig. 1.5

Sol.

  i. When a positively charged rod is brought near *A*, the free electrons in the sphere *A* are attracted towards the rod and moved on the left side of *A*. This movement leaves

unbalanced positive charge on *B*. If the rod is removed before the spheres are separated, the excess electrons on sphere *A* would flow back to *B*. Both the spheres will become uncharged.
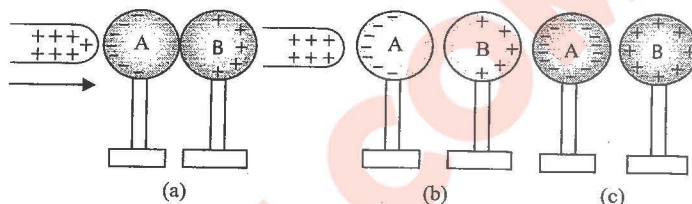
Fig. 1.6

  ii. Yes, net charge is conserved. Before the rod is brought near *A*, both *A* and *B* were neutral. They will remain so even if they have different sizes or materials.

  iii. If the rod is removed after the spheres are separated, then sphere *A* will have net negative charge and sphere *B* will have net positive charge of same magnitude as shown in Fig. 1.6.

Concept Application Exercise 1.1

1. a. How many electrons are in 1 coulomb of negative charge?
   b. Which is the true test of electrification, attraction or repulsion?
   c. Can a body have charge of 0.8×10^{-19} C ?
2. If only one charge is available, can itself be used to obtain a charge many times greater than itself in magnitude?
3. a. Can two bodies having like charges attract each other? (Yes/No)
   b. Can a charged body attract an uncharged body? (Yes/No)
   c. Two identical metallic spheres of exactly equal masses are taken, one is given a positive charge *q* and the other an equal negative charge. Their masses after charging are different. Comment on the statement.
4. A particle has charge of $+10^{-12}$ C.
   a. Does it contain more or less number of electrons as compared with the neutral state?
   b. Calculate the number of electrons transferred to provide this charge.
5. An ebonite rod is rubbed with fur. The ebonite rod is found to have a charge of -3.2×10^{-8} C on it.
   a. Calculate the number of electrons transferred.
   b. What is the charge on fur after rubbing?
6. The electric charge of macroscopic bodies is actually a surplus or deficiency of electrons. Why not protons?
7. A charged rod attracts bits of dry paper which after touching the rod often jump away from it violently. Explain.
8. A person standing on an insulating stool touches a charged insulated conductor. Will the conductor get completely discharged?
9. An electron moves along a metal tube with variable cross section. How will its velocity change when it approaches the neck of the tube (Fig. 1.7)?

Fig. 1.7

10. Define the following statement "If there were only one electrically charged particle in the entire universe, the concept of electric charge would be meaningless".

COULOMB'S LAW

The force of interaction between two point charges is proportional to the product of magnitudes of the two charges and inversely proportional to the square of distance between them.

Fig. 1.8

Let two point electric charges q_1 and q_2 be at rest, separated by a distance r, then they exert a force on each other which is given by

$$F = k\frac{q_1 q_2}{r^2}$$

where k is a proportionality constant known as *electrostatic force constant*

If there is free space (or vacuum) between the two charges, then $k = \dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ Nm^2 C^{-2} (in SI units)

where $\varepsilon_0 = \dfrac{1}{4\pi k} = 8.85 \times 10^{-12}$ C^{2}N^{-1}m^{-2} is the absolute electric permittivity of the free space.

So, force between two charges is given as

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} \qquad (i)$$

Equation (i) is applicable only for point charges placed in vacuum. Now, what happens if the two charges are placed in some medium? In a medium, the force is given as

$$F' = k'\frac{q_1 q_2}{r^2} \qquad (ii)$$

where $k' = \dfrac{1}{4\pi\varepsilon}$ and in this ε is known as absolute electrical permittivity of medium. Then,

$$F' = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2} \qquad (iii)$$

The ratio $\varepsilon/\varepsilon_0 = \varepsilon_r$ is known as relative electrical permittivity of medium. It is also known as *dielectric constant* and denoted by K.

So, $\qquad\qquad \varepsilon/\varepsilon_0 = \varepsilon_r = K$

The value of K for different materials: vacuum = 1, air = 1.006, glass = 3 to 4, water = 81, conductor = ∞.

In general, $K \geq 1$.

Now, from Eqs. (i) and (iii), $\dfrac{F'}{F} = \dfrac{\varepsilon_0}{\varepsilon} = \dfrac{1}{K} \Rightarrow F' = \dfrac{F}{K}$. It means

when the charges are placed in a medium, the force decreases K times. Also, $K = F/F'$.

So, *the dielectric constant of a medium may be defined as the ratio of force between two charges when they are placed in vacuum to that when they are placed in that medium at same separation.*

Note:
- *Coulomb's law is not valid for distances < 10^{-15} m.*
- *Electrostatic forces are comparatively stronger than gravitational forces. Can you show this?*

(As an example—when we hold a book in our hand, electric force between hand and the book is sufficient to balance the gravitational force of earth on the book due to entire earth.)

Some Important Points
- Coulomb's law is applicable only for point charges.
- Coulomb's law is similar to Newton's gravitational law and both obey inverse square law.
- Coulomb's law obeys Newton's third law, i.e., the forces exerted by the two charges on each other are equal and opposite.
- This force acts along the line joining the two particles (called central force).
- Electrostatic force is a conservative force.

COULOMB'S LAW IN VECTOR FORM

Let q_1 and q_2 be two like charges placed at points A and B, respectively, in vacuum.
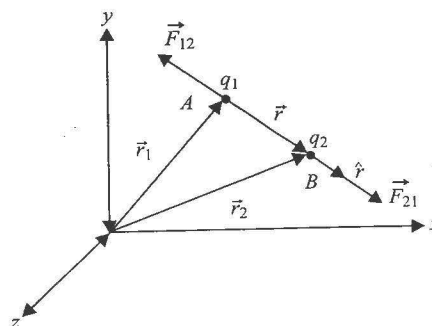
Fig. 1.9

$\vec{r}_1$ is the position vector of point A and $\vec{r}_2$ is the position vector of point B.

Let $\vec{r}$ is vector from A to B, then $\vec{r} = \vec{r}_2 - \vec{r}_1$ and $r = |\vec{r}_2 - \vec{r}_1|$.

$$\Rightarrow \qquad \hat{r} = \frac{\vec{r}}{r} = \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|}$$

Let $\vec{F}_{21}$ be the force on charge q_2 due to q_1; and $\vec{F}_{12}$ be the force on charge q_1 due to q_2.

From Fig. 1.9, it is clear that $\vec{F}_{21}$ and $\hat{r}$ are in the same direction, so

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\frac{\vec{r}}{r} = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{\vec{r}}{r^3}$$

$$\Rightarrow \qquad F_{21} = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|^3}$$

The above equations give the Coulomb's law in vector form.

As we know that charges apply equal and opposite forces on each other, so we have

$$\vec{F}_{12} = -\vec{F}_{21} \Rightarrow \vec{F}_{12} = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|^3}$$

Superposition Principle

It enables us to calculate the force acting on a charge due to more than one charge.

According to superposition principle, the total force on a given charge is vector sum of all the individual forces exerted by each of the other charges.

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots + \vec{F}_n$$

Another important point is that the force between two charges remains unaffected due to the presence of a third charge.
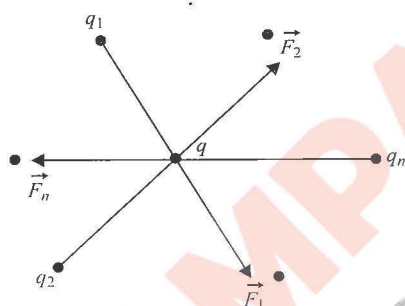
Fig. 1.10

Note:

- *Coulomb's law and principle of superposition together can explain whole of the electrostatics.*
- *Both Coulomb's law and Gravitational law describe inverse square law that involve a property of interacting particles— the charge in one case and mass in the other case.*

Illustration 1.6 Two identical conducting spheres 1 and 2 carry equal amounts of charge and are fixed at a certain distance apart that is large compared with their diameters. The spheres repel each other with an electrical force of 88 mN. Suppose now that a third identical sphere 3, having an insulating handle and initially uncharged, is touched first to sphere 1 and then to sphere 2 and finally removed. Find the force between spheres 1 and 2 as shown in Fig. 1.11(d).

(a)

(b)

(c)

(d)
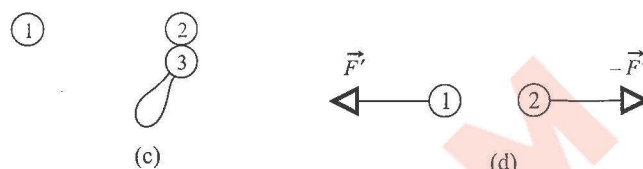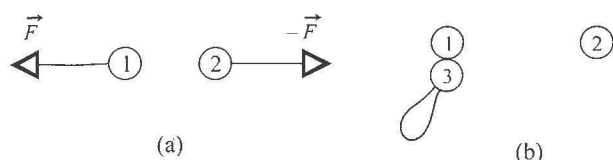
Fig. 1.11

Sol. Initial force between '1' and '2' is $F = kq^2/r^2 = 88$ mN

Charge on '1' after sphere '3' is touched with '1' is $q/2$. Same charge will be on sphere '3' also.

Charge on '2' after sphere '3' is touched with '2' is

$$\frac{q + (q/2)}{2} = \frac{3q}{4}$$

Now, force between '1' and '2' in situation (d) is

$$F' = \frac{K(q/2)(3q/4)}{r^2} = \frac{3}{8}\frac{kq^2}{r^2} = \frac{3}{8}\times 88 = 33 \text{ mN}$$

Illustration 1.7 Two identical He-filled spherical balloons each carrying a charge q are tied to a weight W with strings and float in equilibrium as shown in Fig. 1.12(a). Find

i. the magnitude of q, assuming that the charge on each balloon acts as if it were concentrated at the centre.

ii. the volume of each balloon.

Take density of He as ρ_{He} and density of air as ρ_a. Ignore the weight of the unfilled balloons.
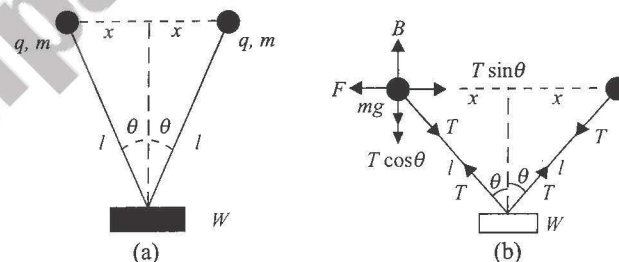
Fig. 1.12

Sol. i. $2T\cos\theta = W, T\sin\theta = F$ [Fig.1.12(b)]

$$\Rightarrow \qquad \frac{\tan\theta}{2} = \frac{F}{W} \Rightarrow F = W\frac{\tan\theta}{2}$$

$$\Rightarrow \qquad \frac{q^2}{4\pi\varepsilon_0(2x)^2} = \frac{W\tan\theta}{2} \Rightarrow q = \sqrt{8W\tan\theta\pi\varepsilon_0 x^2}$$

ii. $T\cos\theta + mg = B \Rightarrow \dfrac{W}{2} + V\rho_{He}g = V\rho_a g$

$$\Rightarrow \qquad V = \frac{W}{2(\rho_a - \rho_{He})g}$$

Illustration 1.8 Two particles, each having a mass of 5 g and charge 10^{-7} C, stay in limiting equilibrium on a horizontal table with a separation of 10 cm between them. Find the coefficient of friction between each particle and the table, which is the same between each particle and table.

Sol. Friction force f will balance the electrostatic repulsion, i.e.,

$$f = F \Rightarrow \mu mg = \frac{q^2}{4\pi\varepsilon_0 r^2}$$
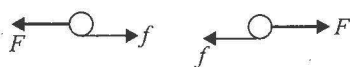
Fig. 1.13

$$\Rightarrow \quad \mu \times \frac{5}{1000} \times 10 = \frac{9 \times 10^9 \times (10^{-7})^2}{(0.10)^2} \Rightarrow \mu = 0.18$$

Illustration 1.9 **A particle of mass m carrying a charge $-q_1$ starts moving around a fixed charge $+q_2$ along a circular path of radius r. Prove that period of revolution T of charge $-q_1$ is given by $T = \sqrt{\dfrac{16\pi^3 \varepsilon_0 m r^3}{q_1 q_2}}$.**

Sol. Electrostatic force on $-q_1$ due to $+q_2$ will provide the necessary centripetal force, hence

$$\frac{kq_1 q_2}{r^2} = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{kq_1 q_2}{mr}}$$

Now, $\qquad T = \dfrac{2\pi r}{v} = \sqrt{\dfrac{16\pi^3 \varepsilon_0 m r^3}{q_1 q_2}}$
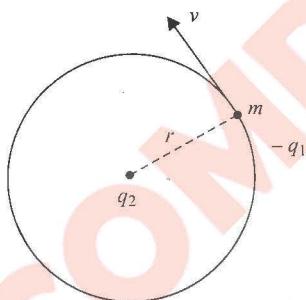
Fig. 1.14

Illustration 1.10 **Consider three charges q_1, q_2 and q_3, each equal to q, at the vertices of an equilateral triangle of side l. What is the force on a charge Q placed at the centroid of the triangle?**

Sol. Method 1. The resultant of three equal coplanar vectors acting at a point will be zero if these vectors form a closed polygon (Fig. 1.15). Hence, the vector sum of the forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ is zero.

Method 2. The forces acting on the charge Q are

$$\vec{F}_1 = \text{force on } Q \text{ due to } q_1 = \frac{1}{4\pi\varepsilon_0} \frac{Qq_1}{AO^2} \widehat{AO}$$

$$\vec{F}_2 = \text{force on } Q \text{ due to } q_2 = \frac{1}{4\pi\varepsilon_0} \frac{Qq_2}{BO^2} \widehat{BO}$$
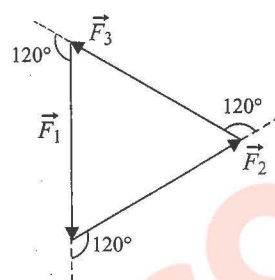
Fig. 1.15
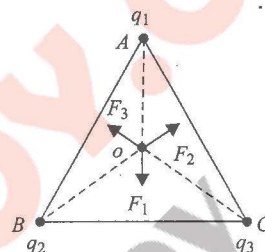
Fig. 1.16

$$\vec{F}_3 = \text{force on } Q \text{ due to } q_3 = \frac{1}{4\pi\varepsilon_0} \frac{Qq_3}{CO^2} \widehat{CO}$$

The resultant force is $\vec{F}_R = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$

$$= \frac{1}{4\pi\varepsilon_0} \frac{Qq}{AO^2} (\widehat{AO} + \widehat{BO} + \widehat{CO}) = 0$$

(as $|q_1| = |q_2| = |q_3|$ and $|\overline{AO}| = |\overline{BO}| = |\overline{CO}|$)

Also, $\overline{AO} + \overline{BO} + \overline{CO} = 0$ because these are three equal vectors in a plane making angles of 120° with each other.

Method 3. The resultant force $\sum \vec{F}$ is the vector sum of individual forces, i.e.,

$$\sum \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 \quad \text{or}$$

$$\sum F_x = F_{1x} + F_{2x} + F_{3x}$$

$$= 0 + F_2 \cos 30° - F_3 \cos 30° \qquad \text{(i)}$$
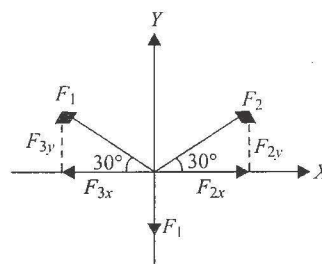
Fig. 1.17

And $\sum F_y = F_{1y} + F_{2y} + F_{3y}$

$$= -F_1 + F_2 \sin 30° + F_3 \sin 30° \qquad \text{(ii)}$$

As $|F_1| = |F_2| = |F_3| = |F|$ (say), Eqs. (i) and (ii) become

$\sum F_x = 0$ and $\sum F_y = 0$. Hence, resultant force $\sum \vec{F} = 0$.

Illustration 1.11 Point charges are placed at the vertices of a square of side a as shown in Fig. 1.18. What should be sign of charge q and magnitude of the ratio $|q/Q|$ so that

i. net force on each Q is zero?
ii. net force on each q is zero?
iii. Is it possible that the entire system could be in electrostatic equilibrium?

Fig. 1.18

Sol.
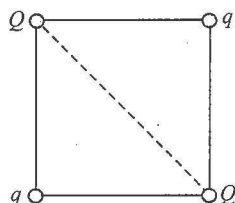
i. Consider the forces acting on charge Q placed at A (shown in Fig. 1.19(a) and (b))

Case 1. Let the charges q and Q are of same sign.
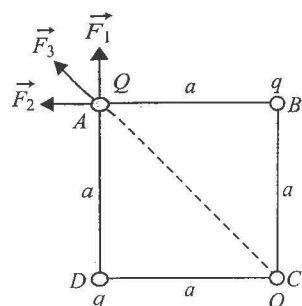
(a)
(q and Q are of same nature)
Here, net force cannot be zero.
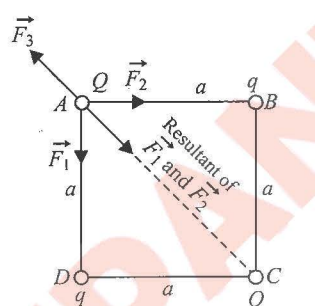
(b)
(q and Q are of opposite nature)
Here, net force can be zero.

Fig. 1.19

Here, $F_1 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{qQ}{a^2}$ (force of q at D on Q at A)

$F_2 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{qQ}{a^2}$ (force of q at B on Q at A)

$F_3 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{QQ}{2a^2}$ (force of Q at C on Q at A)

In Fig. 1.19(a), resultant of forces $\vec{F_1}$ and $\vec{F_2}$ will lie along $\vec{F_3}$ so that net force on Q cannot be zero. Hence, q and Q have to be of opposite signs.

Case II. Let the charges q and Q are of opposite sign.

In this case, as shown in Fig. 1.19(b), resultant of $\vec{F_1}$ and $\vec{F_2}$ will be opposite to $\vec{F_3}$ so that it becomes possible to obtain a condition of zero net force.

Let us write $\vec{F_R} = \vec{F_1} + \vec{F_2}$

$\therefore$ $\sqrt{F_1^2 + F_2^2} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{qQ}{a^2}\sqrt{2}$

Direction of $\vec{F_R}$ will be along AC ($\vec{F_R}$, being resultant of forces of equal magnitude, bisects the angle between the two) $\vec{F_R}$ and

$\vec{F_3}$ are in opposite directions. Net force on Q can be zero if their magnitudes are also equal, i.e.,

$$\dfrac{1}{4\pi\varepsilon_0}\dfrac{qQ}{a^2}\sqrt{2} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{QQ}{2a^2} \text{ or } \dfrac{Q}{4\pi\varepsilon_0 a^2}\left(\sqrt{2}q - \dfrac{Q}{2}\right) = 0$$

$$\Rightarrow q = \dfrac{Q}{2\sqrt{2}} \Rightarrow \left|\dfrac{q}{Q}\right| = \dfrac{1}{2\sqrt{2}} \quad \{Q \neq 0\}$$

Therefore, the sign of q should be negative of Q.

ii. Consider now the forces acting on charge q placed at B (see Fig. 1.20(a) and (b)).

In a similar manner, as discussed in (i), for net force on q to be zero, q and Q have to be of opposite signs. This is also shown in the given figures.
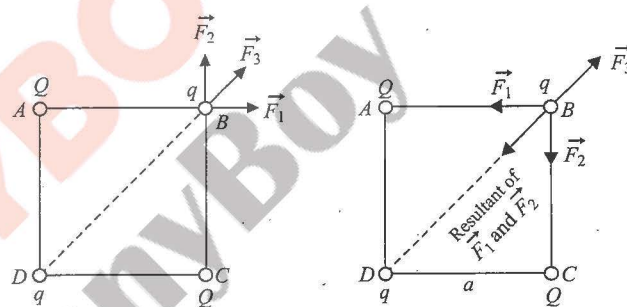
(a)
(q and Q are of same sign)
Here, net force cannot be zero.

(b)
(q and Q are of opposite sign)
Here, net force can be zero.

Fig. 1.20

Now, $F_1 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Qq}{a^2}$ (force of Q at A on q at B)

$F_2 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Qq}{a^2}$ (force of Q at C on q at B)

$F_3 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2}{2a^2}$ (force of q at D on q at B)

Referring to Fig. 1.20(b), let us write $\vec{F_R} = \vec{F_1} + \vec{F_2}$

$\therefore$ $F_R = \sqrt{F_1^2 + F_2^2} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Qq}{a^2}\sqrt{2}$

Resultant of $\vec{F_1}$ and $\vec{F_2}$, i.e., $\vec{F_R}$, is opposite to $\vec{F_3}$. Net force can become zero if their magnitudes are also equal, i.e.,

$$\dfrac{1}{4\pi\varepsilon_0}\dfrac{Qq}{a^2}\sqrt{2} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2}{2a^2} \Rightarrow \dfrac{q}{4\pi\varepsilon_0 a^2}\left(\sqrt{2}Q - \dfrac{q}{2}\right) = 0$$

$$\Rightarrow \quad Q = \dfrac{q}{2\sqrt{2}} \Rightarrow \left|\dfrac{q}{Q}\right| = 2\sqrt{2} \quad \{q \neq 0\}$$

Therefore, the sign of q should be negative of Q.

In this case, we need not to repeat the calculation as the present situation is same as the previous one; we can directly write $|q/Q| = 2\sqrt{2}$.

iii. The entire system cannot be in equilibrium since both conditions, i.e., $q = -\dfrac{Q}{2\sqrt{2}}$ and $Q = -\dfrac{q}{2\sqrt{2}}$ cannot be satisfied together.

Illustration 1.12 Two identical small charged spheres, each having a mass m, hang in equilibrium as shown in Fig.1.21(a). The length of each string is l and the angle made by any string with vertical is θ. Find the magnitude of the charge on each sphere.

Sol. The forces acting on the sphere are tension in the string T; force of gravity, mg; repulsive electric force, F_e, as shown in the free-body diagram of the sphere (Fig. 1.21(b)). The sphere is in equilibrium. The forces in the horizontal and vertical directions must separately add up to zero.

$$\sum F_x = T\sin\theta - F_e = 0 \tag{i}$$

$$\sum F_y = T\cos\theta - mg = 0 \tag{ii}$$
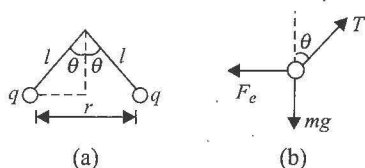
Fig. 1.21

From Eq. (ii), $T = \dfrac{mg}{\cos\theta}$. Thus, we can eliminate T from Eq. (i) to obtain

$$F_e = mg\tan\theta \quad \text{or} \quad \frac{kq^2}{r^2} = mg\tan\theta \tag{iii}$$

where $k = \dfrac{1}{4\pi\varepsilon_0}$ and $r = 2l\sin\theta$.

Equation (iii) now reduces to $\dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2}{(2l\sin\theta)^2} = mg\tan\theta$

or $q = \sqrt{16\pi\varepsilon_0 l^2 mg\tan\theta\sin^2\theta}$

Illustration 1.13 Two identical balls each having a density ρ are suspended from a common point by two insulating strings of equal length. Both the balls have equal mass and charge. In equilibrium, each string makes an angle θ with the vertical. Now, both the balls are immersed in a liquid. As a result, the angle θ does not change. The density of the liquid is σ. Find the dielectric constant of the liquid.

Sol. Let V is the volume of each ball, then mass of each ball is

$$m = \rho V$$

When the balls are in air, from the previous problem,

$$F = mg\tan\theta = \rho Vg\tan\theta \tag{i}$$
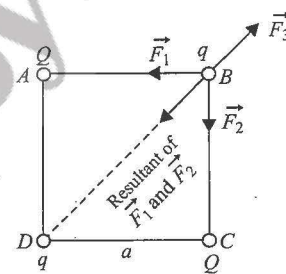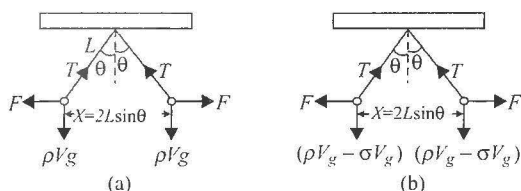

Fig. 1.22

When the balls are suspended in liquid, the Coulombic force is reduced to $F' = F/K$ and apparent weight = weight − upthrust, i.e.,

$$W' = (\rho Vg - \sigma Vg)$$

According to the problem, angle θ is unchanged. Therefore,

$$F' = W'\tan\theta = (\rho Vg - \sigma Vg)\tan\theta \tag{ii}$$

From Eqs. (i) and (ii), we get

$$\frac{F}{F'} = K = \frac{\rho Vg}{\rho Vg - \sigma Vg} = \frac{\rho}{\rho - \sigma}$$

Illustration 1.14 Three particles, each of mass 'm' and carrying a charge q each, are suspended from a common point by insulating massless strings, each of length 'L'. If the particles are in equilibrium and are located at the corners of an equilateral triangle of side 'a', calculate the charge q on each particle. Assume $L \gg a$.

Sol. From Fig. 1.23(a), for equilibrium of a particle along a vertical line, we get

$$T\cos\theta = mg \tag{i}$$

While for equilibrium in the plane of equilateral triangle, we get

$$T\sin\theta = 2F\cos 30° \tag{ii}$$

So, from Eqs. (i) and (ii), we have

$$\tan\theta = \frac{\sqrt{3}F}{mg} \tag{iii}$$

(a)

(b)   (c)
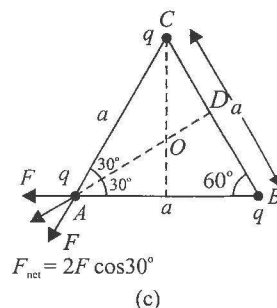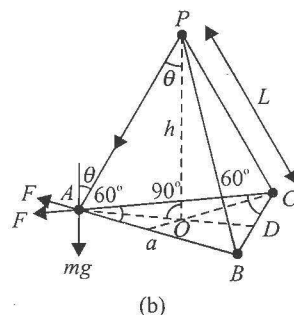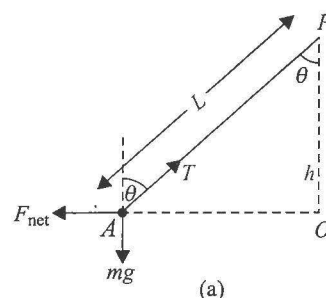
Fig. 1.23

Here, $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2}{a^2}$ and $\tan\theta = \dfrac{OA}{OP} = \dfrac{OA}{\sqrt{L^2 - OA^2}}$

Also, from Fig. 1.23(c), we get

$$OA = \frac{2}{3}AD = \frac{2}{3}a\sin 60° = \frac{a}{\sqrt{3}}$$

So, $\tan\theta = \dfrac{(a/\sqrt{3})}{\sqrt{L^2(a^2/3)}} = \dfrac{a}{(\sqrt{3})L}$ (as $L \gg a$)

On substituting the above values of F and $\tan\theta$ in Eq. (iii), we get

$$\frac{a}{(\sqrt{3})L} = \frac{\sqrt{3}}{mg}\frac{q^2}{4\pi\varepsilon_0 a^2}, \text{ i.e., } q = \left[\frac{4\pi\varepsilon_0 a^3 mg}{3L}\right]^{1/2}$$

Illustration 1.15 A thin fixed ring of radius 'a' has a positive charge 'q' uniformly distributed over it. A particle of mass 'm', having a negative charge 'Q', is placed on the axis at a distance of x ($x \ll a$) from the centre of the ring. Show that the motion of the negatively charged particle is approximately simple harmonic. Calculate the time period of oscillation.
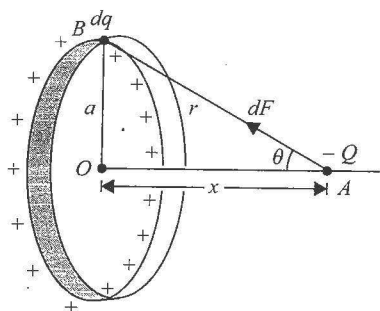
Fig. 1.24

Sol. The force on the point charge Q due to the element dq of the ring is

$$dF = \frac{1}{4\pi\varepsilon_0}\frac{dqQ}{r^2} \text{ along } AB$$

For every element of the ring, there is symmetrically situated diametrically opposite element, the components of forces along the axis will add up while those perpendicular to it will cancel each other. Hence, net force on the charge $-Q$ is $-$ve sign shows that this force will be towards the centre of ring.

$$F = \int dF\cos\theta = \cos\theta \int dF$$

$$= \frac{x}{r}\int \frac{1}{4\pi\varepsilon_0}\left[-\frac{Qdq}{r^2}\right]$$

So, $F = -\dfrac{1}{4\pi\varepsilon_0}\dfrac{Qx}{r^3}\int dq = -\dfrac{1}{4\pi\varepsilon_0}\dfrac{Qqx}{(a^2+x^2)^{3/2}}$ (i)

(as $r = (a^2+x^2)^{1/2}$ and $\int dq = q$)

As the restoring force is not linear, the motion will be oscillatory. However, if $x \ll a$ so that $x^2 \ll a^2$, then

$$F = -\frac{1}{4\pi\varepsilon_0}\frac{Qq}{a^3}x = -kx \text{ with } k = \frac{Qq}{4\pi\varepsilon_0 a^3}$$

i.e., the restoring force will become linear and so the motion is simple harmonic with time period

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{4\pi\varepsilon_0 ma^3}{qQ}}$$

Concept Application Exercise 1.2

1. A negatively charged particle is placed exactly midway between two fixed particles having equal positive charges. What will happen to the charge

   i. if it is displaced at right angle to the line joining the positive charges?

   ii. if it is displaced along the line joining the positive charges?

2. Does an electric charge experience a force due to the field produced by itself? (Yes/No)

3. Two negative charges of a unit magnitude and a positive charge 'q' are placed along a straight line. At what position and value of q will the system be in equilibrium? (Negative charges are fixed.)

4. Figure 1.25 shows three arrangements of an electron e and two protons p (where $D > d$).

   a. Rank the arrangements according to the magnitude of the net electrostatic force on the electron due to the protons, largest first.
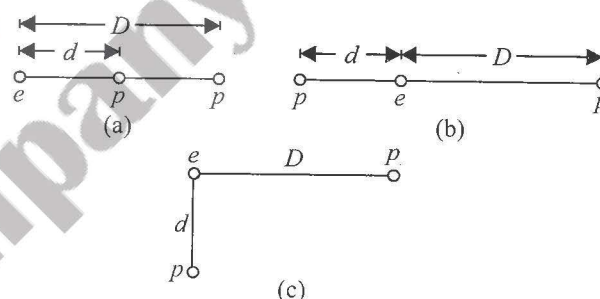
   Fig. 1.25

   b. In situation (c), is the angle between the net force on the electron and the line labeled horizontal less than or more than 45°?

5. Figure 1.26 shows two charge particles on an axis. The charges are free to move. At one point, however, a third charged particle can be placed such that all three particles are in equilibrium.

   Fig. 1.26

   a. Is that point to the left of the first two particles, to their right, or between them?

   b. Should the third particle be positively or negatively charged?

   c. Is the equilibrium stable or unstable?

6. In Fig. 1.27, a central particle of charge $-q$ is surrounded by two circular rings of charged particles of radii r and R, with $R > r$. What is the magnitude and direction of the net electrostatic force on the central particle due to the other particles?
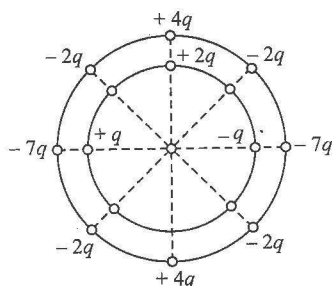
Fig. 1.27

7. Figure 1.28 shows four situations in which particles of charge $+q$ or $-q$ are fixed in place. In each, the particles on the x-axis are equidistant from the y-axis. The particle on the y-axis experiences an electrostatics force F from each of these two particles.

  a. Are the magnitudes F of those forces the same or different?

  b. Is the magnitude of the net force on the particle on the y-axis equal to, greater than or less than $2F$?

  c. Do the x components of the two forces add or cancel?

  d. Do the y components of the forces add or cancel?

  e. Is the direction of the net force on the middle particle that of the canceling components or the adding components?

  f. What is the direction of the net force on the middle particle?
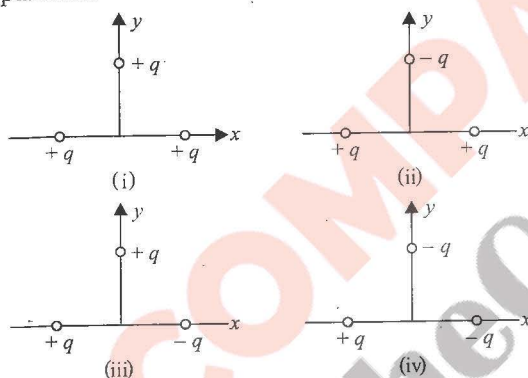
Fig. 1.28

8. Force between two point electric charges kept at a distance 'd' apart in air is F. If these charges are kept at the same distance in water, the force between the charges is F'.
The ratio F'/F is equal to _____.

9. Two small balls each having charge q are suspended by two insulating threads of equal length L from a hook in an elevator. The elevator is freely falling. Calculate the angle between the two threads and tension in each thread.

10. Suppose we have a large number of identical particles, very small in size. Any two of them at 10 cm separation repel with a force of 3×10^{-10} N.

  a. If one of them is at 10 cm from a group (of very small size) of n others, how strongly do you expect it to be repelled?

  b. Suppose you measure the repulsion and find it 6×10^{-6} N. How many particles were there in the group?

ELECTRIC FIELD

If we place a single charge q at some point in space, it will experience no force. But if some other charge (say Q) is placed near it, q will start experiencing a force given by

$$F = \frac{kQq}{r^2}$$

Fig. 1.29

Now, question arises, how does Q apply a force on q or how does q know the presence of Q when there is no direct contact between them.

Basically, the force between two charges can be seen as a two-step process:

1. Firstly, charge Q creates something around itself known as electric field.

2. Secondly, any other charge particle like q if placed at some point in that field experiences a force, or we can say that charges interact with each other through electric field.

So, we can define electric field as the space around a charge in which its influence can be felt by any other charged particle.

How to Measure Electric Field

Strength of electric field at a point in space can be measured in terms of two measureable quantities:

1. Electric field intensity is denoted by E. It is a vector quantity.

2. Electric field potential is denoted by V. It is a scalar quantity.

First, we will discuss them separately and then we will see what is the relation between them and how to obtain one from the other.

Electric Field Intensity E

How to find electric field intensity E at a point?

General method: Electric field intensity, E, is a vector quantity. At a point in a given space, it has both magnitude and direction. Let us calculate E at some point P created due to some charges around P. Bring a small charge q_0 [test charge, generally positive] at point P. Let this charge experiences a force $\vec{F}$ due to charges placed in the vicinity of P. Then we define electric field intensity at P as force experienced per unit test charge (Fig. 1.30).
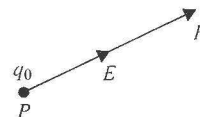
Fig. 1.30

$\vec{E} = \lim\limits_{q_0 \to 0} \dfrac{\vec{F}}{q_0}$. The direction $\vec{E}$ will be same as that of $\vec{F}$.

Note: *Q. Why the magnitude of test charge is kept small?*
Ans. Otherwise, large magnitude may disturb the original charge distribution and then we will get electric field due to disturbed configuration and not original.
Q. What is the minimum possible value of q_0?
Ans. 1.6×10^{-19} C

Unit of E: N/C (newton per coulomb)

Dimensional formula of $E = \dfrac{\text{Force}}{\text{Charge}} = \dfrac{MLT^{-2}}{\text{ampere} \times \text{time}}$

$$= \dfrac{MLT^{-2}}{AT}$$

$$= [MLT^{-3}A^{-1}]$$

Note: *If a test charge experiences no force at a point, the electric field at that point must be zero.*

Electric field due to a point charge is illustrated in Fig. 1.31.

(i) Positive point charge

$E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}$ away from the charge

(ii) Negative point charge

$E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}$ towards the charge

Fig. 1.31

Electric field due to a point charge is spherically symmetric.

Point Charge in an Electric Field

What happens if a point charge q is placed at any point in an electric field which is produced by some other stationary charges. Let this electric field is $\vec{E}$. Charge q will experience a force at this point, let this force is $\vec{F}$. Then, value of electric field at that point must be

$$\vec{E} = \dfrac{\vec{F}}{q} \Rightarrow \vec{F} = q\vec{E}. \text{ This is the force on } q \text{ by } E.$$

Direction of $\vec{F}$: The direction of $\vec{F}$ will be same as that of $\vec{E}$ if q is +ve and opposite if q is −ve (Fig. 1.32).

(a) Positive charge q_0 placed in an electric field: force on q_0 is in same direction as $\vec{E}$.

(b) Negative charge q_0 placed in an electric field: force on q_0 is in opposite direction as $\vec{E}$.
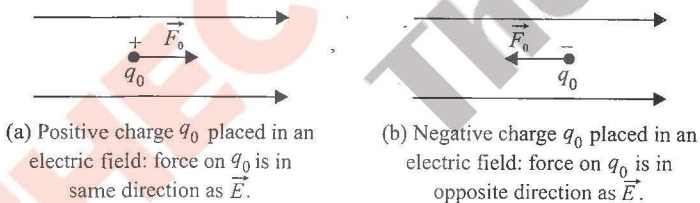
Fig. 1.32

Note: *q has no contribution in $\vec{E}$. A charge particle is not affected due to its own field. It means a charge particle can experience force due to field produced by other charge particles, but not due to field produced by itself.*

Electric Field Intensity due to a Point Charge in Position Vector Form

Electric field at P due to charge Q is

$$\vec{E} = \dfrac{Q(\vec{r} - \vec{r}_0)}{4\pi\varepsilon_0 \left|\vec{r} - \vec{r}_0\right|^3}$$
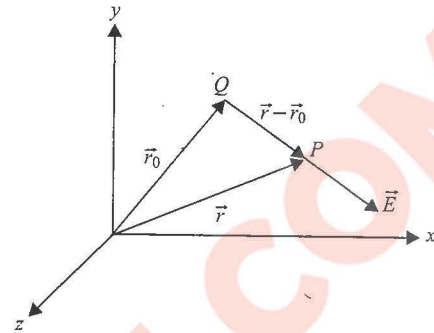
Fig. 1.33

If a charge q is placed at P, then force on this charge by Q is

$$\vec{F} = q\vec{E}$$

$$\Rightarrow \quad \vec{F} = \dfrac{qQ(\vec{r} - \vec{r}_0)}{4\pi\varepsilon_0 \left|\vec{r} - \vec{r}_0\right|^3}$$

Electric Field Intensity due to a Group of Charges

Using the principle of superposition, net field at point P (see Fig. 1.34) is

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \cdots + \vec{E}_n$$

$$\Rightarrow \quad \vec{E} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1}{r_1^2}\hat{r}_1 + \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_2}{r_2^2}\hat{r}_2 + \cdots + \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_n}{r_n^2}\hat{r}_n$$

$$\Rightarrow \quad \vec{E} = \dfrac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\dfrac{q_i}{r_i^2}\hat{r}_i$$

(a)

(b)
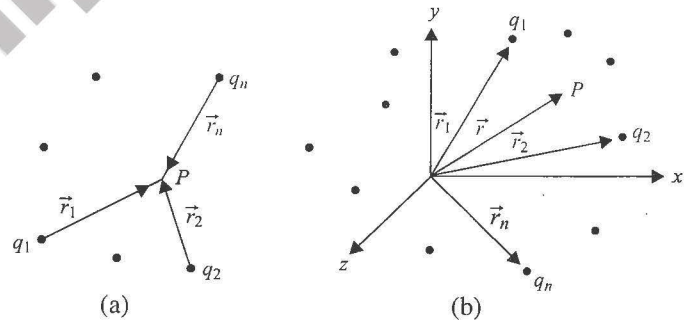
Fig. 1.34

In terms of position vectors:

$$\vec{E} = \dfrac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\dfrac{q_i(\vec{r} - \vec{r}_i)}{\left|\vec{r} - \vec{r}_i\right|^3}$$

Illustration 1.16 Two point-like charges a and b whose strengths are equal in absolute value are positioned at a certain distance from each other. Assuming the field strength is positive in the direction coinciding with the positive direction of the r axis, determine the signs of the charges for each distribution of the field strength between charges shown in Figs. 1.35(a), (b), (c) and (d).

Sol.

a. As electric field tends away at a and towards at b, hence there should be + charge at a and negative charge at b, i.e., q_a is '+' and q_b is '−'.
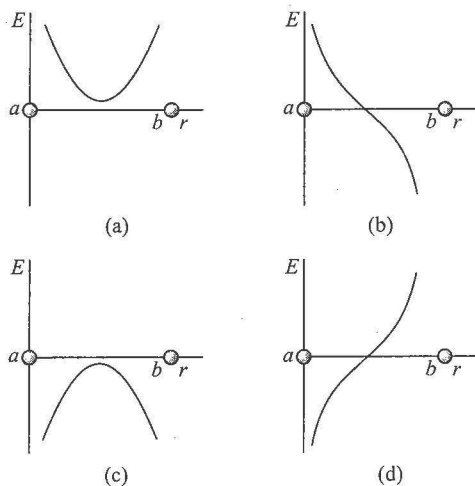
Fig. 1.35

b. The neutral point exists between *a* and *b* only when both q_a and q_b are of same sign. As direction of electric field is away from both, both charges are positive, i.e., q_a is '+' and q_b is '+'.

Similarly, for (c) and (d) in Fig. 1.36:

c. q_a is '−' and q_b is '+'
d. q_a is '−' and q_b is '−'.
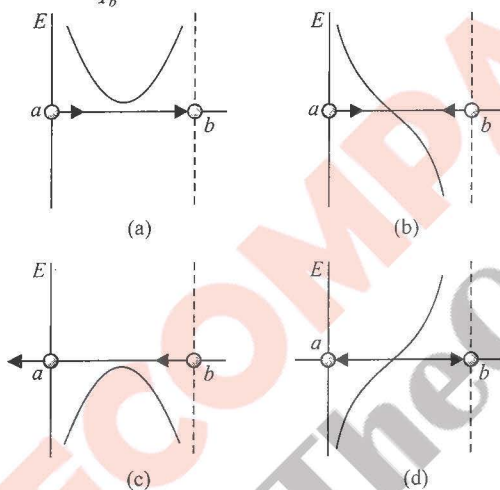
Fig. 1.36

Illustration 1.17 Two point charges $\pm q$ are placed on the axis at $x = -a$ and $x = +a$, as shown in Fig. 1.37.
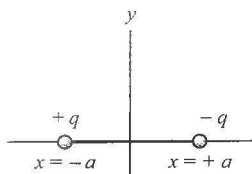
Fig. 1.37

i. Plot the variation of *E* along the *x*-axis.
ii. Plot the variation of *E* along the *y*-axis

Sol.
i. Variation of *E* along the *x*-axis as shown in Fig. 1.38(a).
ii. Variation of *E* along the *y*-axis as shown in Fig. 1.38(b). In this case, field will be maximum at origin because at origin, field due to both charges is directly added.
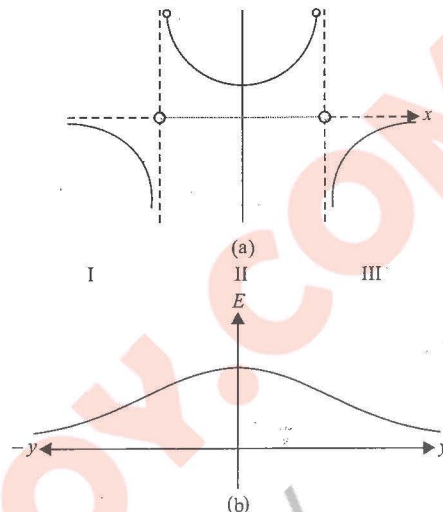
Fig. 1.38

Illustration 1.18 In Fig. 1.39, determine the point (other than infinity) at which the electric field is zero.
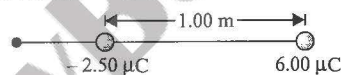
Fig. 1.39

Sol. Electric field will be zero at a point closer to the charge smaller in magnitude. Let electric field is zero at *P* (see Fig. 1.40).
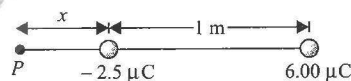
Fig. 1.40

Then, we have

$$\frac{k(2.5 \times 10^{-6})}{x^2} = \frac{k(6 \times 10^{-6})}{(1+x)^2}$$

$$\Rightarrow \qquad x = 1.82 \text{ m}$$

Illustration 1.19 Four charges are arranged as shown in Fig. 1.41. A point *P* is located at distance *r* from the centre of the configuration. Assuming $r \gg l$, find

i. the magnitude of the field at point *P*.
ii. the angle of its vector with the *x*-axis.

Sol. i. Electric field due to charges placed on the *y*-axis [Fig. 1.41(a)].

$$E_y = 2E_1 \sin\theta = 2\frac{1}{4\pi\varepsilon_0}\frac{q}{\left(r^2 + \left(\frac{l}{2}\right)^2\right)}\frac{(l/2)}{\left(r^2 + \frac{l^2}{4}\right)^{1/2}}$$
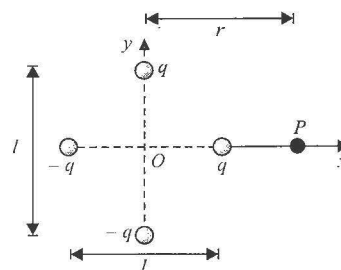
Fig. 1.41

$$E_y = \frac{1}{4\pi\varepsilon_0}\frac{ql}{\left(r^2+\frac{l^2}{4}\right)^{3/2}} = \frac{1}{4\pi\varepsilon_0}\frac{ql}{r^3} \text{ (as } r \gg l)$$

Electric field due to charges placed on x-axis [Fig. 1.42(b)]

$$E_x = E_3 - E_4 = \frac{1}{4\pi\varepsilon_0}\frac{q}{\left(r-\frac{l}{2}\right)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{\left(r+\frac{l}{2}\right)^2}$$

$$= \frac{1}{2\pi\varepsilon_0}\frac{ql}{r^3}$$

(a)

(b)   (c)
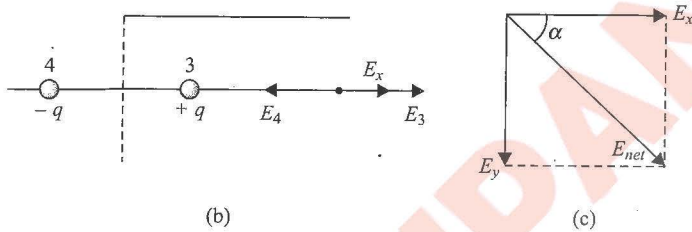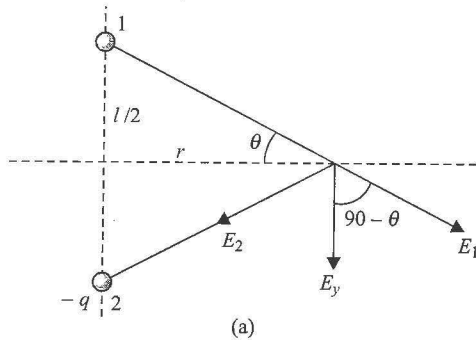
Fig. 1.42

$$\Rightarrow \qquad E_{net} = \sqrt{E_x^2 + E_y^2} = \sqrt{5}\,\frac{ql}{4\pi\varepsilon_0 r^3}$$

ii. The angle that E_{net} makes with x-axis [Fig.1.42(c)] is

$$\alpha = \tan^{-1}\left(\frac{E_y}{E_x}\right) = \tan^{-1}\left(\frac{1}{2}\right) \text{ below } x\text{-axis}$$

Illustration 1.20 A uniform electric field E exists between two metal plates one $-$ve and other $+$ve. The plate length is l and the separation of the plates is d.

   i. An electron and a proton start from the negative plate and positive plate, respectively, and go to opposite plates. Which one of them wins this race?
   ii. An electron and a proton start moving parallel to the plates towards the other end from the midpoint of the separation of plates at one end of the plates. Which of the two will have greater deviation when they come out of the plates if they start with the
   a. same initial velocity,
   b. same initial kinetic energy and
   c. same initial momentum.

Sol.

i. $a_c = \dfrac{eE}{m_e}$, $a_p = \dfrac{eE}{m_p}$; $d = \dfrac{1}{2}at^2$

$t = \sqrt{\dfrac{2d}{a}}$ or $t = \sqrt{\dfrac{2md}{eE}}$; therefore, we have $\dfrac{t_e}{t_p} = \sqrt{\dfrac{m_e}{m_p}}$.

As $m_e < m_p$, therefore $t_e < t_p$. Hence, electron will take less time, i.e., the electron wins the race.
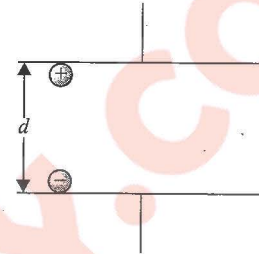
Fig. 1.43

ii. Time to cross the plates $t = \dfrac{l}{u}$

Deviation: $y = \dfrac{1}{2}at^2 = \dfrac{1}{2}\dfrac{eE}{m}\left(\dfrac{l}{u}\right)^2$

$$\frac{y_e}{y_p} = \frac{m_p}{m_e}\left(\frac{u_p}{u_e}\right)^2 \qquad\qquad \text{(i)}$$

a. If $u_p = u_e$, then $\dfrac{y_e}{y_p} = \dfrac{m_p}{m_e}$.

As $m_p > m_e$, therefore $y_e > y_p$.

Hence, deviation of electron will be more.
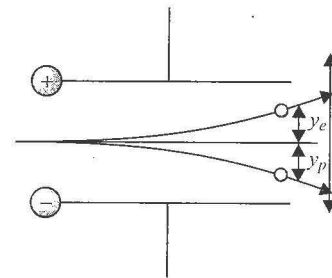
Fig. 1.44

b. From Eq. (i), $\dfrac{y_e}{y_p} = \left(\dfrac{m_p u_p^2}{m_e u_e^2}\right) = 1$ (as given)

Hence, deviation of both electron and proton will be the same.

c. From Eq. (i), $\dfrac{y_e}{y_p} = \left(\dfrac{m_p u_p}{m_e u_e}\right)^2 \dfrac{m_e}{m_p} = \dfrac{m_e}{m_p}$

As $m_e < m_p$, $y_e < y_p$.

Hence, the deviation of proton will be more.

Illustration 1.21 A charge 10^{-9} coulomb is located at origin in free space and another charge Q at $(2, 0, 0)$. If the x-component of the electric field at $(3, 1, 1)$ is zero, calculate the value of Q. Is the y-component zero at $(3, 1, 1)$?

Sol. The electric field due to a point charge q_i at position $\bar{r}_i$ in vector form is given by

$$\bar{E}_i = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_i^3}\bar{r}_i$$

Here $\vec{r}_i = (3-0)\hat{i} + (1-0)\hat{j} + (1-0)\hat{k} = 3\hat{i} + \hat{j} + \hat{k}$

$= \sqrt{(3^2 + 1^2 + 1^2)} = \sqrt{11}$ m

$\vec{r}_2 = (3-2)\hat{i} + (1-0)\hat{j} + (1-0)\hat{k} = \hat{i} + \hat{j} + \hat{k}$

$= \sqrt{(1^2 + 1^2 + 1^2)} = \sqrt{3}$ m

So,

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0} \frac{10^{-9}}{(11)^{3/2}} [3\hat{i} + \hat{j} + \hat{k}] \text{ and}$$

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0} \frac{Q}{(3)^{3/2}} [\hat{i} + \hat{j} + \hat{k}]$$

Hence, net field is $\vec{E} = \vec{E}_1 + \vec{E}_2$

$$= \frac{1}{4\pi\varepsilon_0} \left[\left(\frac{3 \times 10^{-9}}{11\sqrt{11}} + \frac{Q}{3\sqrt{3}} \right)\hat{i} \right.$$

$$\left. + \left(\frac{10^{-9}}{11\sqrt{11}} + \frac{Q}{3\sqrt{3}} \right)\hat{j} + \left(\frac{10^{-9}}{11\sqrt{11}} + \frac{Q}{3\sqrt{3}} \right)\hat{k} \right]$$
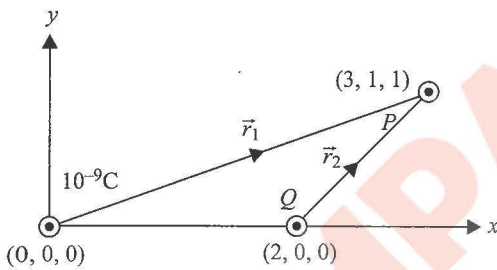
Fig. 1.45

According to the given problem

$E_x = 0$, i.e., $\dfrac{1}{4\pi\varepsilon_0} \left(\dfrac{3 \times 10^{-9}}{11\sqrt{11}} + \dfrac{Q}{3\sqrt{3}} \right) = 0$

So, $Q = -\left[\dfrac{3}{11} \right]^{3/2} \times 3 \times 10^{-9}$ coulomb

And for this value of Q

$$E_y = \frac{1}{4\pi\varepsilon_0} \left[\frac{10^{-9}}{11\sqrt{11}} - \frac{(3/11)^{3/2} \times 3 \times 10^{-9}}{3\sqrt{3}} \right]$$

$$= -\frac{1}{4\pi\varepsilon_0} \frac{2 \times 10^{-9}}{11\sqrt{11}} \neq 0 \text{ i.e., } E_y \text{ is not zero.}$$

Lines of Force

This idea was given by Michael Faraday. The lines of force provide a nice idea to visualise the pattern of electric field in a given space. We assume that space around a charged body is filled with some lines known as electric lines of force. *These lines of force are drawn in space in such a way that tangent to the line at any point gives the direction of electric field at that point.* It has been found quite convenient to visualize the electric field in terms of lines of force.
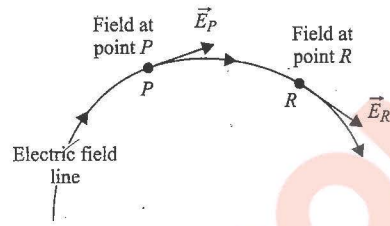
Fig. 1.46

Properties of Electric Lines of Force

- Electric lines of force start (or diverge out) from a positive charge and end (or converge) on a negative charge.
- The tangent drawn at any point on the line of force gives the direction of force acting on a positive charge at that point (see Fig. 1.47).
- In S.I. system of units, the number of electric lines of force originating or terminating on a charge of q coulomb is equal to q/ε_0.
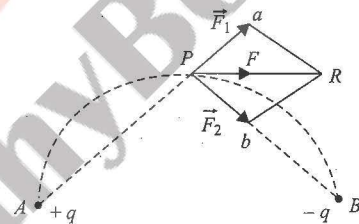
Fig. 1.47

- Two electric lines of force never cross each other, because if they do so then at the point of intersection, intensity will have two directions which is absurd.
- Electric lines of force can never be closed loops, as a line can never start and end on the same charge.
- The electric lines of force do not pass through a conductor as electric field inside a conductor is always zero.
- Lines of force have a tendency to contract longitudinally like a stretched elastic string producing attraction between opposite charges and repel each other laterally resulting in repulsion between similar charges and edge effect (curving of lines of force near the edges of a charged conductor).
- Electric lines of force end or start normally on the surface of a conductor.
- Tangent to the line of force at a point in an electric field gives the direction of intensity or force or acceleration which a positive charge will experience there but not the direction of motion always, therefore a positive point charge free to move in an electric field may or may not follow the line of force. It will follow the line of force if it is a straight line (as direction of velocity and acceleration will be same) and will not follow the line if it is curved as the direction of motion will be different from that of acceleration and the particle will move in the direction of neither motion nor acceleration (line of force).

The use of the electric lines of force is that we can compare the intensities at two points just by looking at the distribution of lines of force. Where the field lines are close together, E is large and where they are far apart, E is small.
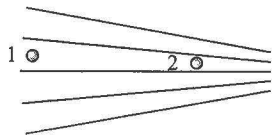
Fig. 1.48

In Fig. 1.48, electric lines of forces are shown. The electric field intensity at point 2 will be greater in comparison to that at point 1.

DIFFERENT PATTERNS OF ELECTRIC FIELD LINES

Magnitude is not constant

Direction is not constant

Both magnitude and direction are not constant

Both magnitude and direction are constant

(a)
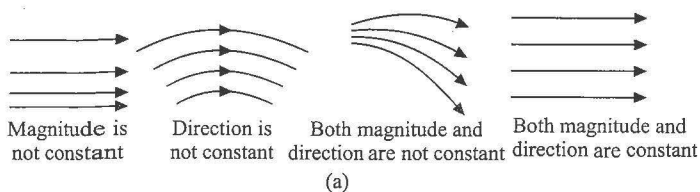
(b) A single positive charge
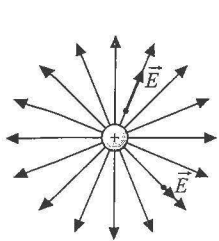
(c) A positive charge and a negative charge of equal magnitude (an electric dipole)
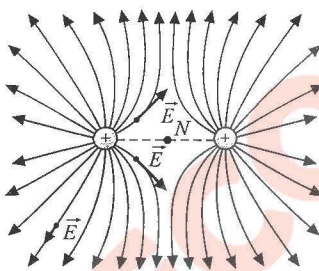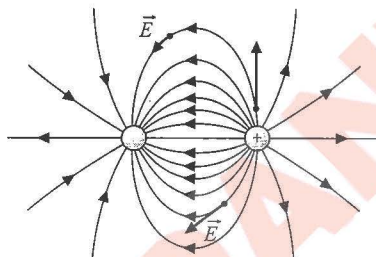
(d) Two equal positive charges. N is the neutral point lying at the middle of the charges.
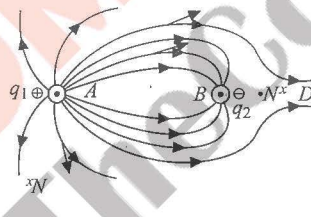
(e) A is a positive charge and B a negative charge of different magnitudes ($|q_2| < |q_1|$)
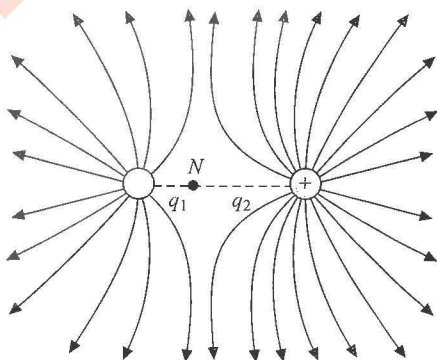
(f) Two positive charges of different magnitudes ($q_1 < q_2$)

Fig. 1.49

Note: *Neutral point (N) is the location where the net electric field due to charges is zero. It lies near the charge of smaller magnitude.*

A metal plate and a point charge

Parallel metal plates having dissimilar charges
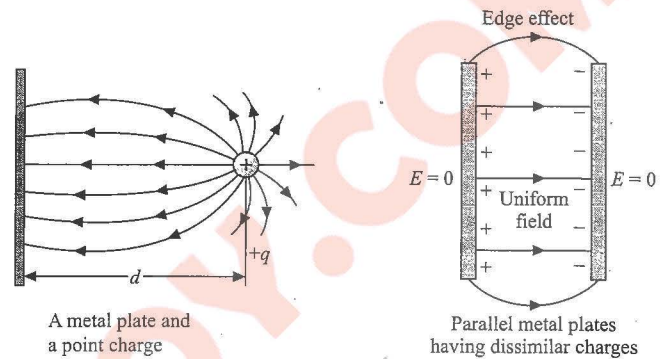
Fig. 1.50

Illustration 1.22 Figure 1.51 shows the sketch of field lines for two point charges $2Q$ and $-Q$. The pattern of field lines can be deduced by considering the following points:
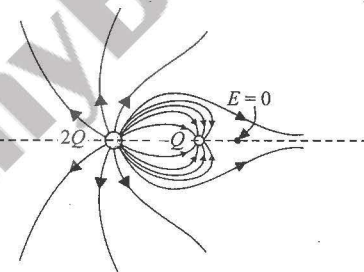
Fig. 1.51

Sol.

i. Symmetry: For every point above the line joining the two charges, there is an equivalent point below it. Therefore, the pattern must be symmetrical about the line joining the two charges.

ii. Near field: Very close to a charge, its field predominates. Therefore, the lines are radial and spherically symmetric.

iii. Far field: Far from the system of charges, the pattern should look like that of a single point charge of value $(2Q - Q) = +Q$, i.e., the lines should be radially outward.

iv. Null point or neutral point: There is one point at which $E = 0$. No lines should pass through this point. Neutral point lies near the position of charge of smaller magnitude.

v. Number of lines: Twice as many lines leave $+2Q$ as enter $-Q$.

Note: *Excess lines from 2Q charge will meet at infinity.*

Illustration 1.23 Charges $+q$ and $-2q$ are fixed at distance d apart as shown in the figure.
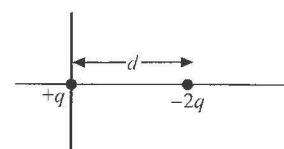
Fig. 1.52

i. **Sketch roughly the pattern of electric field lines, showing position of neutral point.**

ii. **Where should a charge particle q be placed so that it experiences no force?**

Sol. Let net force on q at P is zero, then

$$\frac{kq^2}{x^2} = \frac{kq\, 2q}{(d+x)^2} \Rightarrow x = \frac{d}{\sqrt{2}-1}$$
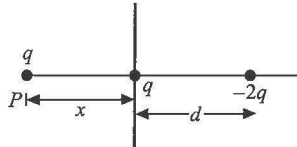
Fig. 1.53

P is the neutral potential where electric field will be zero.
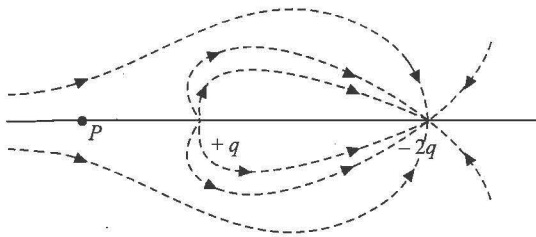
Fig. 1.54

Illustration 1.24 The field lines for two point charges are shown in Fig. 1.55.
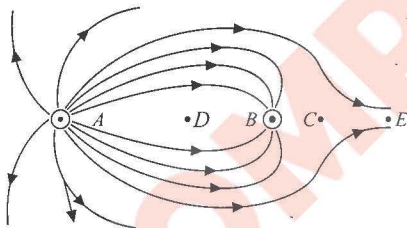
Fig. 1.55

i. **Is the field uniform?**
ii. **Determine the ratio q_A/q_B.**
iii. **What are the sign of q_A and q_B?**
iv. **Apart from infinity, where is the neutral point?**
v. **If q_A and q_B are separated by a distance $10(\sqrt{2}-1)$ cm, find the position of neutral point.**
vi. **Where will the lines meet which are coming from A and are not meeting at q_B?**
vii. **Will a positive charge follow the line of force if free to move?**

Sol.

i. No

ii. Number of lines coming from or coming to a charge is proportional to magnitude of charge, so

$$\frac{q_A}{q_B} = \frac{12}{6} = 2$$

iii. q_A is positive and q_B is negative.

iv. C is the other neutral point.

v. For neutral point $E_A = E_B$

$$\frac{1}{4\pi\varepsilon_0}\frac{q_A}{(l+x)^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_B}{x^2}$$

Fig. 1.56

$$\left(\frac{l+x}{x}\right)^2 = \frac{q_A}{q_B} = 2 \Rightarrow x = 10 \text{ cm}$$

vi. At infinity.

vii. No. As lines of force are curved, the direction of velocity and acceleration will be different. Hence, a charge cannot follow strictly the line of force. Also to move on some curved path, centripetal force is required, whereas lines of force will provide only tangential force.

FIELD OF RING CHARGE

A ring-shaped conductor with radius a carries a total charge Q uniformly distributed around it. Let us calculate the electric field at a point P that lies on the axis of the ring at a distance x from its centre.
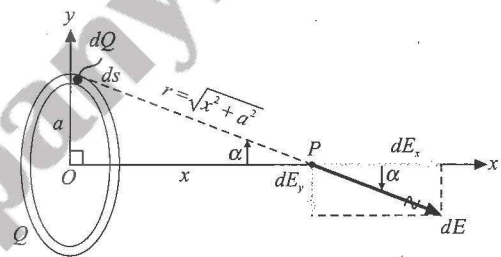
Fig. 1.57

As shown in the figure, the ring is divided into infinitesimal segments each of length ds. Each segment has charge dQ and acts as a point charge source of electric field. Let $d\vec{E}$ be the electric field from one such segment; the net electric field at P is then the sum of all contributions $d\vec{E}$ from all the segments that make up the ring. (The same technique works for any situation in which charge is distributed along a line or a curve.) The calculation of $\vec{E}$ is greatly simplified because the field point P is on the symmetry axis of the ring. If we consider two ring segments at the top and bottom of the ring, we see that the contributions $d\vec{E}$ to the field at P from these segments have the same x-component but opposite y-components. Hence, the total y-component of field due to this pair of segments is zero. When we add up the contributions from all such pairs of segments, the total field $\vec{E}$ will have only a component along the ring's symmetry axis (the x-axis), with no component perpendicular to that axis (that is, no y-component or z-component). So, the field at P is described completely by its x-component E_x.

To calculate E_x, note that the square of the distance r from a ring segment to the point P is $r^2 = x^2 + a^2$. Hence, the magnitude of this segment's contribution to the electric field at P is

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{x^2 + a^2}$$

Using $\cos\alpha = \dfrac{x}{r} = \dfrac{x}{(x^2+a^2)^{1/2}}$ the component dE_x of this field along the x-axis is

$$dE_x = dE \cos \alpha = \frac{1}{4\pi\varepsilon_0} \frac{dQ}{x^2 + a^2} \frac{x}{\sqrt{x^2 + a^2}} = \frac{1}{4\pi\varepsilon_0} \frac{x \, dQ}{(x^2 + a^2)^{3/2}}$$

To find the total x-component E_x of the field at P, we integrate this expression over all segments of the ring, i.e.,

$$E_x = \int \frac{1}{4\pi\varepsilon_0} \frac{x \, dQ}{(x^2 + a^2)^{3/2}}$$

Since x does not vary as we move from point to point around the ring, all the factors on the right side except dQ are constant and can be taken outside the integral. The integral of dQ is just the total charge Q and we finally get

$$\vec{E} = E_x \hat{i} = \frac{1}{4\pi\varepsilon_0} \frac{xQ}{(x^2 + a^2)^{3/2}} \hat{i} \qquad (i)$$

- Electric field is directed away from positively charged ring.
- For $x = 0$, $E = 0$. This conclusion may be arrived at by the symmetry consideration.
- At a large distance from the ring, the electric field will be zero. Hence, it should have certain maximum value between $x = 0$ and $x = \infty$ (or $x = -\infty$).
- If we maximize Eq. (i), we can get the value of x_m as well as E_{max}.

For the maximum value of E_x, we get

$$\frac{d}{dx}\left\{ \frac{1}{4\pi\varepsilon_0} Q \frac{x}{(x^2 + a^2)^{3/2}} \right\} = 0$$

$$\frac{(x^2 + a^2)^{3/2}(1-x)\frac{3}{2}(x^2+a^2)^{1/2}(2x)}{(x^2+a^2)^3} = 0$$

$$(x^2 + a^2) - 3x^2 = 0 \Rightarrow x = \pm \frac{a}{\sqrt{2}}$$

and the maximum value of the electric field is

$$E_{a(max)} = \frac{1}{4\pi\varepsilon_0}\left(\frac{2Q}{3\sqrt{3}\,a^2} \right)$$
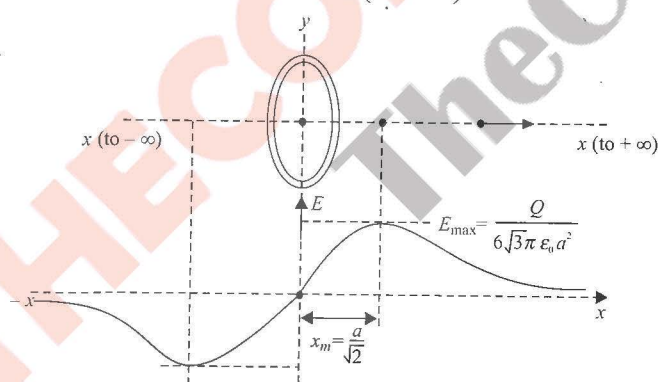
Fig. 1.58

Illustration 1.25 Two identical point charges having magnitude q each are placed as shown in the figure.
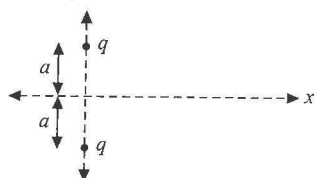
Fig. 1.59

i. Plot the variation of electric field on x-axis.
ii. Where will the magnitude of electric field be maximum on x-axis? Find the maximum value of electric field on x-axis.
iii. If we place a negative charge (of magnitude $-q$ and mass m) at the mid point of charges and displaced along x-axis, examine whether it will perform simple harmonic motion. If yes, then find the time period of oscillation of the particle.

Sol. i.
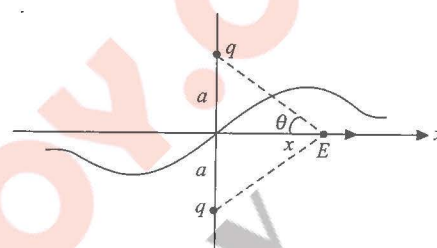
Fig. 1.60

ii. Field at $x = x$: $E = 2\left[\frac{1}{4\pi\varepsilon_0} \frac{q}{(a^2 + x^2)} \right] \cos\theta$

$$\Rightarrow \qquad E = \frac{q}{2\pi\varepsilon_0} \frac{x}{(a^2 + x^2)^{3/2}}$$

For E to be maximum, $\dfrac{dE}{dx} = 0$

Solving, we get $x = \pm \dfrac{a}{\sqrt{2}} \Rightarrow E_{max} = \dfrac{q}{3\sqrt{3}\pi\varepsilon_0 a^2}$

iii. Force on particle: $F = -qE = \dfrac{-q^2}{2\pi\varepsilon_0} \dfrac{x}{(a^2 + x^2)^{3/2}}$

For $x \ll a$, particle will execute S.H.M. with time period

$$T = 2\pi \sqrt{\frac{2\pi\varepsilon_0 m a^3}{q^2}}$$

Electric Field due to an Infinite Line Charge

Positive electric charge Q is distributed uniformly along a line, lying along the y-axis. Let us find the electric field at point D on the x-axis at a distance r_0 from the origin.

We divide the line charge into infinitesimal segments, each of which acts as a point charge; let the length of a typical segment at height l be dl. If the charge is distributed uniformly with the linear charge density λ, then the charge dQ in a segment of length dl is $dQ = \lambda dl$. At point D, the differential electric field dE created by this element is

$r = r_0 \sec\theta$
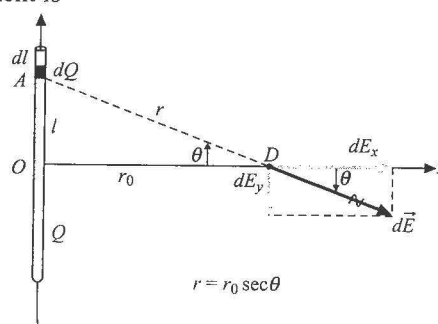
Fig. 1.61

$$dE = \frac{dQ}{4\pi\varepsilon_0 r^2} = \frac{\lambda dl}{4\pi\varepsilon_0 r^2} = \frac{\lambda dl}{4\pi\varepsilon_0 r_0^2 \sec^2\theta} \qquad \text{(i)}$$

In triangle AOD; $OA = OD\tan\theta$, i.e., $l = r_0\tan\theta$.

Differentiating this equation with respect to θ, we get $dl = r_0\sec^2\theta\, d\theta$.

Substituting the value of dl in Eq. (i), we get

$$dE = \frac{\lambda d\theta}{4\pi\varepsilon_0 r_0}$$

Field dE has components $dE_x\, dE_y$ given by

$$dE_x = \frac{\lambda\cos\theta\, d\theta}{4\pi\varepsilon_0 r_0} \quad\text{and}\quad dE_y = \frac{\lambda\sin\theta\, d\theta}{4\pi\varepsilon_0 r_0}$$

On integrating expression for dE_x and dE_y in limits $\theta = -\pi/2$ to $\theta = +\pi/2$, we obtain E_x and E_y. Note that as the length of wire increases, the angle θ also increases. For a very long wire (infinitely long wire), θ approaches $\pi/2$.

$$E_x = \int_{-\pi/2}^{+\pi/2} \frac{\lambda\cos\theta\, d\theta}{4\pi\varepsilon_0 r_0} = \frac{\lambda}{2\pi\varepsilon_0 r_0} \quad\text{and}$$

$$E_y = \int_{-\pi/2}^{+\pi/2} \frac{\lambda\sin\theta\, d\theta}{4\pi\varepsilon_0 r_0} = 0$$

Thus, $E = E_x = \dfrac{\lambda}{4\pi\varepsilon_0 r_0}$

Note: *Using a symmetry argument, we could have guessed that E_y would be zero; if we place a positive test charge at D, the upper half of the line of charge pushes downward on it, and the lower half pushes upward with equal magnitude.*

- If the wire has finite length and the angles subtended by ends of wire at a point are θ_1 and θ_2, the limits of integration would change.

$$E_x = \int_{-\theta_1}^{+\theta_2} \frac{\lambda\cos\theta\, d\theta}{4\pi\varepsilon_0 r_0}$$

$$= \frac{\lambda}{4\pi\varepsilon_0 r_0}(\sin\theta_1 + \sin\theta_2)$$

$$E_y = \int_{-\theta_1}^{+\theta_2} \frac{\lambda\sin\theta\, d\theta}{4\pi\varepsilon_0 r_0}$$

$$= \frac{\lambda}{4\pi\varepsilon_0 r_0}(\cos\theta_1 - \cos\theta_2)$$
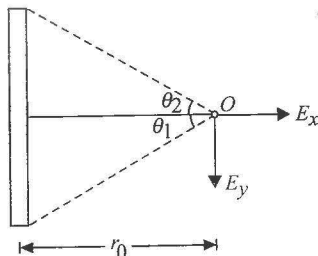
Fig. 1.62

- If we wish to determine field at the end of a long wire, we may substitute $\theta_1 = 0$ and $\theta_2 = \pi/2$ in the expressions for E_x and E_y, i.e.,

$$E_x = \frac{\lambda}{4\pi\varepsilon_0 r_0}\left[\sin(0) + \sin\left(\frac{\pi}{2}\right)\right] = \frac{\lambda}{4\pi\varepsilon_0 r_0} \quad\text{and}$$

$$E_y = \frac{\lambda}{4\pi\varepsilon_0 r_0}\left[\cos(0) - \cos\left(\frac{\pi}{2}\right)\right] = \frac{\lambda}{4\pi\varepsilon_0 r_0}$$
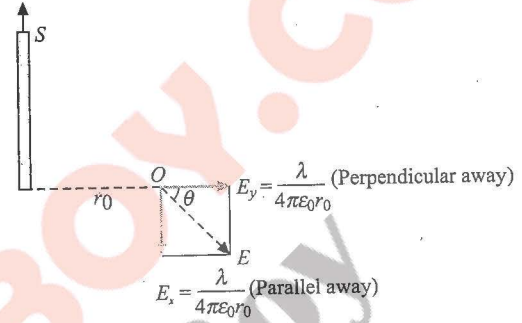
Fig. 1.63

Magnitude of resultant field $\vec{E}$ is

$$|\vec{E}| = \sqrt{E_x^2 + E_y^2} = \frac{\sqrt{2}\lambda}{4\pi\varepsilon_0 r_0}$$

$\vec{E}$ makes an angle θ with the x-axis, where $\tan\theta = |E_y|/|E_x| = 1; \theta = 45°$

FIELD OF UNIFORMLY CHARGED DISK

Let us find the electric field caused by a disk of radius R with a uniform positive surface charge density (charge per unit area) σ at a point on the axis of the disk at a distance x from its centre.
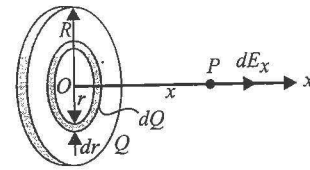
Fig. 1.64

The situation is shown in Fig.1.64. We can represent this charge distribution as a collection of concentric rings of charge. We already know how to find the field of a single ring on its axis of symmetry, therefore we will add the contribution of all the rings. As shown in the figure, a typical ring has charge dQ, inner radius r and outer radius $r + dr$. Its area dA is approximately equal to its width dr times its circumference $2\pi r$, or $dA = 2\pi r\, dr$. The charge per unit area is $\sigma = dQ/dA$, so the charge of ring is $dQ = \sigma(2\pi r dr)$, or $dQ = 2\pi\sigma r dr$. The field component dE_x at point P due to charge dQ of a ring of radius r is

$$dE_x = \frac{1}{4\pi\varepsilon_0}\frac{(2\pi\sigma r dr)x}{(x^2 + r^2)^{3/2}}$$

To find the total field due to all the rings, we integrate dE_x over r. To include the whole disk, we must integrate from 0 to R (not from $-R$ to R), i.e.,

$$E_x = \int dE_x = \int_0^R dE_x = \int_0^R \frac{1}{4\pi\varepsilon_0} \frac{(2\pi\sigma r\,dr)x}{(x^2 + r^2)^{3/2}}$$

Remember that x is a constant during the integration and that the integration variable is r. The integral can be evaluated by the use of the substitution $z = x^2 + r^2$. We will let you work out the details; the result is

$$E_x = \frac{\sigma x}{2\varepsilon_0}\left[-\frac{1}{\sqrt{x^2 + R^2}} + \frac{1}{x}\right] = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right] \quad \text{(i)}$$

In this figure, the charge is assumed to be positive. At a point on the symmetry axis of a uniformly charged ring, the electric field due to the ring has no components perpendicular to the axis. Hence, at point P in the figure, $dE_y = dE_z = 0$ for each ring, and thus the total field has $E_y = E_z = 0$.

Again, we can ask what happens if the charge distribution gets very large. Suppose we keep increasing the radius R of the disk, simultaneously adding charge so that the surface charge density σ (charge per unit area) is constant. In the limit that R is much larger than the distance x of the field point from the disk ($R \gg x$), i.e., the situation becomes the electric field near infinite plane sheet of charge.

From Eq. (i), we get

$$E_x = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{x\sqrt{1 + \dfrac{R^2}{x^2}}}\right] = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{1}{\sqrt{1 + \dfrac{R^2}{x^2}}}\right];$$

As $R \gg x$, then the term $\dfrac{1}{\sqrt{1 + \dfrac{R^2}{x^2}}} \to 0$

And we get $E_x = \dfrac{\sigma}{2\varepsilon_0}$

Our final result does not contain the distance x from the plane. This is a correct but rather surprising result.

We observe the following.

- The electric field produced by an infinite plane sheet of charge is independent of the distance from the sheet.
- The electric field is uniform; everywhere its direction is perpendicular to the sheet and away from it.
- Infinite plane sheet of charge is a hypothetical case. In real practice, there is no such infinite plane sheet of charge. Again, there is no such thing as an infinite sheet of charge, but if the dimensions of the sheet are much larger than the distance x of the observation point P from the sheet, the field is very nearly the same as for an infinite sheet.

FIELD OF TWO OPPOSITELY CHARGED SHEETS

Two infinite plane sheets are placed parallel to each other, separated by a distance d (as shown in Fig. 1.65). The lower sheet has a uniform positive surface charge density σ, and the upper sheet

has a uniform negative surface charge density $-\sigma$ with the same magnitude. Let us find the electric field between the two sheets, above the upper sheet and below the lower sheet.
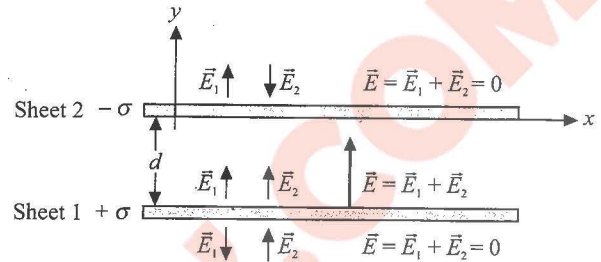
Fig. 1.65

The situation described in this example in an idealization of two finite, oppositely charged sheets, like the plates shown in the figures. If the dimensions of the sheets are large in comparison to the separation d, then to good approximation we can consider the sheets to be infinite in extent. We know the field due to a single infinite plane sheet of charge. We can then find the total field by using the principle of superposition of electric fields. Let sheet 1 be the lower sheet of positive charge, and let sheet 2 be the upper sheet of negative charge; the fields due to each sheet are $\vec{E}_1$ and $\vec{E}_2$, respectively, and both have the same magnitude at all points, no matter how far from either sheet, i.e., $E_1 = E_2 = \sigma/2\varepsilon_0$.
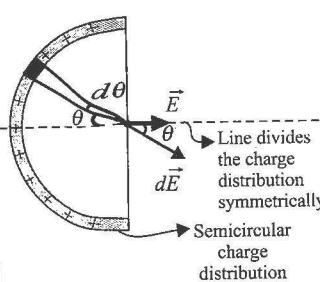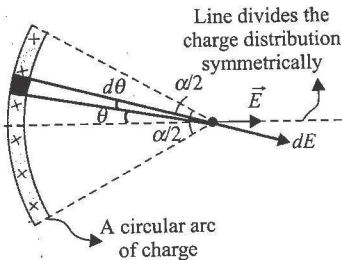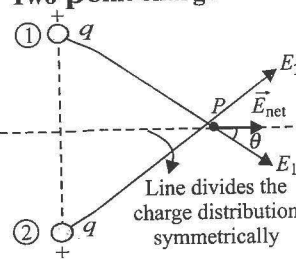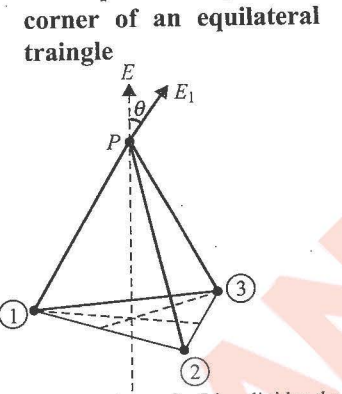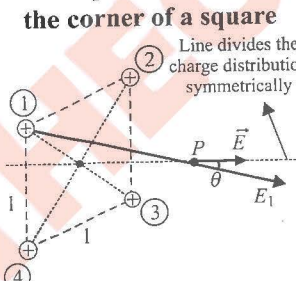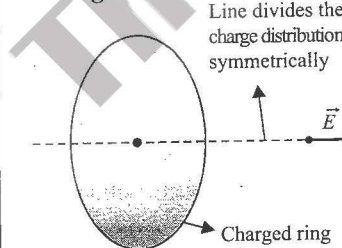
At all points, the direction of $\vec{E}_1$ is away from the positive charge of sheet 1, and the direction of $\vec{E}_2$ is towards the negative charge of sheet 2. These fields, as well as the x- and y-axes, are shown in the figure. At points between the sheets, the fields add each other and at points above the upper sheet or below the lower sheet cancel each other. Thus, the total field is

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \begin{cases} 0 & \text{above the upper sheet} \\[2mm] \dfrac{\sigma}{\varepsilon_0}\hat{j} & \text{between the sheets} \\[2mm] 0 & \text{above the upper sheet} \end{cases}$$
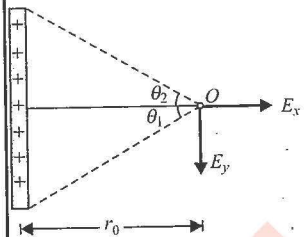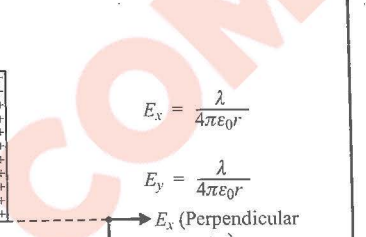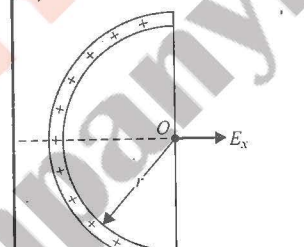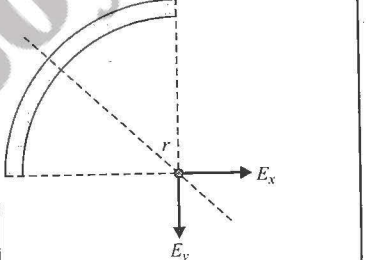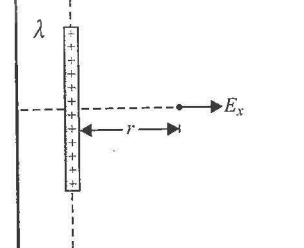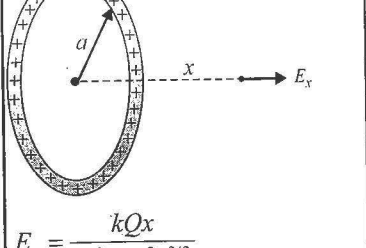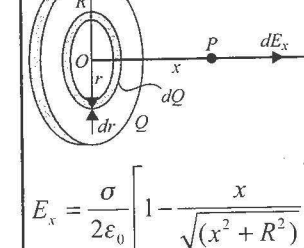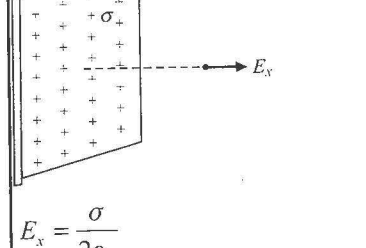
Because we considered the sheets to be infinite, our result does not depend on the separation d.

Symmetry plays very important role in problem solving. Electric field is in the direction along the line which divides the charge distribution symmetrically.

$$E_{net} = \int dE \cos\theta$$

Semicircular charge distribution. Line divides the charge distribution symmetrically.

A circular arc of charge. Line divides the charge distribution symmetrically.

$$E_{net} = \int dE \cos\theta$$

Two point charge

Line divides the charge distribution symmetrically.

Here, $|\vec{E}_1| = |\vec{E}_2|$

$$E_{net} = 2|\vec{E}_1|\cos\theta$$

Three point charge at the corner of an equilateral traingle

Line divides the charge distribution symmetrically.

Here, electric fields at P due to charges (1), (2) and (3) are equal,

i.e., $|\vec{E}_1| = |\vec{E}_2| = |\vec{E}_3|$.

Hence, $E_{net} = 3|\vec{E}_1|\cos\theta$

Four point charges at the corner of a square

Line divides the charge distribution symmetrically.

The electric field at point P due to charges (1), (2), (3) and (4) is

$$|\vec{E}_1| = |\vec{E}_2| = |\vec{E}_3| = |\vec{E}_4|$$

Hence, net electric field at P is

$$|\vec{E}_{net}| = 4|\vec{E}_1|\cos\theta$$

Charged disk

Line divides the charge distribution symmetrically.

Charged ring

Net field at a point on the axis is along the axis of disk.

Some Useful Results

A charged rod of fixed length having charge density λ

$$E_x = \frac{\lambda}{4\pi\varepsilon_0 r_0}(\sin\theta_1 + \sin\theta_2)$$

$$E_y = \frac{\lambda}{4\pi\varepsilon_0 r_0}(\cos\theta_1 - \cos\theta_2)$$

Semi-infinite rod having charge density λ

$$E_x = \frac{\lambda}{4\pi\varepsilon_0 r}$$

$$E_y = \frac{\lambda}{4\pi\varepsilon_0 r}$$

E_x (Perpendicular away)

E_y (Parallel away)

Semicircular ring having charge density λ

$$E_x = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$E_y = 0$$

Quarter circular ring having charge density λ

$$E_x = \frac{\lambda}{4\pi\varepsilon_0 r}, \quad E_y = \frac{\lambda}{4\pi\varepsilon_0 r}$$

Infinite line charge

$$E_x = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$E_y = 0$$

Charged ring

$$E_x = \frac{kQx}{(x^2 + a^2)^{3/2}}$$

$$E_y = 0$$

Charged disk

$$E_x = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{(x^2 + R^2)}}\right]$$

$$E_y = 0$$

Infinite sheet of charge

$$E_x = \frac{\sigma}{2\varepsilon_0}$$

$$E_y = 0$$

20. A droplet of ink in an industrial ink-jet printer carries a charge of 1.6×10^{-10} C and is deflected onto the paper by a force of 3.2×10^{-4} N. Find the strength of the electric field to produce this force.

ELECTRIC DIPOLE

- An electric dipole is a system of two equal and opposite point charges separated by a very small and finite distance.
- Figure 1.77 shows an electric dipole consisting of two equal and opposite point charges $-q$ and $+q$ separated by a small distance $2l$. The strength of an electric dipole is measured by a vector quantity known as *electric dipole moment*. Its magnitude is equal to the product of the magnitude of either charge and the distance between the two charges, i.e.,

$$p = 2ql$$
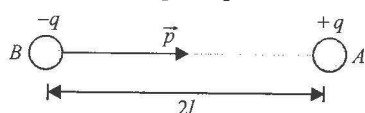
Fig. 1.77

The direction of p is from negative charge to positive charge.

- In S.I. system of units, p is measured in coulomb-metre.

ELECTRIC FIELD DUE TO A DIPOLE

Electric Field Intensity due to an Electric Dipole at a Point on the Axial Line

- A line passing through the negative and positive charges of the electric dipole is called the axial line of the electric dipole.
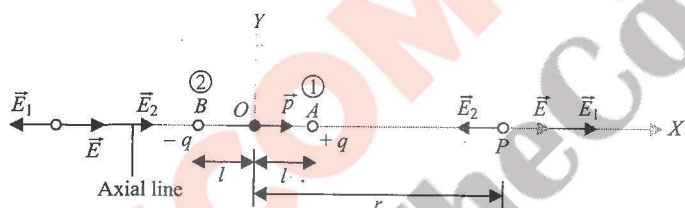
Fig. 1.78

- Suppose an electric dipole AB is located in a medium of dielectric constant K (as shown in Fig.1.78). Let the dipole consists of two point charges $-q$ and $+q$ separated by a short distance $2l$ metre. Let P be an observation point on the axial line such that its distance from the midpoint O of the electric dipole is r. We are interested to calculate the intensity of electric field at P.

- $E_1 = \dfrac{1}{4\pi\varepsilon_0 K} \dfrac{q}{(r-l)^2}$ due to q at P

    (along the direction OX)

    and $E_2 = \dfrac{1}{4\pi\varepsilon_0 K} \dfrac{q}{(r+l)^2}$ due to $-q$ at P
    (along the direction OB)

The intensities E_1 and E_2 are along the same line but in opposite directions. Since $E_1 > E_2$, the resultant intensity E

at the point P will be equal to their differences and in the direction $\vec{AP}$. Thus,

$$E = E_1 - E_2 = \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r-l)^2} - \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r+l)^2}$$

$$= \frac{q}{4\pi\varepsilon_0 K}\left[\frac{4lr}{(r^2-l^2)^2}\right] = \frac{1}{4\pi\varepsilon_0 K}\left[\frac{2(2ql)r}{(r^2-l^2)^2}\right]$$

But $2ql = p$ = electric dipole moment;

$$\Rightarrow \qquad E = \frac{1}{4\pi\varepsilon_0 K}\frac{2pr}{(r^2-l^2)^2}$$

- If l is very small as compared to $r(l \ll r)$, then l^2 can be neglected in comparison to r^2. Then, the electric field intensity at the point P due to a short dipole is given by

$$E = \frac{1}{4\pi\varepsilon_0 K}\frac{2pr}{r^4} = \frac{1}{4\pi\varepsilon_0 K}\frac{2p}{r^3}$$

$$= \frac{1}{4\pi\varepsilon_0 K}\frac{2p}{r^3}$$

- If dipole is placed in air or vacuum, then $K = 1$ and

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$$

Note: *The direction of electric field E is in the direction of $\vec{p}$, i.e., parallel to the axis of dipole from the negative charge towards the positive charge.*

In vector form, we can write

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{r^3}$$

Electric Field Intensity due to an Electric Dipole at a Point on the Equatorial Line

An equatorial line of the electric dipole is a line perpendicular to the axial line and it passes through a point midway between charges.

- Let us now suppose that the observation point P is situated on the equatorial line of dipole such that its distance from midpoint O of the electric dipole is r (as shown in Fig.1.79). Let us assume again that the medium between the electric dipole and the observation point has dielectric constant K.

- $E_1 = \dfrac{1}{4\pi\varepsilon_0 K} \dfrac{q}{(r^2+l^2)}$ (along the direction PD)

    and $E_2 = \dfrac{1}{4\pi\varepsilon_0 K} \dfrac{q}{(r^2+l^2)}$ (along the direction PC)

The magnitude of E_1 and E_2 is equal but directions are different.

Net intensity: $E = E_1 \cos\theta + E_2 \cos\theta$

[since components cancel out]

$$= \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r^2+l^2)}\cos\theta + \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r^2+l^2)}\cos\theta$$

$$= \frac{1}{4\pi\varepsilon_0 K} \frac{q}{(r^2+l^2)} \times 2\cos\theta \ \text{ along } PR$$

But from the figure,

$$\cos\theta = \frac{OA}{PA} = \frac{OA}{(OP^2+OA^2)^{1/2}} = \frac{l}{(r^2+l^2)^{1/2}}$$

$$\therefore E = \frac{1}{4\pi\varepsilon_0 K} \frac{q}{(r^2+l^2)} \times \frac{2l}{(r^2+l^2)^{1/2}} = \frac{1}{4\pi\varepsilon_0 K} \times \frac{2ql}{(r^2+l^2)^{3/2}}$$

But $2ql = p =$ electric dipole moment

$$\therefore \qquad E = \frac{1}{4\pi\varepsilon_0 K} \times \frac{p}{(r^2+l^2)^{3/2}}$$

- If l is very small as compared to $r (l \ll r)$, then l^2 can be neglected in comparison to r^2.
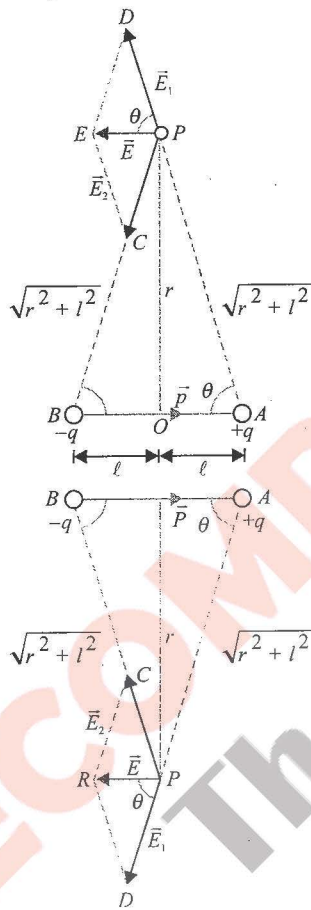
Fig. 1.79

Then, the electric field intensity at the point P due to a short dipole is given by

$$E = \frac{1}{4\pi\varepsilon_0 K} \times \frac{p}{(r^2)^{3/2}} = \frac{1}{4\pi\varepsilon_0 K} \frac{p}{r^3}$$

- If dipole is placed in air or vacuum, then $K=1$ and $E = \frac{1}{4\pi\varepsilon_0} \frac{p}{r^3}$.

The direction of resultant electric field is opposite to the direction of dipole moment, so we can write

$$\vec{E} = -\frac{1}{4\pi\varepsilon_0} \times \frac{\vec{p}}{r^3}$$

Note: *The direction of electric field E is opposite to the direction of $\vec{p}$, i.e., antiparallel to the axis of dipole from the positive charge towards the negative charge.*

ELECTRIC FIELD INTENSITY DUE TO A SHORT DIPOLE AT SOME GENERAL POINT

- Let AB be a short electric dipole of dipole moment $\vec{p}$ (directed from B to A). We are interested to find the electric field at some general point P. The distance of observation point P w.r.t. midpoint O of the dipole is r and the angle made by the line OP w.r.t. axis of dipole is θ.
- We know that dipole moment of a dipole is a vector quantity. It can be resolved into two rectangular components $\vec{p}_1$ and $\vec{p}_2$ as shown in Fig. 1.80, so that $\vec{p} = \vec{p}_1 + \vec{p}_2$. The magnitudes of $\vec{p}_1$ and $\vec{p}_2$ are $p_1 = p\cos\theta$ and $p_2 = p\sin\theta$.
- It is clear from figure that point P lies on the axial line of dipole with moment $\vec{p}_1$. Hence, the magnitude of the electric field intensity $\vec{E}_1$ at P due to $\vec{p}_1$ is
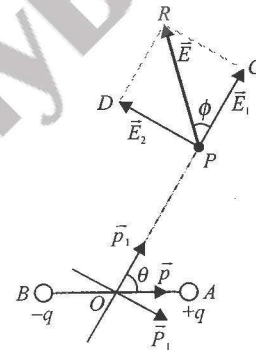
Fig. 1.80

$$E_1 = \frac{1}{4\pi\varepsilon_0} \times \frac{2p\cos\theta}{r^3} \ \text{ (along } OC) \qquad \text{(i)}$$

Similarly, P lies on the equatorial line of dipole with moment $\vec{p}_2$. Hence, the magnitude of electric field intensity $\vec{E}_2$ at P due to $\vec{p}_2$ is

$$E_2 = \frac{1}{4\pi\varepsilon_0} \frac{p\sin\theta}{r^3} \ \text{ (opposite to } p_2) \qquad \text{(ii)}$$

Hence, the resultant intensity at P is $\vec{E} = \vec{E}_1 + \vec{E}_2$.

The magnitude of $\vec{E}$ is $E = \sqrt{(E_1^2 + E_2^2)}$ (as $\vec{E}_1$ and $\vec{E}_2$ are mutually perpendicular).

or

$$E = \sqrt{\left(\frac{2p\cos\theta}{4\pi\varepsilon_0 r^3}\right)^2 + \left(\frac{p\sin\theta}{4\pi\varepsilon_0 r^3}\right)^2}$$

$$= \frac{p}{4\pi\varepsilon_0 r^3} \sqrt{4\cos^2\theta + \sin^2\theta} = \frac{p}{4\pi\varepsilon_0 r^3} \sqrt{1 + 3\cos^2\theta}$$

- If the resultant field intensity vector $\vec{E}$ makes an angle ϕ with the direction of $\vec{E}_1$, then

$$\tan\phi = \frac{E_2}{E_1} = \frac{(p\sin\theta/4\pi\varepsilon_0 r^3)}{(2p\cos\theta/4\pi\varepsilon_0 r^3)} = \frac{1}{2}\tan\theta$$

Illustration 1.26 Three charges $-q, +2q$ and $-q$ are arranged on a line as shown in Fig.1.81. Calculate the field at a distance $r \gg a$ on the line.

Sol. The field at point P is superposition of fields $\vec{E}_1, \vec{E}_2, \vec{E}_3$ due to each charge.
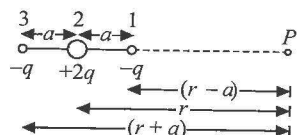
Fig. 1.81

$$\vec{E}_1 = -\frac{q}{4\pi\varepsilon_0(r-a)^2}\hat{i}; \quad \vec{E}_2 = +\frac{2q}{4\pi\varepsilon_0 r^2}\hat{i}; \quad \vec{E}_3 = -\frac{q}{4\pi\varepsilon_0(r+a)^2}\hat{i}$$

Now,

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = \frac{q}{4\pi\varepsilon_0}\left[-\frac{1}{(r-a)^2} + \frac{2}{r^2} - \frac{1}{(r+a)^2}\right]\hat{i}$$

$$E = \frac{q}{4\pi\varepsilon_0 r^2}\left[-\left\{1-\left(\frac{a}{r}\right)\right\}^{-2} + 2 - \left\{1+\left(\frac{a}{r}\right)\right\}^{-2}\right]$$

If $r \gg a$, we can use binomial approximation:

$$(1+\alpha)^n \simeq 1 + n\alpha + \frac{n(n-1)}{\lfloor 2}\alpha^2 + \cdots \text{ for } \alpha \ll 1$$

Therefore,

$$E = \frac{q}{4\pi\varepsilon_0 r^2}\left[-\left\{\left(1-2\left(-\frac{a}{r}\right)\right) + \frac{-2(-2-1)}{2}\left(-\frac{a}{r}\right)^2\right\}\right.$$
$$\left. + 2 - \left\{1 - 2\frac{a}{r} + \frac{-2(-2-1)}{2}\left(\frac{a}{r}\right)^2\right\}\right] = \frac{6a^2 q}{4\pi\varepsilon_0 r^4}$$

The charge in this problem may be considered as two dipoles placed close together. Such an arrangement of charge is called an electric quadrupole.

Illustration 1.27 What is the force on a dipole of dipole moment p placed as shown in Fig.1.82.
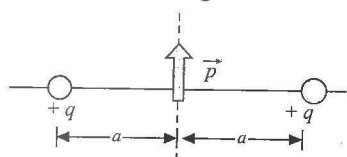
Fig. 1.82

Sol. Force on any q by dipole is

$$F = qE_{\text{dipole}} = \frac{q}{4\pi\varepsilon_0}\frac{p}{a^3} \text{ downwards}$$

So from the third law, force on dipole due to both charges is

$$2F = \frac{qp}{2\pi\varepsilon_0 a^3} \text{ upwards}$$

Net Force on a Dipole in a Non-Uniform Field

Suppose an electric dipole with dipole moment $\vec{P}$ is placed in a non-uniform electric field $\vec{E} = E\hat{i}$ that points along the x-axis (Fig.1.83). Let E depends only on x. The electric field at the position of negative charge is E and at the position of positive charge $(E+\Delta E)$. Then, the net force acting on the dipole is
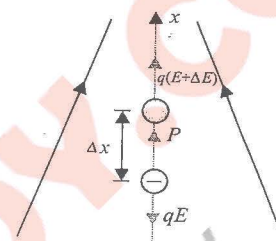
Fig. 1.83

$$F = q(E+\Delta E) - qE = q\Delta E = q\left[\frac{\Delta E}{\Delta x}\Delta x\right]$$

$$\left[\text{as } \frac{\Delta E}{\Delta x} = \frac{dE}{dx}\right]$$

$$F = q\Delta x\frac{dE}{dx} = p\frac{dE}{dx}$$

$$|\vec{F}| = \left|P\frac{d\vec{E}}{dx}\right|$$

where dE/dx is the gradient of the field in the x-direction.

Illustration 1.28 Find the force on a small electric dipole of dipole moment $\vec{P}$ due to a point charge Q placed at a distance r.

Fig. 1.84

Sol. Electric field of a point charge is a non-uniform electric field. Electric field at a distance x from the point charge is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{x^2} \Rightarrow \frac{dE}{dx} = -\frac{1}{4\pi\varepsilon_0}\frac{2Q}{x^3}$$

The magnitude of force on the dipole is

$$F = \left|p\frac{dE}{dx}\right|_{x=r} = \frac{-1}{4\pi\varepsilon_0}\frac{2pQ}{r^3}, \text{ negative sign indicates that force is}$$

towards Q or it is attractive.

Alternatively: The same result can be calculated as force on the point charge due to dipole which is the same as the force on dipole due to the point charge (Newton's third law). The electric field of small dipole at a distance r is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3} \text{ Hence, force on the point charge } Q \text{ is}$$

$$F = EQ = \frac{1}{4\pi\varepsilon_0}\frac{2pQ}{r^3}$$

DIPOLE IN A UNIFORM ELECTRIC FIELD

Torque: When a dipole is placed in a uniform field as shown in Fig.1.85, the net force on it is $F_R = \left[q\vec{E} + (-q)\vec{E} \right] = 0$
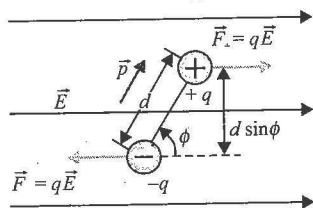
Fig. 1.85

Hence, net force on a dipole is zero in a uniform electric field. While the torque $\tau = qE \times d \sin\phi$

i.e., $\tau = pE \times \sin\phi$ {as $p = qd$}

or $\vec{\tau} = \vec{p} \times \vec{E}$ (by electric field)

and $\vec{\tau} = \vec{E} \times \vec{p}$ (by us if the dipole is in equilibrium)

From the expression, it is clear that couple acting on a dipole is maximum $(= pE)$ when dipole is perpendicular $(\phi = 90°)$ to the field and minimum $(= 0)$ when dipole is parallel $(\phi = 0°)$ or antiparallel $(\phi = 180°)$ to the field.

By applying a torque, electric field tends to align a dipole in its own direction.

Illustration 1.29 An electric dipole consists of two charges of 0.1 µC separated by a distance of 2.0 cm. The dipole is placed in an external field of 10^5 NC^{-1}. What maximum torque does the field exert on the dipole?

Sol. $\tau = pE \sin\theta = q \times 2a \times E \sin\theta$. The value of τ will be maximum when $\sin\theta = 1$

$$\therefore \tau_{max} = 10^{-7} \times 2 \times 10^{-2} \times 10^5 \times 1 = 2 \times 10^{-4} \text{ N m}$$

Concept Application Exercise 1.4

1. State the following statements as true/false:
   a. An electric dipole is kept in a uniform electric field at some angle with it. It experiences a force but no torque.
   b. An electric dipole may experience a net force when it is placed in a non-uniform electric field.
   c. An electric dipole is kept in a non-uniform electric field. It can experience a force and a torque.
2. Electric intensity due to an electric dipole varies with distance as $E \propto r^n$, where n is _____.
3. An electric dipole of moment $\vec{p}$ is placed at the origin along the x-axis. The electric field E at a point P, whose position vector makes an angle θ with the x-axis, will make an angle with x-axis which is _____.
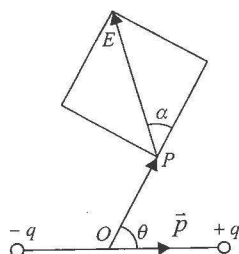
Fig. 1.86

4. Two point charges of $+1$ µC and -1 µC are separated by a distance of 100 Å. A point P is at a distance of 10 cm from the midpoint and on the perpendicular bisector of the line joining the two charges. Find the electric field at P.
5. An electric dipole consists of two opposite charges of magnitude 2×10^{-6} C each and separated by a distance of 3 cm. It is placed in an electric field of 2×10^5 NC^{-1}. Determine the maximum torque on the dipole.
6. Three charges are arranged on the vertices of an equilateral triangle as shown in Fig.1.87. Find the dipole moment of the combination.
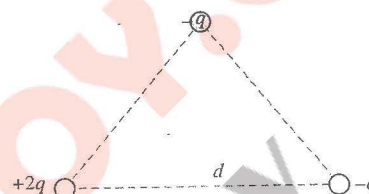
Fig. 1.87

7. The electric field at A due to dipole p is perpendicular to p. the angle θ is _____.
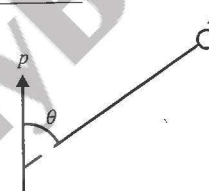
Fig. 1.88

8. An electric dipole is formed by two particles fixed at the end of a light rod of length l. The mass of each particle is m and the charges are $-q$ and $+q$. The system is placed in such a way that the dipole axis is parallel to a uniform electric field E that exists in region. The dipole is slightly rotated about its centre and released. Show that for small angular displacement, the motion is S.H.M. Evaluate its time period.
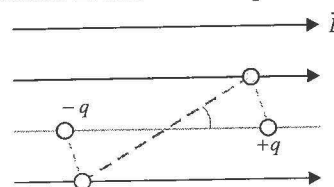
Fig. 1.89

9. A dipole consists of two particles carrying charges $+2$ µC and -2 µC and masses 1 and 2 kg, respectively, separated by a distance of 6 m. It is placed in a uniform electric field of 8×10^4 Vm^{-1}. For small oscillations about its equilibrium position, find the angular frequency.
10. A small electric dipole of dipole moment P is placed near a point charge $+Q$ as shown. Then, the net force on the dipole is towards _____.
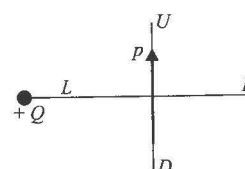
Fig. 1.90

Solved Examples

Example 1.1 A uniformly charged wire with linear charge density λ is laid in the form of a semicircle of radius R. Find the electric field generated by the semicircle at the centre.

Sol. We consider a differential element dl on the ring that subtends an angle $d\theta$ at the centre of the ring, i.e., $dl = R d\theta$.

Charge on this element $= dQ = \lambda R d\theta$.

This element creates a field dE which makes an angle θ at the centre as shown in Fig.1.91.
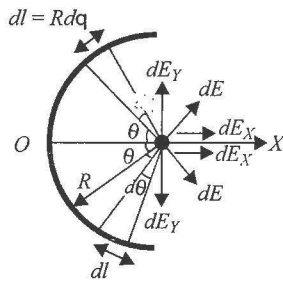
Fig. 1.91

For each differential element in the upper half of the ring, there corresponds a symmetrically placed charge element in the lower half. The y-components of the field due to these symmetric elements cancel out and the x-components remain. We get

$$dE_x = dE \cos\theta = \frac{dQ}{4\pi\varepsilon_0 R^2}\cos\theta = \frac{\lambda(Rd\theta)\cos\theta}{4\pi\varepsilon_0 R^2}$$

On integrating the expression for dE_x w.r.t. angle θ in limits $\theta = -\pi/2$ to $\theta = +\pi/2$, we obtain

$$E = \int\limits_{-\pi/2}^{+\pi/2} \frac{\lambda R}{4\pi\varepsilon_0 R^2}\cos\theta\, d\theta = \frac{\lambda}{2\pi\varepsilon_0 R}$$

In terms of the total charge, say Q, on the ring, $\lambda = Q/\pi R$ and we get $E = Q/(2\pi^2\varepsilon_0 R^2)$.

If we consider the wire in the form of an arc as shown in Fig. 1.92, the symmetry consideration is not useful in cancelling out x- and y-components of the fields, if θ_1 and θ_2 are different. We will integrate dE_x as well as dE_y in limits $\theta = -\theta_1$ to $\theta = +\theta_2$.
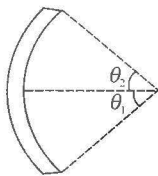
Fig. 1.92

$$E_x = \int\limits_{-\theta_1}^{+\theta_2} \frac{\lambda R}{4\pi\varepsilon_0 R^2}\cos\theta\, d\theta = \frac{\lambda}{4\pi\varepsilon_0 R}(\sin\theta_1 + \sin\theta_2)$$

$$E_y = -\int\limits_{-\theta_1}^{+\theta_2} \frac{\lambda R}{4\pi\varepsilon_0 R^2}\sin\theta\, d\theta = \frac{\lambda}{4\pi\varepsilon_0 R}(\cos\theta_2 - \cos\theta_1)$$

For a symmetrical arc, $\theta_1 = \theta_2$. Thus, E_y vanishes and

$$E_x = \frac{\lambda\sin\theta}{2\pi\varepsilon_0 R}$$

Example 1.2 A long wire with a uniform charge density λ is bent in two configurations shown in Fig. 1.93 (a) and (b). Determine the electric field intensity at point O.

Sol. In Fig. 1.93(a),

a. Field due to segment (1):

$$\vec{E}_1 = \left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)\hat{i} + \left(-\frac{\lambda}{4\pi\varepsilon_0 R}\right)\hat{j}$$

(a)   (b)
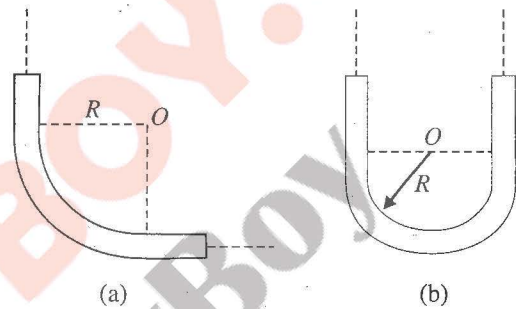
Fig. 1.93

Field due to segment (2):

$$\vec{E}_2 = \left(-\frac{\lambda}{4\pi\varepsilon_0 R}\right)\hat{i} + \left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)\hat{j}$$

Field due to quarter shape wire segment (3):

$$\vec{E}_3 = \left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)\hat{i} + \left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)\hat{j} \qquad (\because \theta_1 = 90°, \theta_2 = 0°)$$
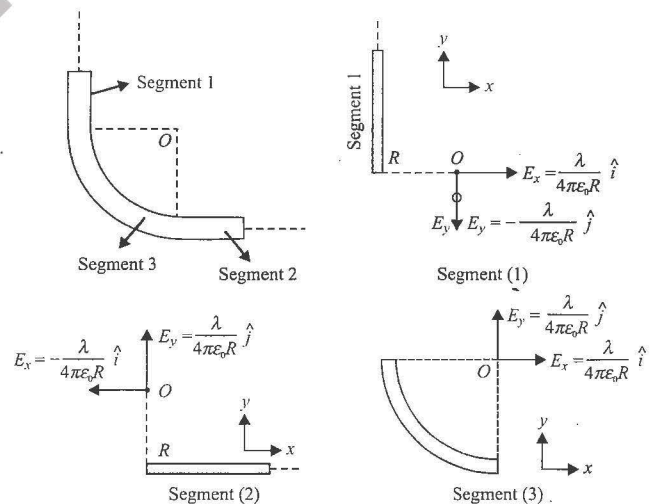
Fig. 1.94

Resultant field is superposition of the fields due to each part, i.e.,

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 \tag{i}$$

Substituting the values of $\vec{E}_1$, $\vec{E}_2$ and $\vec{E}_3$ in Eq. (i), we get

$$\vec{E} = \left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)\hat{i} + \left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)\hat{j}$$

1.30   Electrostatics and Current Electricity

Segment (1)

Fig. 1.95

$$|\vec{E}| = \left[\left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)^2 + \left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)^2\right]^{1/2} = \frac{\sqrt{2}\lambda}{4\pi\varepsilon_0 R}$$

Here, $E_x = E_y = \dfrac{\lambda}{4\pi\varepsilon_0 R}$. Hence, the resultant field will make an angle of $45°$ with the axis.

b. Field due to segment 1

$$\vec{E}_1 = \frac{\lambda}{4\pi\varepsilon_0 R}[\hat{i} - \hat{j}]$$

Field due to segment 2, $\vec{E}_{x_2} = -\dfrac{\lambda}{4\pi\varepsilon_0 R}\hat{i}$

$$\vec{E}_{y_2} = -\frac{\lambda}{4\pi\varepsilon_0 R}\hat{j}$$

$$\vec{E}_2 = -\frac{\lambda}{4\pi\varepsilon_0 R}[\hat{i} + \hat{j}]$$
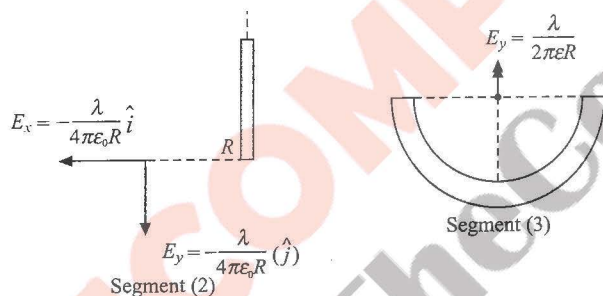
Segment (2)

Fig. 1.96

Field due to segment 3,

$$\vec{E}_3 = \frac{\lambda}{2\pi\varepsilon_0 R}\hat{j}$$

From the principle of superposition of electric fields,

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = \frac{\lambda}{4\pi\varepsilon_0 R}[\hat{i} - \hat{j}] - \frac{\lambda}{4\pi\varepsilon_0 R}[\hat{i} + \hat{j}]$$

$$+ \frac{\lambda}{2\pi\varepsilon_0 R}\hat{j} = 0$$

Hence, the net field is zero.

Example 1.3   A particle having charge that of an electron and mass 1.6×10^{-30} kg is projected with an initial speed u at an angle $45°$ to the horizontal from the lower plate of a parallel-plate capacitor as shown in the figure. The plates are sufficiently long and have separation of 2 cm. Find the maximum value of the velocity of the particle so that it does not hit the upper plate. Take electric field between the plates $= 10^3$ Vm^{-1} directed upwards.

Sol. Resolving the velocity of the particle parallel and perpendicular to the plate, we get
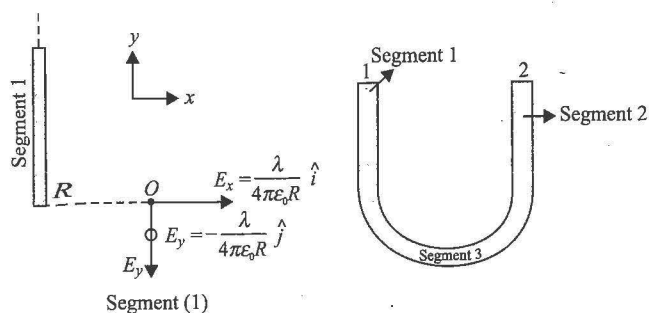
$$u_{\parallel} = u\cos 45° = \frac{u}{\sqrt{2}} \text{ and } u_{\perp} = u\sin 45° = \frac{u}{\sqrt{2}}$$

Force on the charged particle in downward direction normal to the plate $= eE$.
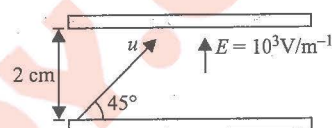
Fig. 1.97

Therefore, acceleration $a = eE/m$, where m is the mass of the charged particle.

The particle will not hit the upper plate, if the velocity component normal to the plate becomes zero before reaching it, i.e., $0 = -u_{\perp}^2 - 2ay$ with $y \le d$, where d is the distance between the plates.

Therefore, the maximum velocity for the particle not to hit the upper plate (for this $y = d = 2$ cm) is

$$u_{\perp} = \sqrt{2ay} = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 10^3 \times 2 \times 10^{-2}}{1.6 \times 10^{-30}}}$$

$$= 2 \times 10^6 \text{ ms}^{-1}$$

$$\Rightarrow \qquad u_{max} = u_{\perp}/\cos 45° = 2\sqrt{2} \times 10^6 \text{ ms}^{-1}$$

Example 1.4   A particle of mass m and charge q is released from rest in a uniform field of magnitude E. The uniform field is created between two parallel-plates of charge densities $+\sigma$ and $-\sigma$, respectively. The particle accelerates towards the other plate a distance d apart. Determine the speed at which it strikes the opposite plate.
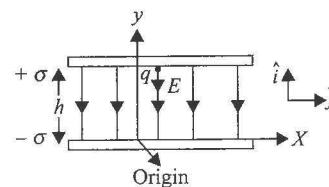
Fig. 1.98

Sol. The applied electric field is $\vec{E} = -E_0\hat{J}$.

The force experienced by the charge q is, $\vec{F} = q\vec{E} = -qE_0\hat{J}$.

The force is constant, and so the acceleration is constant as well, i.e.,

$$\vec{a} = \frac{\vec{F}}{m} = -\frac{qE_0}{m}\hat{j}$$

Because of constant acceleration, the particle moves in the $-$ve y-direction; the problem is analogous to motion of a mass released from rest in a gravitational field.

From equations of motion, we get

$$v_y = v_{y0} + a_y t = 0 - \frac{qE_0}{m}t \qquad \text{(i)}$$

And

$$y = y_0 + v_0 t + \frac{1}{2}a_y t^2; 0 = d + 0 - \frac{1}{2}\frac{qE_0}{m}t^2 \qquad \text{(ii)}$$

Particle starts at $y_0 = d$ and impact occurs at $y = 0$

From Eq. (ii), we get $t = \left(\frac{2dm}{qE_0}\right)^{1/2}$

From Eq. (i), we get $v_y = -\frac{qE_0}{m}\left(\frac{2dm}{qE_0}\right)^{1/2} = -\sqrt{\frac{2qE_0 d}{m}}$

Example 1.5 Two balls of charges q_1 and q_2 initially have a velocity of the same magnitude and direction. After a uniform electric field has been applied for a certain time interval, the direction of first ball changes by 60° and the velocity magnitude is reduced by half. The direction of velocity of the second ball changes by 90°. In what ratio will the velocity of the second ball change? Determine the magnitude of the charge-to-mass ratio of the second ball if it is equal to α_1 for the first ball. Ignore the electrostatic interaction between the balls.
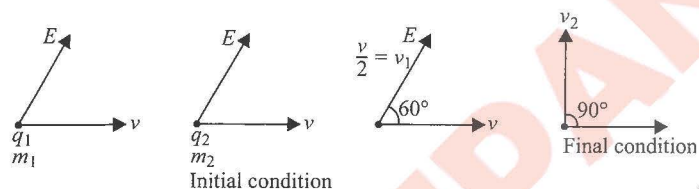
Fig. 1.99

Sol. Let the electric field on each ball be given by

$$E = E_x \hat{i} + E_y \hat{j}$$

From impulse–momentum equation, we have

Impulse = Change in momentum

Let the final velocities of the balls be v_1 and v_2. Noting that $v_1 = v/2$, we have

$$q_1(E_x \hat{i} + E_y \hat{j})\Delta t = m_1\left(\frac{v}{2}\cos 60° \hat{i} + \frac{v}{2}\sin 60° \hat{j}\right) - m_1 v\hat{i} \qquad \text{(i)}$$

$$q_2(E_x \hat{i} + E_y \hat{j})\Delta t = m_2\left(v_2 \cos 90° \hat{i} + v_2 \sin 90° \hat{j}\right) - m_2 v\hat{i} \qquad \text{(ii)}$$

On comparing the x- and y-components on both sides of Eq. (i), we get

$$\frac{q_1}{m_1}E_x \Delta t = -\frac{3}{4}v \text{ and } \frac{q_1}{m_1}E_y \Delta t = \frac{\sqrt{3}}{4}v \qquad \text{(iii)}$$

Similarly, for Eq. (ii), we get

$$\frac{q_2}{m_2}E_x \Delta t = -v \text{ and } \frac{q_2}{m_2}E_y \Delta t = v_2 \qquad \text{(iv)}$$

From Eq. (iii) and (iv), by dividing the equations expression for x-components, we get

$$\frac{q_1/m_1}{q_2/m_2} = \frac{3}{4} \qquad \text{(v)}$$

or

$$\frac{q_2}{m_2} = \frac{4}{3}\frac{q_1}{m_1} = \frac{4}{3}\alpha_1$$

Also

$$\frac{q_1/m_1}{q_2/m_2} = \frac{\sqrt{3}v}{4v_2} \Rightarrow \frac{\sqrt{3}v}{4v_2} = \frac{3}{4} \Rightarrow v_2 = \frac{v}{\sqrt{3}}$$

Example 1.6 A rigid insulated wire frame in the form of a right-angled triangle ABC is set in a vertical plane as shown in the figure. Two beads of equal masses m each and carrying charges q_1 and q_2 are connected by a cord of length l and can slide without friction on the wires. **(IIT-JEE, 1978)**
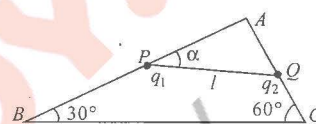
Fig. 1.100

Considering the case when the beads are stationary determine

(i) the angle α
(ii) the tension in the cord
(iii) the normal reaction on the beads

If the cord is now cut what are the value of the charges for which the beads continue to remain stationary?

Sol. Tension and electrostatic force are in opposite direction and along the string. Now each bead is in equilibrium under the concurrent forces: Normal reaction (N), weight (mg) and the resultant of tension and electrostatic force, i.e., $T - F_e$

where

$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{\ell^2}$$

Applying Lami's theorem for both the beads, we get

$$\frac{N_1}{\sin(120-\alpha)} = \frac{mg}{\cos\alpha} = \frac{T-F_e}{\cos 60°} \qquad \text{(i)}$$
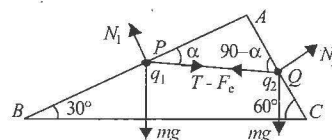
Fig. 1.101

$$\frac{N_2}{\sin(60°+\alpha)} = \frac{mg}{\sin\alpha} = \frac{T-F_e}{\cos 30°} \qquad \text{(ii)}$$

Dividing Eq. (i) by Eq. (ii), we have

(i) $\tan\alpha = \frac{\cos 30°}{\cos 60°} = \sqrt{3}$, therefore $\alpha = 60°$

(ii) $T = F_e + mg = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q_1 q_2}{\ell^2} + mg \qquad \text{(iii)}$

(iii) $N_1 = \sqrt{3}\,mg$ and $N_2 = mg$

From Eq. (iii), $T = 0$ when string is cut

or $q_1 q_2 = -(4\pi\varepsilon_0)mg\,\ell^2$

EXERCISES

1. Calculate the number of electrons in a small, electrically neutral silver pin that has a mass of 10.0 g. Silver has 47 electrons per atom, and its molar mass is 107.87 gmol^{-1}.

2. A charged particle of radius 5×10^{-7} m is located in a horizontal electric field of intensity 6.28×10^5 Vm^{-1}. The surrounding medium has coefficient of viscosity $\eta = 1.6 \times 10^5$ Nsm^{-2}. The particle starts moving under the effect of electric field and finally attains a uniform horizontal speed of 0.02 ms^{-1}. Find the number of electrons on it. Assume gravity free space.

3. Suppose that 1.00 g of hydrogen is separated into electrons and protons. Suppose also that the protons are placed at the Earth's north pole and the electrons are placed at the south pole. What is the resulting compression force on the Earth? (Given that radius of earth is 6400 km).

4. Two identical conducting small spheres are placed with their centres 0.300 m apart. One is given a charge of 12.0 nC and the other a charge of -18.0 nC.

   a. Find the electric force exerted by one sphere on the other?

   b. If the spheres are connected by a conducting wire, find the electric force between the two after they attain equilibrium.

5. Four equal point charges, each of magnitude $+Q$, are to be placed in equilibrium at the corners of a square. What should be the magnitude and sign of the point charge that should be placed at the centre of square to do this job?

6. Two point electric charges of values q and $2q$ are kept at a distance d apart from each other in air. A third charge Q is to be kept along the same line in such a way that the net force acting on q and $2q$ is zero. Find the location of the third charge from charge 'q'.

7. Two fixed point charges $+4e$ and $+e$ unit are separated by a distance 'a'. Where the third point charge should be placed from $+4e$ charge for it to be in equilibrium?

8. Two identical particles are charged and held at a distance of 1 m from each other. They are found to be attracting each other with a force of 0.027 N. Now, they are connected by a conducting wire, so that charge flows between them. When the charge flow stops, they are found to be repelling each other with a force of 0.009 N. Find the initial charge on each particle.

9. Two similarly and equally charged identical metal spheres A and B repel each other with a force of 2×10^{-5} N. A third identical uncharged sphere C is touched with A and then placed at the mid-point between A and B. Find the net electric force on C.

10. Three point charges of $+2$ μC, -3 μC and -3 μC are kept at the vertices A, B and C, respectively, of an equilateral triangle of side 20 cm as shown in the figure. What should be the sign and magnitude of the charge (q) to be placed at

the midpoint (M) of side BC so that the charge at A remains in equilibrium?
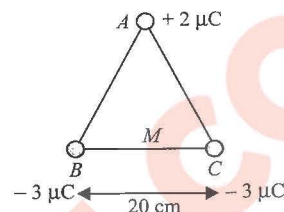
Fig. 1.102

11. Two small beads having positive charges $3q$ and q are fixed at the opposite ends of a horizontal, insulating rod, extending from the origin to the point $x = d$. As shown in the figure, a third small charged bead is free to slide on the rod. At what position is the third bead in equilibrium? Can it be in stable equilibrium?
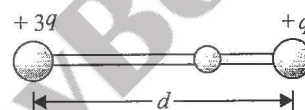
Fig. 1.103

12. A copper atom consists of copper nucleus surrounded by 29 electrons. The atomic weight of copper is 63.5 gmol^{-1}. Let us now take two pieces of copper each weighing 10 g. Let us consider one electron from one piece is transferred to another for every 1000 atoms in a piece.

   a. Find the magnitude of charge appearing on each piece.

   b. What will be the Coulomb force between the two pieces after the transfer of electrons if they are 10 cm apart?

   [Avogradro's number $= 6 \times 10^{23}$ mol^{-1}]

13. A flat square sheet of charge of side 50 cm carries a uniform surface charge density. An electron 0.5 cm from a point near the centre of the sheet experiences a force of 1.8×10^{-12} N directed away from the sheet. Determine the total charge on the sheet.

14. A particle of mass 9×10^{-31} kg having a negative charge of 1.6×10^{-19} C is projected horizontally with a velocity of 10^6 ms^{-1} into a region between two infinite horizontal parallel plates of metal. The distance between the plates is $d = 0.3$ cm and the particle enters 0.1 cm below the top plate. The top and bottom plates are connected, respectively, to the positive and negative terminals of a 30 V battery. Find the components of the velocity of the particle just before it hits one of the plates.
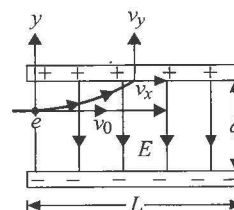
Fig. 1.104

15. Point charges q and $-q$ are located at the vertices of a square with diagonals $2l$ as shown in the figure. Evaluate the magnitude of the electric field strength at a point located symmetrically with respect to the vertices of the square at a distance x from the centre.
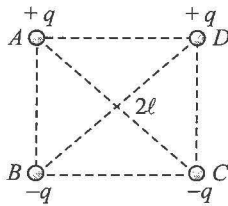
Fig. 1.105

16. Two mutually perpendicular long straight conductors carrying uniformly distributed charges of linear charge densities λ_1 and λ_2 are positioned at a distance a from each other. How does the interaction between the rods depend on a?
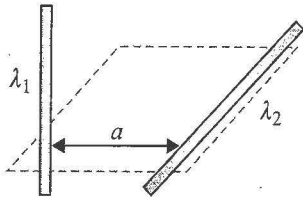
Fig. 1.106

17. A ring of radius 0.1 m is made out of a thin metallic wire of area of cross section 10^{-6} m². The ring has a uniform charge of π coulombs. Find the change in the radius of the ring when a charge of 10^{-8} C is placed at the centre of the ring. Young's modulus of the metal is 2×10^{11} Nm⁻².

18. A charged cork ball of mass m is suspended on a light string in the presence of a uniform electric field as shown in the figure. When $E = (A\hat{i} + B\hat{j})$ NC⁻¹, where A and B are positive numbers, the ball is in equilibrium at the θ. Find (a) the charge on the ball and (b) the tension in the string.
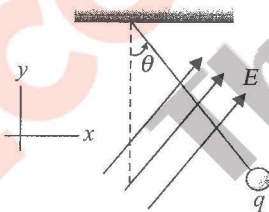
Fig. 1.107

**19.** A ring of radius R has charge $-Q$ distributed uniformly over it. Calculate the charge that should be placed at the centre of the ring such that the electric field becomes zero at a point on the axis of the ring distant 'R' from the centre of the ring.

20. Two identical small equally charged conducting balls are suspended from long threads secured at one point. The charges and masses of the balls are such that they are in equilibrium when the distance between them is a (the length of thread $L \gg a$). Then one of the balls is discharged. What will be the distance b ($b \ll L$) between the balls when equilibrium is restored?

21. Two point charges Q_a and Q_b are positioned at points A and B. The field strength to the right of charge Q_b on the line that passes through the two charges varies according to a law that is represented schematically in the figure accompanying the problem (without employing a definite scale). The field strength is assumed to be positive if its direction coincides with the positive direction of the x-axis. The distance between the charges is $l = 21$ cm (Fig. 1.108). Find
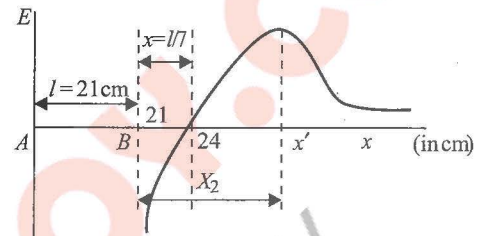
Fig. 1.108

a. the signs of the charges

b. the ratio between the absolute values of charges Q_a and Q_b

c. the coordinate x of the point where the field strength is maximum

22. Two semicircular wires ABC and ADC, each of radius 'R', are lying on $x-y$ and $x-z$ plane, respectively, as shown in Fig. 1.109.

If the linear charge density of the semicircular parts and straight parts is λ, find the electric field intensity $\vec{E}$ at the origin.
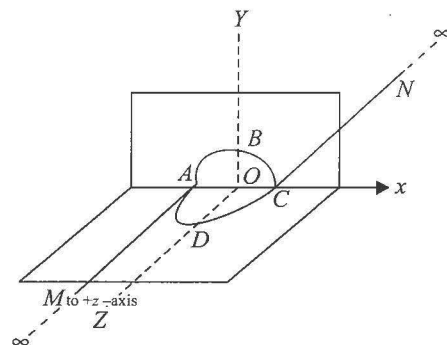
Fig. 1.109

23. An infinite wire having linear charge density λ is arranged as shown in Fig 1.110. A charge particle of mass m and charge q is released from point P. Find the initial acceleration of the particle (at $t = 0$) just after the particle is released.

Fig. 1.110

1.34   Electrostatics and Current Electricity
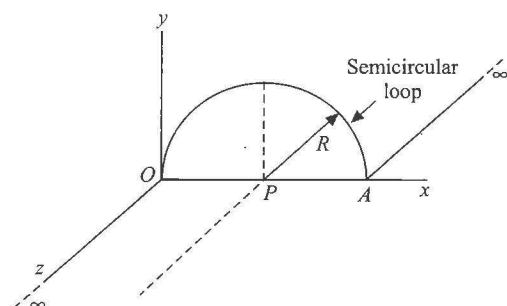
24. Two similar balls, each of mass m and charge q, are hung from a common point by two silk threads, each of length l (Fig.1.111). Prove that separation between the balls is

$$x = \left[\frac{q^2 l}{2\pi\varepsilon_0 mg}\right]^{1/3}, \text{ if } \theta \text{ is small.}$$
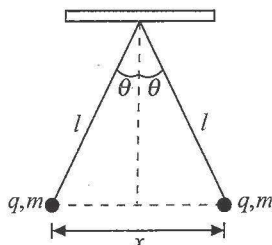
Fig. 1.111

Find the rate dq/dt with which the charge should leak off each sphere if the velocity of approach varies as $v = a/\sqrt{x}$, where a is a constant.

25. Three equal negative charges, $-q_1$ each, form the vertices of an equilateral triangle. A particle of mass m and a positive charge q_2 is constrained to move along a line perpendicular to the plane of triangle and through its centre which is at a distance r from each of the negative charges as shown in the figure. The whole system is kept in gravity free space. Find the time period of vibration of the particle for small displacement from equilibrium position.
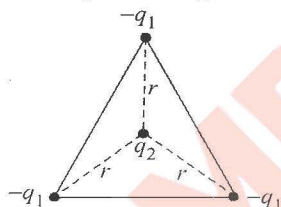
Fig. 1.112

26. A ball of radius R carries a positive charge whose volume density at a point is given as $\rho = \rho_0(1 - r/R)$, where ρ_0 is a constant and r is the distance of the point from the centre. Assuming the permittivities of the ball and the environment to be equal to unity, find

a. the magnitude of the electric field strength as a function of the distance r both inside and outside the ball

b. the maximum intensity E_{max} and the corresponding distance r_m.

Objective Type

Solutions on page 1.46

1. If a body is charged by rubbing it, its weight
   a. always decreases slightly
   b. always increases slightly
   c. may increase slightly or may decrease slightly
   d. remains precisely the same

2. In S.I. system, the value of ε_0 is
   a. $C^2 N^{-1} m^{-2}$
   b. $9 \times 10^9 \, C^2 N^{-1} m^{-2}$
   c. $\frac{1}{9 \times 10^9} \, C^2 N^{-1} m^{-2}$
   d. $\frac{1}{4\pi \times 9 \times 10^9} \, C^2 N^{-1} m^{-2}$

3. Dimensions of ε_0 are
   a. $M^{-1} L^{-3} T^4 A^2$
   b. $M^0 L^{-3} T^3 A^3$
   c. $M^{-1} L^{-3} T^3 A$
   d. $M^{-1} L^{-3} T A^2$

4. The dimensional formula of electric intensity is
   a. $MLT^{-2} A^{-1}$
   b. $MLT^{-3} A^{-1}$
   c. $ML^2 T^{-3} A^{-1}$
   d. $ML^2 T^{-3} A^{-2}$

5. The dielectric constant K of an insulator can be
   a. -1
   b. 0
   c. 0.5
   d. 5

6. Choose the correct statement:
   a. The total charge of the universe is constant.
   b. The total number of the charged particles is constant.
   c. The total positive charge of the universe remains constant.
   d. The total negative charge of the universe remains constant.

7. Two neutrons are placed at some distance apart from each other. They will
   a. attract each other
   b. repel each other
   c. neither attract nor repel each other
   d. cannot say

8. When a soap bubble is charged, its size
   a. increases
   b. decreases
   c. remains the same
   d. increases if it is given positive charge and decreases if it is given negative charge

9. Two point charges at certain distance apart in air repel each other with a force F. A glass plate is introduced between the charges. The force becomes F_1, where
   a. $F_1 < F$
   b. $F_1 = F$
   c. $F_1 > F$
   d. data is insufficient

10. There are two charges $+1 \, \mu C$ and $+5 \, \mu C$. The ratio of the forces (force on one due to other) acting on them will be
    a. 1:1
    b. 1:2
    c. 1:3
    d. 1:4

11. Two point charges Q_1 and Q_2 are 3 m apart, and their sum of charges is 10 μC. If force of attraction between them is 0.075 N, then the values of Q_1 and Q_2, respectively, are
    a. $5 \, \mu C, 5 \, \mu C$
    b. $15 \, \mu C, -5\mu C$
    c. $5\mu C, 15\mu C$
    d. $-15 \, \mu C, 5 \, \mu C$

12. A certain charge 'Q' is to between divided into two parts q and $Q - q$. What is the relationship 'Q' and 'q' if the two parts, placed at a given distance 'r' apart, are to have the maximum Coulomb repulsion?
    a. $q = Q/2$
    b. $q = Q/3$
    c. $q = 2Q/2$
    d. $q = Q/4$

13. Three charged particles are placed on a straight line as shown in the figure. q_1 and q_2 are fixed but q_3 can be moved. Under the action of the forces from q_1 and q_2, q_3 is in equilibrium. What is the relation between q_1 and q_2?
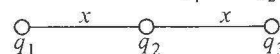
Fig. 1.113

    a. $q_1 = 4q_2$
    b. $q_1 = -q_2$
    c. $q_1 = -4q_2$
    d. $q_1 = q_2$

14. Two particles A and B (B is right of A) having charges 8×10^{-6} C and -2×10^{-6} C, respectively, are held fixed with separation of 20 cm. Where should a third charged particle be placed so that it does not experience a net electric force.
    a. 5 cm right of B
    b. 5 cm left of A
    c. 20 cm left of A
    d. 20 cm right of B

15. Five balls numbered 1, 2, 3, 4, 5 are suspended using separate threads. The balls (1, 2), (2, 4) and (4, 1) show electrostatic attraction, while balls (2, 3) and (4, 5) show repulsion. Therefore, ball 1 must be
   a. negatively charged   b. positively charge
   c. neutral   d. made of metal

16. Electric charges A and B repel each other. Electric charges B and C also repel other. If A and C are held close together, they will
   a. attract   b. repel
   c. not affect each other   d. none of these

17. Two point charges repel each other with a force of 100 N. One of the charges is increased by 10% and the other is reduced by 10%. The new force of repulsion at the same distance would be
   a. 100 N   b. 121 N
   c. 99 N   d. none of these

18. Three charges $+Q_1$, $+Q_2$ and q are placed on a straight line such that q is somewhere in between $+Q_1$ and $+Q_2$. If this system of charges is in equilibrium, what should be the magnitude and sign of charge q?
   a. $\dfrac{Q_1 Q_2}{\left(\sqrt{Q_1}+\sqrt{Q_2}\right)^2}$, +ve   b. $\dfrac{Q_1+Q_2}{2}$, +ve
   c. $\dfrac{Q_1 Q_2}{\left(\sqrt{Q_1}+\sqrt{Q_2}\right)^2}$, −ve   d. $\dfrac{Q_1+Q_2}{2}$, −ve

19. Two positive and equal charges are fixed at a certain distance. A third small charge is placed in between the two charges and it experiences zero net force due to the other two.
   a. The equilibrium is stable if small charge is positive
   b. The equilibrium is stable if small charge is negative
   c. The equilibrium is always stable
   d. The equilibrium is not stable

20. An isolated charge q_1 of mass m is suspended freely by a thread of length l. Another charge q_2 is brought near it ($r \gg l$). When q_1 is in equilibrium, tension in thread will be
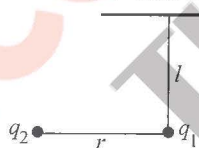

Fig. 1.114

   a. mg b. $> mg$
   c. $< mg$ d. none of these

21. Three equal charges, each $+q$, are placed on the corners of an equilateral triangle of side a. Then, the coulomb force experienced by one charge due to the rest of the two is
   a. kq^2/a^2 b. $2kq^2/a^2$ c. $\sqrt{3}\,kq^2/a^2$ d. zero

22. A positively charged ball hangs from a long silk thread. Electric field at a certain point (at the same horizontal level of ball) due to this charge is E. Let us put a positive test charge q_0 at this point and measure F/q_0 on this charge. Then, E
   a. $> F/q_0$ b. $< F/q_0$

   c. $= F/q_0$ d. none of these

23. Electric field near a straight wire carrying a steady current is
   a. proportional to the distance from the wire
   b. proportional to inverse square of the distance from the wire
   c. inversely proportional to the distance from the wire
   d. zero

24. A force of 2.25 N acts on a charge of 15×10^{-4} C. Calculate the intensity of electric field at the point.
   a. 1500 NC^{-1} b. 150 NC^{-1}
   c. 15000 NC^{-1} d. none of these

25. An α particle is situated in an electric field of strength 15×10^4 NC^{-1}. Force acting on it is
   a. 4.8×10^{-12} N   b. 4.8×10^{-14} N
   c. 48×10^{-14} N   d. none of these

26. Two particles of masses in the ratio 1 : 2, with charges in the ratio 1 : 1, are placed at rest in a uniform electric field. They are released and allowed to move for the same time. The ratio of their kinetic energies will be finally
   a. 2 : 1   b. 8 : 1   c. 4 : 1   d. 1 : 4

27. Three equal charges, each $+q$, are placed on the corners of an equilateral triangle. The electric field intensity at the centroid of the triangle is
   a. kq/r^2 b. $3kq/r^2$ c. $\sqrt{3}\,kq/r^2$ d. zero

28. A point charge of 100 μC is placed at $3\hat{i}+4\hat{j}$ m. Find the electric field intensity due to this charge at a point located at $9\hat{i}+12\hat{j}$ m.
   a. 8000 Vm^{-1} b. 9000 Vm^{-1}
   c. 2250 Vm^{-1} d. 4500 $V\,m^{-1}$

29. Electric lines of force
   a. exist everywhere
   b. exist only in the immediate vicinity of electric charges
   c. exist only when both positive and negative charges are near one another
   d. are imaginary

30. Two charges $Q_1 = 18$ μC and $Q_2 = -2$ μC are separated by a distance R and Q_1 is to the left of Q_2. The distance of the point where the net electric field is zero is
   a. between Q_1 and Q_2 b. left of Q_1 at $R/2$
   c. right of Q_2 at R d. right of Q_2 at $R/2$

31. An oil drop, carrying six electronic charges and having a mass of 1.6×10^{-12} g, falls with some terminal velocity in a medium. What magnitude of vertical electric field is required to make the drop move upwards with the same speed as it was formerly moving downwards with? Ignore buoyancy.
   a. 10^5 NC^{-1} b. 10^4 NC^{-1}
   c. 3.3×10^4 NC^{-1} d. 3.3×10^5 NC^{-1}

32. What is the largest charge a metal ball of 1 mm radius can hold? Dielectric strength of air is 3×10^6 Vm^{-1}.
   a. 3 nC   b. 1/3 nC   c. 2 nC   d. 1/2 nC

33. Five point charges, $+q$ each, are placed at the five vertices of a regular hexagon. The distance of centre of the hexagon from any of the vertices is a. The electric field at the centre of the hexagon is

a. $\dfrac{q}{4\pi\varepsilon_0 a^2}$ **b.** $\dfrac{q}{8\pi\varepsilon_0 a^2}$ **c.** $\dfrac{q}{16\pi\varepsilon_0 a^2}$ **d.** zero

34. A ring of charge with radius 0.5 m has $0.002\,\pi$ m gap. If the ring carries a charge of +1 C, the electric field at the centre is
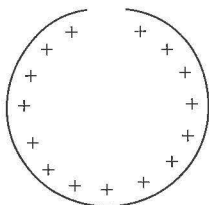

Fig. 1.115

a. $7.5 \times 10^7\,\text{NC}^{-1}$ **b.** $7.2 \times 10^7\,\text{NC}^{-1}$
c. $6.2 \times 10^7\,\text{NC}^{-1}$ **d.** $6.5 \times 10^7\,\text{NC}^{-1}$

35. A block of mass m containing a net negative charge $-q$ is placed on a frictionless horizontal table and is connected to a wall through an unstretched spring of spring constant k as shown. If horizontal electric field E parallel to the spring is switched on, then the maximum compression of the spring is
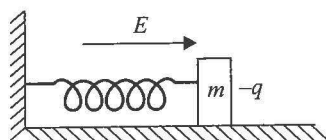

Fig. 1.116

a. $\sqrt{qE/k}$ **b.** $2qE/k$ **c.** qE/k **d.** zero

36. Figure 1.117 shows the electric lines of force emerging from a charged body. If the electric fields at A and B are E_A and E_B, respectively, and if the distance between A and B is r, then
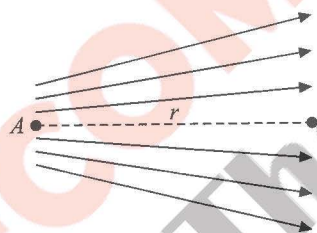

Fig. 1.117

a. $E_A > E_B$ **b.** $E_A < E_B$ **c.** $E_A = E_B/r$ **d.** $E_A = E_B/r^2$

37. If an electron has an initial velocity in a direction different from that of a uniform electric field, the path of the electron is
   a. a straight line   **b.** a circle
   c. an ellipse   **d.** a parabola

38. A point charge q_1 is moved along a circular path of radius r in the electric field of another point charge q_2 at the centre of the path. The work done by the electric field on the charge q_1 in half revolution is
   a. zero   **b.** positive
   c. negative   **d.** none of these

39. A spherical conducting ball is suspended by a grounded conducting thread. A positive point charge is moved near the ball. The ball will

a. be attracted to the point charge and swing toward it.
b. be repelled from the point charge and swing away from it.
c. not be affected by the point charge
d. none of these

40. Three +ve charges of equal magnitude 'q' are placed at the vertices of an equilateral triangle of side 'l'. How can the system of charges be placed in equilibrium?
   a. By placing a charge $Q = (-q/\sqrt{3})$ at the centroid of the triangle
   b. By placing a charge $Q = (q/\sqrt{3})$ at the centroid of the triangle
   c. By placing a charge $Q = q$ at a distance l from all the three charges
   d. By placing a charge $Q = -q$ above the plane of the triangle at a distance l from all the three charges

41. In the figure, two equal positive point charges $q_1 = q_2 = 2.0$ μC interact with a third point charge $Q = 4.0$ μC. The magnitude as well as direction of the net force on Q is

Fig. 1.118

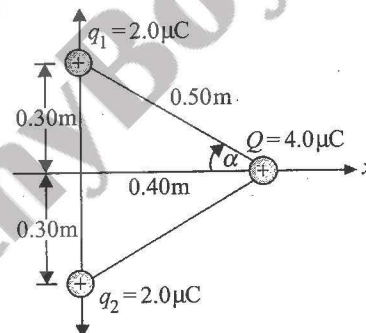**a.** 0.23 N in $+x$ direction   **b.** 0.46 N in $+x$ direction
c. 0.23 N in $-x$ direction   **d.** 0.46 N in $-x$ direction

42. Three identical spheres, each having a charge q and radius R, are kept in such a way that each touches the other two. The magnitude of the electric force on any sphere due to other two is

a. $\dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{q}{R}\right)^2$ **b.** $\dfrac{\sqrt{3}}{4\pi\varepsilon_0}\left(\dfrac{q}{R}\right)^2$

c. $\dfrac{\sqrt{3}}{16\pi\varepsilon_0}\left(\dfrac{q}{R}\right)^2$ **d.** $\dfrac{\sqrt{5}}{16\pi\varepsilon_0}\left(\dfrac{q}{R}\right)^2$

43. Five point charges, each of value $+q$, are placed on five vertices of a regular hexagon of side L. The magnitude of the force on a point charge of value $-q$ coulomb placed at the centre of the hexagon is

a. $\dfrac{1}{\pi\varepsilon_0}\left(\dfrac{q}{L}\right)^2$ **b.** $\dfrac{2}{\pi\varepsilon_0}\left(\dfrac{q}{L}\right)^2$

c. $\dfrac{1}{2\pi\varepsilon_0}\left(\dfrac{q}{L}\right)^2$ **d.** $\dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{q}{L}\right)^2$

44. It is required to hold equal charges q in equilibrium at the corners of a square. What charge when placed at the center of the square will do this?

a. $-\dfrac{q}{2}(1 + 2\sqrt{2})$ **b.** $\dfrac{q}{2}(1 + 2\sqrt{2})$

c. $\dfrac{q}{4}(1+2\sqrt{2})$

d. $-\dfrac{q}{4}(1+2\sqrt{2})$

45. A point charge $q = -8.0$ nC is located at the origin. The electric field (in NC^{-1}) vector at the point $x = 1.2$ m, $y = -1.6$ m as shown in Fig.1.119 is

a. $-14.4\hat{i}+10.8\hat{j}$

b. $-14.4\hat{i}-10.8\hat{j}$

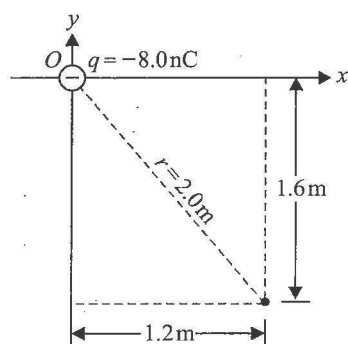c. $-10.8\hat{i}+14.4\hat{j}$

d. $-10.8\hat{i}-14.4\hat{j}$

Fig. 1.119

46. A positive point charge 50 μC is located in the plane xy at a point with radius vector $\vec{r}_0 = 2\hat{i}+3\hat{j}$. The electric field vector $\vec{E}$ at a point with radius vector $\vec{r} = 8\hat{i}-5\hat{j}$, where r_0 and r are expressed in metre, is

a. $(1.4\hat{i}-2.6\hat{j})$ kNC^{-1}

b. $(1.4\hat{i}+2.6\hat{j})$ kNC^{-1}

c. $(2.7\hat{i}-3.6\hat{j})$ kNC^{-1}

d. $(2.7\hat{i}+3.6\hat{j})$ kNC^{-1}

47. Four identical charges Q are fixed at the four corners of a square of side a. The electric field at a point P located symmetrically at a distance $a/\sqrt{2}$ from the centre of the square is

a. $\dfrac{Q}{2\sqrt{2}\pi\varepsilon_0 a^2}$

b. $\dfrac{Q}{\sqrt{2}\pi\varepsilon_0 a^2}$

c. $\dfrac{2\sqrt{2}Q}{\pi\varepsilon_0 a^2}$

d. $\dfrac{\sqrt{2}Q}{\pi\varepsilon_0 a^2}$

48. A thin glass rod is bent into a semicircle of radius r. A charge $+Q$ is uniformly distributed along the upper half and a charge $-Q$ is uniformly distributed along the lower half, as shown in Fig.1.120. The electric field E at P, the centre of the semicircle, is
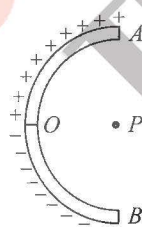

Fig. 1.120

a. $\dfrac{Q}{\pi^2\varepsilon_0 r^2}$

b. $\dfrac{2Q}{\pi^2\varepsilon_0 r^2}$

c. $\dfrac{4Q}{\pi^2\varepsilon_0 r^2}$

d. $\dfrac{Q}{4\pi^2\varepsilon_0 r^2}$

49. A system consists of a thin charged wire ring of radius r and a very long uniformly charged wire oriented along the axis of the ring, with one of its ends coinciding with the centre of the ring. The total charge on the ring is q and the linear charge density on the straight wire is λ. The interaction force between the ring and the wire is

a. $\dfrac{\lambda q}{4\pi\varepsilon_0 r}$

b. $\dfrac{\lambda q}{2\sqrt{2}\pi\varepsilon_0 r}$

c. $\dfrac{2\sqrt{2}\lambda q}{\pi\varepsilon_0 r}$

d. $\dfrac{4\lambda q}{\pi\varepsilon_0 r}$

50. Find the electric field vector at $P\,(a, a, a)$ due to three infinitely long lines of charges along the x-, y- and z-axes, respectively. The charge density, i.e., charge per unit length of each wire is λ.
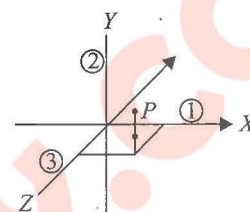

Fig. 1.121

a. $\dfrac{\lambda}{3\pi\varepsilon_0 a}(\hat{i}+\hat{j}+\hat{k})$

b. $\dfrac{\lambda}{2\pi\varepsilon_0 a}(\hat{i}+\hat{j}+\hat{k})$

c. $\dfrac{\lambda}{2\sqrt{2}\pi\varepsilon_0 a}(\hat{i}+\hat{j}+\hat{k})$

d. $\dfrac{\sqrt{2}\lambda}{\pi\varepsilon_0 a}(\hat{i}+\hat{j}+\hat{k})$

51. A particle of mass m and charge $-q$ moves diametrically through a uniformly charged sphere of radius R with total charge Q. The angular frequency of the particle's simple harmonic motion, if its amplitude $< R$, is given by

a. $\sqrt{\dfrac{1}{4\pi\varepsilon_0}\dfrac{qQ}{mR}}$

b. $\sqrt{\dfrac{1}{4\pi\varepsilon_0}\dfrac{qQ}{mR^2}}$

c. $\sqrt{\dfrac{1}{4\pi\varepsilon_0}\dfrac{qQ}{mR^3}}$

d. $\sqrt{\dfrac{1}{4\pi\varepsilon_0}\dfrac{m}{qQ}}$

52. A particle of mass m carrying a positive charge q moves simple harmonically along x-axis under the action of a varying electric field E directed along x-axis. The motion of the particle is confined between $x = 0$ and $x = 2l$. The angular frequency of the motion is ω. Then, which of the following is correct?

a. $qE = -m\omega^2(x-l)$

b. $qE = m\omega^2(x-l)$

c. Electric field to the right of origin is directed along +ve x-axis for all values of x.

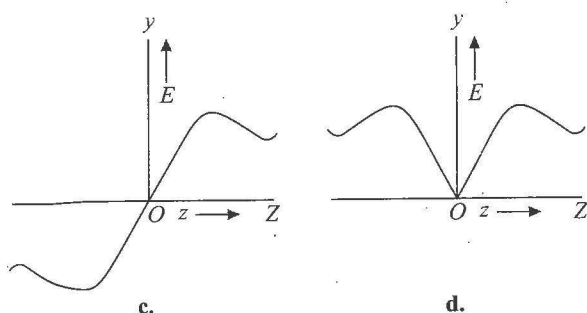d. Electric field to the right of origin is directed along −ve x-axis for all values of x.

53. A circular ring carries a uniformly distributed positive charge and lies in $X - Y$ plane with centre at origin of coordinate system. If at a point $(0, 0, z)$ the electric field is E, then which of the following graphs is correct?

a.

b.

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c.

d.
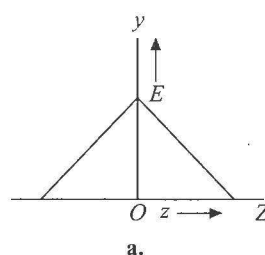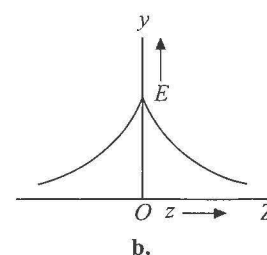
1. For the arrangement shown in Fig.1.122, the two positive charges, $+Q$ each, are fixed. Mark out the correct statement(s) regarding a third charged particle q placed at midpoint P that can be displaced along or perpendicular to the line connecting the charges.
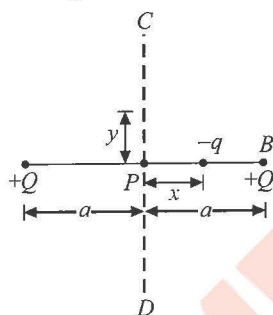

Fig. 1.122

a. The particle will perform S.H.M. for $x << a$.
b. The particle will oscillate about P but not harmonically for any x.
c. The particle will perform S.H.M. for $y << a$.
d. The particle will oscillate about P but not harmonically for y comparable to a.

2. A particle of mass m and charge $-q$ has been projected from ground as shown in Fig.1.123. Mark out the correct statement(s). XY plane is vertical.
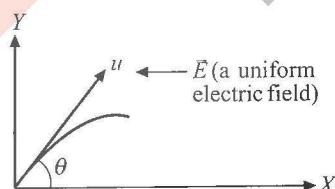

Fig. 1.123

a. The path of motion of the particle may be parabolic.
b. The path of motion of the particle may be a straight line.
c. Time of flight of the particle is $(2u\sin\theta)/g$.
d. Range of motion of the particle can be less than, greater than or equal to $(u^2\sin 2\theta)/g$.

3. For the arrangement shown in Fig.1.124, the two point charges are in equilibrium. The infinite wire is fixed in the horizontal plane and the two point charges are placed one above and the other below the wire.
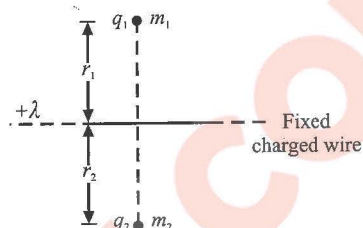

Fig. 1.124

Considering the gravitational effect of the earth, the nature of q_1 and q_2 can be

a. $q_1 \to$ +ve, $q_2 \to$ +ve
b. $q_1 \to$ +ve, $q_2 \to$ −ve
c. $q_1 \to$ −ve, $q_2 \to$ −ve
d. $q_1 \to$ −ve, $q_2 \to$ +ve

In the following questions, each question contains Statement I (Assertion) and Statement II (Reason). Each question has four choices (a), (b), (c) and (d). out of which **ONLY ONE** is correct.

a. Statement I is True, Statement II is True; Statement II is a correct explanation for Statement I.
b. Statement I is True, Statement II is True; Statement II is **NOT** a correct explanation for Statement I.
c. Statement 1 is True, Statement II is False.
d. Statement 1 is False, Statement II is True.

1. **Statement I:** If a point charge is rotated in a circle around another charge at the centre of circle, the work done by electric field will be zero.
   Statement II: Work done is equal to dot product of force and displacement.

2. **Statement I:** A positive point charge initially at rest in a uniform electric field starts moving along electric lines of forces. (Neglect all other forces except electric forces.)
   Statement II: A point charge released from rest in an electric field always moves along the lines of force.

3. **Statement I:** When a neutral body acquires +ve charge, its mass decreases.
   Statement II: A body acquires +ve charge when it loses electrons.

4. **Statement I:** Two similarly charged bodies may attract each other.
   Statement II: When charge on one body (Q) is much greater than that on another (q) and they are close enough to each other, then force of attraction between Q and induced charges exceeds the force of repulsion between Q and q.

5. **Statement I:** Charge is quantized because only integral number of electrons can be transferred.
   Statement II: There is no possibility of transfer of some fraction of electron.

Comprehension Type

For Problems 1–2

Two small identical conducting balls A and B of charges of $+10$ μC and $+30$ μC, respectively, are kept at a separation of 50 cm. These balls have been connected by a wire for a short time.

1. The final charge on each of the balls A and B will be
   a. 10 μC and 30 μC, respectively
   b. 20 μC on each ball
   c. 30 μC and 10 μC, respectively
   d. -40 μC and 80 μC, respectively
2. The force of interaction between the balls is
   a. 28.8 N   b. 32.6 N   c. 14.4 N   d. 72 N

For Problems 3–4

Two free point charges A and B having charges $+q$ and $+4q$, respectively, are a distance l apart. A third charge is so placed that the entire system is in equilibrium.

3. The third charge should be placed
   a. left of A at a distance $l/3$ from A
   b. right of A at a distance $l/3$ from B
   c. between A and B at a distance $2l/3$ from A
   d. between A and B at a distance $l/3$ from A
4. The third charge has magnitude and sign

   a. $Q = \left(-\dfrac{4}{9}\right) q$ b. $Q = \left(\dfrac{4}{9}\right) q$

   c. $Q = \left(\dfrac{3}{5}\right) q$ d. $Q = \left(-\dfrac{3}{5}\right) q$
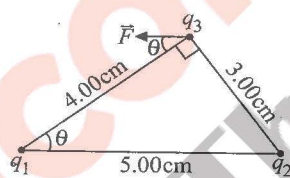
For Problems 5–7

Fig. 1.125

Three charges are placed as shown in Fig.1.125. The magnitude of $q1$ is 2.00 μC, but its sign and the value of the charge q_2 are not known. Charge q_3 is $+4.00$ μC, and the net force on q_3 is entirely in the negative x-direction.

5. As per the condition given in the problem, the sign of q_1 and q_2 will be
   a. $+, +$ b. $+, -$ c. $-, +$ d. $-, -$
6. The magnitude of q_2 is

   a. $\dfrac{27}{64} \mu C$ b. $\dfrac{27}{32} \mu C$ c. $\dfrac{13}{32} \mu C$ d. $\dfrac{13}{64} \mu C$

7. The magnitude of force acting on q_3 is
   a. 25.25 N   b. 32.5 N   c. 56.25 N   d. 13.5 N
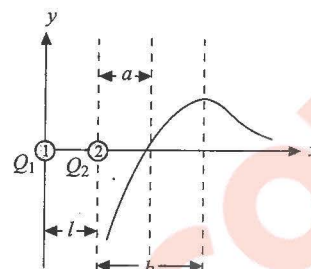
For Problems 8–10

Fig. 1.126

Two point like charges Q_1 and Q_2 are positioned at points 1 and 2. The field intensity to the right of the charge Q_2 on the line that passes through the two charges varies according to a law that is represented schematically in Fig.1.126. The field intensity is assumed to be positive if its direction coincides with the positive direction on the x-axis. The distance between the charges is l.

8. The sign of each charge Q_1 and Q_2 is
   a. $+, -$ b. $-, +$ c. $+, +$ d. $-, -$
9. The ratio of the absolute values of the charges $|Q_1/Q_2|$ is

   a. $\left(\dfrac{a+l}{a}\right)^2$ b. $\left(\dfrac{l}{a}\right)^2$ c. $\left(\dfrac{a}{a+l}\right)^2$ d. $\left(\dfrac{a}{l}\right)^2$

10. The value of b, where the field intensity is maximum, is

   a. $\dfrac{l}{(Q_1 / Q_2)^{1/3} + 1}$ b. $\dfrac{l}{(Q_1 / Q_2)^{1/3} - 1}$

   c. $\dfrac{l}{(Q_2 / Q_1)^{1/3} + 1}$ d. $\dfrac{l}{(Q_2 / Q_1)^{1/3} - 1}$

For Problems 11–12

Four equal positive charges, each of value Q, are arranged at the four corners of a square of diagonal $2a$. A small body of mass m carrying a unit positive charge is placed at a height h above the centre of the square.

11. What should be the value of Q in order that this body may be in equilibrium ?

   a. $\pi\varepsilon_0 \dfrac{mg}{2h}(h^2 + 2a^2)^{3/2}$ b. $\pi\varepsilon_0 \dfrac{mg}{h}(h^2 + a^2)^{3/2}$

   c. $\pi\varepsilon_0 \dfrac{2mg}{h}(h^2 + 2a^2)^{3/2}$ d. $\pi\varepsilon_0 \dfrac{mg}{2h}(h^2 - a^2)^{3/2}$

12. The type of equilibrium of the point mass is (consider only vertical displacement)
   a. stable equilibrium   b. unstable equilibrium
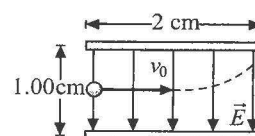   c. neutral equilibrium   d. cannot be determined

For Problems 13–16

Fig. 1.127

An electron is projected with an initial speed $v_0 = 1.60 \times 10^6$ ms^{-1} into the uniform field between the parallel plates as shown in Fig. 1.127. Assume that the field between the plates is uniform and directed vertically downwards, and that the field outside the

plates is zero. The electron enters the field at a point midway between the plates. Mass of electron = 9.1×10^{-31} kg.

13. If the electron just misses the upper plate, the time of flight of electron up to this instant is
   a. 1.25×10^{-9} s   b. 32.5×10^{-6} s
   c. 1.25×10^{-8} s   d. 32.5×10^{-8} s

14. For condition of previous situation, the magnitude of electric field is
   a. 124 NC^{-1} b. 364 NC^{-1}
   c. 224 NC^{-1} d. 520 NC^{-1}

15. If instead of electron, a proton were projected with the same speed, then compare the paths travelled by the electron and the proton.
   a. The proton will hit the upper plate.
   b. The proton will hit the lower plate.
   c. The proton will not hit either plate.
   d. None of these.

16. The vertical displacement travelled by the proton as it exits the region between the plates is. Mass of proton = 1.67×10^{-27} kg.
   a. 1.6×10^{-8} m   b. 3.25×10^{-8} m
   c. 5.25×10^{-6} m   d. 2.73×10^{-6} m

For Problems 17–18

An electron is projected as shown in Fig. 1.128, with kinetic energy K, at an angle $\theta = 45°$ between two charged plates. Ignore the gravity.

17. The magnitude of the electric field, so that the electron just fails to strike the upper plate is
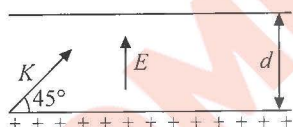   a. K/qd b. $2K/qd$ c. $K/2qd$ d. infinite

Fig. 1.128

18. At what distance from the starting point will the electron strike the lower plate?
   a. d b. $2d$ c. $3d$ d. $4d$

For Problems 19–20

In 1909, Robert Millikan was the first to find the charge of an electron in his now-famous oil-drop experiment. In that experiment, tiny oil drops were sprayed into a uniform electric field between a horizontal pair of oppositely charged plates. The drops were observed with a magnifying
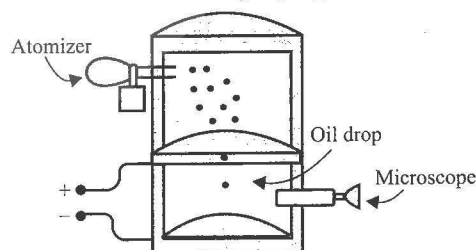

Fig. 1.129

eyepiece, and the electric field was adjusted so that the upward force on some negatively charged oil drops was just sufficient to balance the downward force of gravity. That is, when suspended, upward force qE just equalled mg. Millikan accurately measured the charges on many oil drops and found the values to be whole number multiples of 1.6×10^{-19} C— the charge of the electron. For this, he won the Nobel prize.

19. If a drop of mass 1.08×10^{-14} kg remains stationary in an electric field of 1.68×10^{5} NC^{-1}, then the charge of this drop is
   a. 6.40×10^{-19} C   b. 3.2×10^{-19} C
   c. 1.6×10^{-19} C   d. 4.8×10^{-19} C

20. Extra electrons on this particular oil drop (given the presently known charge of the electron) are
   a. 4   b. 3   c. 5   d. 8

Matching Column Type

Solutions on page 1.52

1. In Fig.1.130, charges, each $+q$, are fixed at L and M. O is the midpoint of distance LM. X- and Y- axes are as shown. Consider the situations given in column I and match them with the information in Column II:
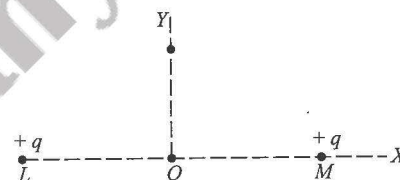

Fig. 1.130

Column I	Column II
i. Let us place a charge $+q$ at O, displace it slightly along X-axis and release. Assume that it is allowed to move only along X-axis. At position O,	a. force on the charge is zero
ii. Place a charge $-q$ at O. Displace it slightly along X-axis and release. Assume that it is allowed to move only along X-axis. At position O,	b. potential energy of the system is maximum
iii. Place a charge $+q$ at O. Displace it slightly along Y-axis and release. Assume that it is allowed to move only along Y-axis. At position O,	c. potential energy of the system is minimum
iv. Place a charge $-q$ at O. Displace it slightly along Y-axis and release. Assume that it is allowed to move only along Y-axis. At position O,	d. the charge is in equilibrium

2. Match the forces given in Column I with the properties given in Column II: