# Contents

# 1 Basic Mathematics used in Physics

- **Quadratic equation**

Roots of $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Sum of roots $x_1 + x_2 = -\dfrac{b}{a}$; Product of roots $x_1 x_2 = \dfrac{c}{a}$

- **Binomial theorem**

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2}x^2 + \dfrac{n(n-1)(n-2)}{6}x^3 + \ldots$

$(1-x)^n = 1 - nx + \dfrac{n(n-1)}{2}x^2 - \dfrac{n(n-1)(n-2)}{6}x^3 + \ldots$

If $x << 1$ then $(1+x)^n \approx 1 + nx$ & $(1-x)^n \approx 1 - nx$

- **Logarithm**

$\log mn = \log m + \log n$

$\log \dfrac{m}{n} = \log m - \log n$

$\log m^n = n \log m$

$\log_e m = 2.303 \log_{10} m$

$\log 2 = 0.3010$

$\log 3 = 0.4771$

- **Componendo and dividendo theorem**

If $\dfrac{p}{q} = \dfrac{a}{b}$ then $\dfrac{p+q}{p-q} = \dfrac{a+b}{a-b}$

- **Arithmetic progression-AP**

$a, a+d, a+2d, a+3d, \ldots a+(n-1)d$ here $d$ = common difference

Sum of n terms $S_n = \dfrac{n}{2}[2a+(n-1)d]$

**Note :** (i) $1+2+3+4+5 \ldots + n = \dfrac{n(n+1)}{2}$

(ii) $1^2+2^2+3^2+\ldots+n^2 = \dfrac{n(n+1)(2n+1)}{6}$

- **Geometrical progression-GP**

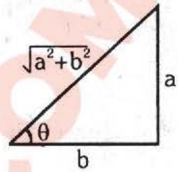
$a, ar, ar^2, ar^3, \ldots$ here, $r$ = common ratio

Sum of n terms $S_n = \dfrac{a(1-r^n)}{1-r}$

Sum of $\infty$ terms $S_\infty = \dfrac{a}{1-r}$

- **Trigonometry**

$2\pi$ radian $= 360° \Rightarrow 1$ rad $= 57.3°$




$$\sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{base}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{perpendicular}}{\text{base}} \qquad \cot\theta = \frac{\text{base}}{\text{perpendicular}}$$

$$\sec\theta = \frac{\text{hypotenuse}}{\text{base}} \qquad \text{cosec}\,\theta = \frac{\text{hypotenuse}}{\text{perpendicular}}$$

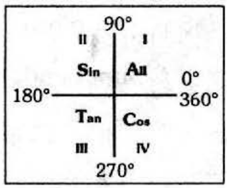
$$\sin\theta = \frac{a}{\sqrt{a^2+b^2}} \qquad \cos\theta = \frac{b}{\sqrt{a^2+b^2}} \qquad \tan\theta = \frac{a}{b}$$

$$\text{cosec}\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$\sin^2\theta + \cos^2\theta = 1$ $\qquad$ $1 + \tan^2\theta = \sec^2\theta$ $\qquad$ $1 + \cot^2\theta = \text{cosec}^2\theta$

$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$ $\qquad$ $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$




$\sin 2A = 2\sin A\cos A$

$\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1$

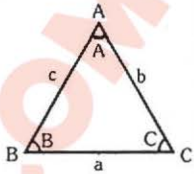
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

| | | | |
|---|---|---|---|
| $\sin(90°+\theta) = \cos\theta$ | $\sin(180°-\theta) = \sin\theta$ | $\sin(-\theta) = -\sin\theta$ | $\sin(90°-\theta) = \cos\theta$ |
| $\cos(90°+\theta) = -\sin\theta$ | $\cos(180°-\theta) = -\cos\theta$ | $\cos(-\theta) = \cos\theta$ | $\cos(90°-\theta) = \sin\theta$ |
| $\tan(90°+\theta) = -\cot\theta$ | $\tan(180°-\theta) = -\tan\theta$ | $\tan(-\theta) = -\tan\theta$ | $\tan(90°-\theta) = \cot\theta$ |
| $\sin(180°+\theta) = -\sin\theta$ | $\sin(270°-\theta) = -\cos\theta$ | $\sin(270°+\theta) = -\cos\theta$ | $\sin(360°-\theta) = -\sin\theta$ |
| $\cos(180°+\theta) = -\cos\theta$ | $\cos(270°-\theta) = -\sin\theta$ | $\cos(270°+\theta) = \sin\theta$ | $\cos(360°-\theta) = \cos\theta$ |
| $\tan(180°+\theta) = \tan\theta$ | $\tan(270°-\theta) = \cot\theta$ | $\tan(270°+\theta) = -\cot\theta$ | $\tan(360°-\theta) = -\tan\theta$ |

| $\theta \rightarrow$ | 0° (0) | 30° $\left(\frac{\pi}{6}\right)$ | 45° $\left(\frac{\pi}{4}\right)$ | 60° $\left(\frac{\pi}{3}\right)$ | 90° $\left(\frac{\pi}{2}\right)$ | 120° $\left(\frac{2\pi}{3}\right)$ | 135° $\left(\frac{3\pi}{4}\right)$ | 150° $\left(\frac{5\pi}{6}\right)$ | 180° ($\pi$) | 270° $\left(\frac{3\pi}{2}\right)$ | 360° ($2\pi$) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\sin\theta$ | 0 | $\frac{1}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{3}}{2}$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$ | 0 | $-1$ | 0 |
| $\cos\theta$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | $-\frac{1}{\sqrt{2}}$ | $-\frac{\sqrt{3}}{2}$ | $-1$ | 0 | 1 |
| $\tan\theta$ | 0 | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ | $\infty$ | $-\sqrt{3}$ | $-1$ | $-\frac{1}{\sqrt{3}}$ | 0 | $\infty$ | 0 |

- **sine law**

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$




- **cosine law**

$$\cos A = \frac{b^2+c^2-a^2}{2bc}, \cos B = \frac{c^2+a^2-b^2}{2ca}, \cos C = \frac{a^2+b^2-c^2}{2ab}$$

- **For small θ**

$\sin\theta \approx \theta$ $\cos\theta \approx 1$ $\tan\theta \approx \theta \sin\theta \approx \tan\theta$

- **Differentiation**

- $y = x^n \rightarrow \frac{dy}{dx} = nx^{n-1}$
- $y = \ell nx \rightarrow \frac{dy}{dx} = \frac{1}{x}$
- $y = \sin x \rightarrow \frac{dy}{dx} = \cos x$
- $y = \cos x \rightarrow \frac{dy}{dx} = -\sin x$
- $y = e^{\alpha x+\beta} \rightarrow \frac{dy}{dx} = \alpha e^{\alpha x+\beta}$
- $y = uv \rightarrow \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
- $y = f(g(x)) \Rightarrow \frac{dy}{dx} = \frac{df(g(x))}{dg(x)} \times \frac{d(g(x))}{dx}$
- $y=k$ = constant $\Rightarrow \frac{dy}{dx} = 0$
- $y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

- **Integration**

- $\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
- $\int \frac{1}{x} dx = \ell nx + C$
- $\int \sin x dx = -\cos x + C$
- $\int \cos x dx = \sin x + C$
- $\int e^{\alpha x+\beta} dx = \frac{1}{\alpha} e^{\alpha x+\beta} + C$
- $\int (\alpha x+\beta)^n dx = \frac{(\alpha x+\beta)^{n+1}}{\alpha(n+1)} + C$

- **Maxima & Minima of a function y = f(x)**

  - For maximum value $\frac{dy}{dx} = 0$ & $\frac{d^2y}{dx^2} = -ve$

  - For minimum value $\frac{dy}{dx} = 0$ & $\frac{d^2y}{dx^2} = +ve$

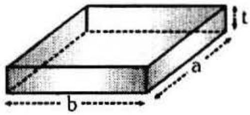
- **Average of a varying quantity**

If y = f(x) then $< y > = \bar{y} = \frac{\int_{x_1}^{x_2} y\,dx}{\int_{x_1}^{x_2} dx} = \frac{\int_{x_1}^{x_2} y\,dx}{x_2 - x_1}$

**Formulae for determination of area**

- Area of a square = $(\text{side})^2$
- Area of rectangle = length $\times$ breadth
- Area of a triangle = $\frac{1}{2} \times$ base $\times$ height
- Area of a trapezoid = $\frac{1}{2} \times$ (distance between parallel sides) $\times$ (sum of parallel sides)
- Area enclosed by a circle = $\pi r^2$ (r = radius)
- Surface area of a sphere = $4\pi r^2$ (r = radius)
- Area of a parallelogram = base $\times$ height
- Area of curved surface of cylinder = $2\pi r\ell$ (r = radius and $\ell$ = length)
- Area of whole surface of cylinder = $2\pi r (r + \ell)$ ($\ell$ = length)
- Area of ellipse = $\pi$ ab (a & b are semi major and semi minor axis respectively)
- Surface area of a cube = $6(\text{side})^2$
- Total surface area of a cone = $\pi r^2 + \pi r\ell$ where $\pi r\ell = \pi r\sqrt{r^2 + h^2}$ = lateral area

**Formulae for determination of volume :**




- Volume of a rectangular slab = length × breadth × height = abt
- Volume of a cube = $(\text{side})^3$
- Volume of a sphere = $\frac{4}{3}\pi r^3$ (r = radius)
- Volume of a cylinder = $\pi r^2 \ell$ (r = radius and $\ell$ = length)
- Volume of a cone = $\frac{1}{3}\pi r^2 h$ (r = radius and h = height)

**KEY POINTS:**

- To convert an angle from degree to radian, we have to multiply it by $\frac{\pi}{180°}$ and to convert an angle from radian to degree, we have to multiply it by $\frac{180°}{\pi}$.
- By help of differentiation, if y is given, we can find $\frac{dy}{dx}$ and by help of integration, if $\frac{dy}{dx}$ is given, we can find y.
- The maximum and minimum values of function $A\cos\theta + B\sin\theta$ are $\sqrt{A^2+B^2}$ and $-\sqrt{A^2+B^2}$ respectively.
- $(a+b)^2 = a^2 + b^2 + 2ab$ $(a-b)^2 = a^2 + b^2 - 2ab$

  $(a+b)(a-b) = a^2 - b^2$ $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

  $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

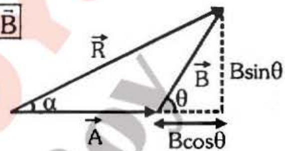
# Important Notes

# 2 Vectors

- **Vector Quantities**

  A physical quantity which requires magnitude and a particular direction, when it is expressed.

- **Triangle law of Vector addition** $\boxed{\vec{R}=\vec{A}+\vec{B}}$



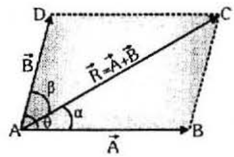

$R=\sqrt{A^2+B^2+2AB\cos\theta}$

$\tan\alpha=\dfrac{B\sin\theta}{A+B\cos\theta}$ If A = B then $R=2A\cos\dfrac{\theta}{2}$ & $\alpha=\dfrac{\theta}{2}$

$R_{max}$ = A+B for θ=0° ; $R_{min}$ = A–B for θ=180°

- **Parallelogram Law of Addition of Two Vectors**

  If two vectors are represented by two adjacent sides of a parallelogram which are directed away from their common point then their sum (i.e. resultant vector) is given by the diagonal of the parallelogram passing away through that common point.

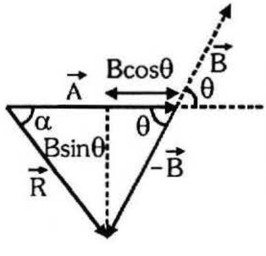



$\overline{AB}+\overline{AD}=\overline{AC}=\vec{R}$ or $\vec{A}+\vec{B}=\vec{R}\Rightarrow R=\sqrt{A^2+B^2+2AB\cos\theta}$

$\tan\alpha=\dfrac{B\sin\theta}{A+B\cos\theta}$ and $\tan\beta=\dfrac{A\sin\theta}{B+A\cos\theta}$

- **Vector subtraction**

$\boxed{\vec{R}=\vec{A}-\vec{B}}\Rightarrow\vec{R}=\vec{A}+(-\vec{B})$

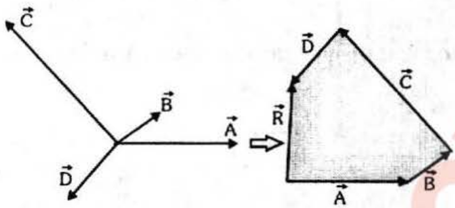



$R=\sqrt{A^2+B^2-2AB\cos\theta}$ , $\tan\alpha=\dfrac{B\sin\theta}{A-B\cos\theta}$

If A = B then $R=2A\sin\dfrac{\theta}{2}$

- **Addition of More than Two Vectors (Law of Polygon)**

If some vectors are represented by sides of a polygon in same order, then their resultant vector is represented by the closing side of polygon in the opposite order.




$\vec{R}=\vec{A}+\vec{B}+\vec{C}+\vec{D}$

- **Rectangular component of a 3–D vector**

□ $\vec{A}=A_x\hat{i}+A_y\hat{j}+A_z\hat{k}$

Angle made with x-axis $\cos\alpha=\dfrac{A_x}{A}=\dfrac{A_x}{\sqrt{A_x^2+A_y^2+A_z^2}}=\ell$

Angle made with y-axis $\cos\beta=\dfrac{A_y}{A}=\dfrac{A_y}{\sqrt{A_x^2+A_y^2+A_z^2}}=m$

Angle made with z-axis $\cos\gamma=\dfrac{A_z}{A}=\dfrac{A_z}{\sqrt{A_x^2+A_y^2+A_z^2}}=n$

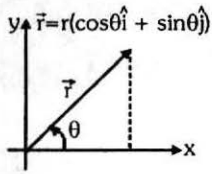
□ $\ell$, m, n are called direction cosines

$$\ell^2+m^2+n^2=\cos^2\alpha+\cos^2\beta+\cos^2\gamma=\frac{A_x^2+A_y^2+A_z^2}{\left(\sqrt{A_x^2+A_y^2+A_z^2}\right)^2}=1$$

or $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=2$

- **General Vector in x-y plane**

$\vec{r}=x\hat{i}+y\hat{j}=r\left(\cos\theta\hat{i}+\sin\theta\hat{j}\right)$



**Examples**

**1.** Construct a vector of magnitude 6 units making an angle of 60° with x-axis.

**Sol.** $\vec{r} = r(\cos 60\hat{i} + \sin 60\hat{j}) = 6\left(\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right) = 3\hat{i} + 3\sqrt{3}\hat{j}$

**2.** Construct an unit vector making an angle of 135° with x axis.

**Sol.** $\hat{r} = 1(\cos 135°\hat{i} + \sin 135°\hat{j}) = \frac{1}{\sqrt{2}}(-\hat{i} + \hat{j})$

- **Scalar product (Dot Product)**

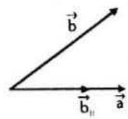
  - $\vec{A}.\vec{B} = AB\cos\theta \Rightarrow \boxed{\text{Angle between two vectors } \theta = \cos^{-1}\left(\frac{\vec{A}\cdot\vec{B}}{AB}\right)}$

  - If $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ & $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ then

    $\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z$ and angle between $\vec{A}$ & $\vec{B}$ is given by

    $$\cos\theta = \frac{\vec{A}.\vec{B}}{AB} = \frac{A_xB_x + A_yB_y + A_zB_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}\sqrt{B_x^2 + B_y^2 + B_z^2}}$$

  - $\hat{i}.\hat{i} = 1$, $\hat{j}.\hat{j} = 1$, $\hat{k}.\hat{k} = 1$, $\hat{i}.\hat{j} = 0$, $\hat{i}.\hat{k} = 0$, $\hat{j}.\hat{k} = 0$

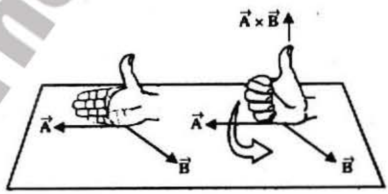



  - Component of vector $\vec{b}$ along vector $\vec{a}$, $\vec{b}_{\parallel} = (\vec{b}\,.\,\hat{a})\hat{a}$

  - Component of $\vec{b}$ perpendicular to $\vec{a}$, $\vec{b}_{\perp} = \vec{b} - \vec{b}_{\parallel} = \vec{b} - (\vec{b}\cdot\hat{a})\hat{a}$

- **Cross Product (Vector product)**

  - $\vec{A} \times \vec{B} = AB\sin\theta\,\hat{n}$ where $\hat{n}$ is a vector perpendicular to $\vec{A}$ & $\vec{B}$ or their plane and its direction given by right hand thumb rule.



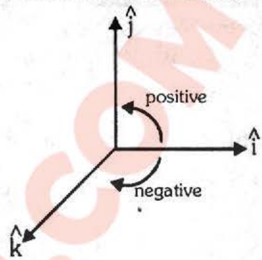

  - $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{i}(A_yB_z - A_zB_y) - \hat{j}(A_xB_z - B_xA_z) + \hat{k}(A_xB_y - B_xA_y)$

  - $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

- $(\vec{A}\times\vec{B}).\vec{A} = (\vec{A}\times\vec{B}).\vec{B} = 0$
- $\hat{i}\times\hat{i}=\vec{0},\ \hat{j}\times\hat{j}=\vec{0},\ \hat{k}\times\hat{k}=\vec{0}$
- $\hat{i}\times\hat{j}=\hat{k};\ \hat{j}\times\hat{k}=\hat{i},$

  $\hat{k}\times\hat{i}=\hat{j};\ \hat{j}\times\hat{i}=-\hat{k}$

  $\hat{k}\times\hat{j}=-\hat{i},\ \hat{i}\times\hat{k}=-\hat{j}$



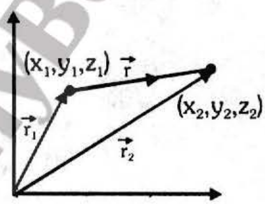

- **Differentiation**

- $\dfrac{d}{dt}(\vec{A}.\vec{B}) = \dfrac{d\vec{A}}{dt}.\vec{B} + \vec{A}.\dfrac{d\vec{B}}{dt}$
- $\dfrac{d}{dt}(\vec{A}\times\vec{B}) = \dfrac{d\vec{A}}{dt}\times\vec{B} + \vec{A}\times\dfrac{d\vec{B}}{dt}$

- **When a particle moved from $(x_1, y_1, z_1)$ to $(x_2, y_2, z_2)$ then its displacement vector**

$$\vec{r} = \vec{r}_2 - \vec{r}_1 = (x_2\hat{i} + y_2\hat{j} + z_2\hat{k}) - (x_1\hat{i} + y_1\hat{j} + z_1\hat{k})$$

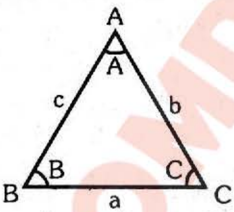
$$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

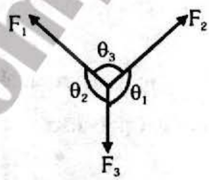



Magnitude

$$r = |\vec{r}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

- **Lami's theorem**

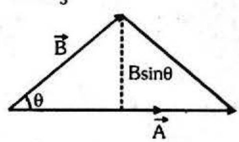


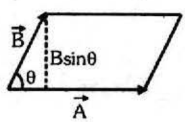

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{F_1}{\sin\theta_1} = \frac{F_2}{\sin\theta_2} = \frac{F_3}{\sin\theta_3}$$

- **Area of triangle** Area $= \dfrac{|\vec{A}\times\vec{B}|}{2} = \dfrac{1}{2}AB\sin\theta$




- **Area of parallelogram** Area $= |\vec{A}\times\vec{B}| = AB\sin\theta$




- **For Parallel vectors** $\vec{A}\times\vec{B} = \vec{0}$
- **For perpendicular vectors** $\vec{A}.\vec{B} = 0$
- **For coplanar vectors** $\vec{A}.(\vec{B}\times\vec{C}) = 0$

**Examples of dot products :**

- Work, $W = \vec{F}.\vec{d} = Fd\cos\theta$ where $F \rightarrow$ force, $d \rightarrow$ displacement
- Power, $P = \vec{F}.\vec{v} = Fv\cos\theta$ where $F \rightarrow$ force, $v \rightarrow$ velocity
- Electric flux, $\phi_E = \vec{E}.\vec{A} = EA\cos\theta$ where $E \rightarrow$ electric field, $A \rightarrow$ Area
- Magnetic flux, $\phi_B = \vec{B}.\vec{A} = BA\cos\theta$ where $B \rightarrow$ magnetic field, $A \rightarrow$ Area
- Potential energy of dipole in uniform field, $U = -\vec{p}.\vec{E}$ where $p \rightarrow$ dipole moment, $E \rightarrow$ Electric field

**Examples of cross products :**

- Torque $\vec{\tau} = \vec{r} \times \vec{F}$ where $r \rightarrow$ position vector, $F \rightarrow$ force
- Angular momentum $\vec{J} = \vec{r} \times \vec{p}$ where $r \rightarrow$ position vector, $p \rightarrow$ linear momentum
- Linear velocity $\vec{v} = \vec{\omega} \times \vec{r}$ where $r \rightarrow$ position vector, $\omega \rightarrow$ angular velocity
- Torque on dipole placed in electric field $\vec{\tau} = \vec{p} \times \vec{E}$

  where $p \rightarrow$ dipole moment, $E \rightarrow$ electric field

**KEY POINTS :**

- **Tensor :** A quantity that has different values in different directions is called tensor.

  **Ex.** Moment of Inertia

  In fact tensors are merely a generalisation of scalars and vectors; a scalar is a zero rank tensor, and a vector is a first rank tensor.
- Electric current is not a vector as it does not obey the law of vector addition.
- A unit vector has no unit.
- To a vector only a vector of same type can be added and the resultant is a vector of the same type.
- A scalar or a vector can never be divided by a vector.

# Important Notes

# 3 Units, Dimension, Measurements and Practical Physics

- **Fundamental or base quantities :**

The quantities which do not depend upon other quantities for their complete definition are known as *fundamental or base quantities.*

e.g. : length, mass, time, etc.

- **Derived quantities :**

The quantities which can be expressed in terms of the fundamental quantities are known as *derived quantities* .e.g.

Speed (=distance/time), volume, acceleration, force, pressure, etc.

- **Units of physical quantities**

The chosen reference standard of measurement in multiples of which, a physical quantity is expressed is called the *unit* of that quantity.

Physical Quantity = Numerical Value × Unit

**Systems of Units**

| | MKS | CGS | FPS | MKSQ | MKSA |
|---|---|---|---|---|---|
| (i) | Length (m) | Length (cm) | Length (ft) | Length (m) | Length (m) |
| (ii) | Mass (kg) | Mass (g) | Mass (pound) | Mass (kg) | Mass (kg) |
| (iii) | Time (s) | Time (s) | Time (s) | Time (s) | Time (s) |
| (iv) | – | – | – | Charge (Q) | Current (A) |

**Fundamental Quantities in S.I. System and their units**

| S.N. | Physical Qty. | Name of Unit | Symbol |
|---|---|---|---|
| 1 | Mass | kilogram | kg |
| 2 | Length | meter | m |
| 3 | Time | second | s |
| 4 | Temperature | kelvin | K |
| 5 | Luminous intensity | candela | Cd |
| 6 | Electric current | ampere | A |
| 7 | Amount of substance | mole | mol |

## SI Base Quantities and Units

| Base Quantity | SI Units | | |
|---|---|---|---|
| | Name | Symbol | Definition |
| Length | meter | m | The meter is the length of the path traveled by light in vacuum during a time interval of 1/(299, 792, 458) of a second (1983) |
| Mass | kilogram | kg | The kilogram is equal to the mass of the international prototype of the kilogram (a platinum-iridium alloy cylinder) kept at International Bureau of Weights and Measures, at Sevres, near Paris, France. (1889) |
| Time | second | s | The second is the duration of 9, 192, 631, 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom (1967) |
| Electric Current | ampere | A | The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to $2 \times 10^{-7}$ Newton per metre of length. (1948) |
| Thermodynamic Temperature | kelvin | K | The kelvin, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water. (1967) |
| Amount of Substance | mole | mol | The mole is the amount of substance of a system, which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon-12. (1971) |
| Luminous Intensity | candela | Cd | The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency $540 \times 10^{12}$ hertz and that has a radiant intensity in that direction of 1/683 watt per steradian (1979). |

- **Supplementary Units**
  - Radian (rad) - for measurement of plane angle
  - Steradian (sr) - for measurement of solid angle
- **Dimensional Formula**

Relation which express physical quantities in terms of appropriate powers of fundamental units.

- **Use of dimensional analysis**
  - To check the dimensional correctness of a given physical relation
  - To derive relationship between different physical quantities
  - To convert units of a physical quantity from one system to another

$$n_1u_1 = n_2u_2 \Rightarrow n_2 = n_1\left(\frac{M_1}{M_2}\right)^a\left(\frac{L_1}{L_2}\right)^b\left(\frac{T_1}{T_2}\right)^c \text{ where } u = M^aL^bT^c$$

- **Limitations of this method :**
  - In Mechanics the formula for a physical quantity depending on more than three other physical quantities cannot be derived. It can only be checked.
  - This method can be used only if the dependency is of multiplication type. The formulae containing exponential, trigonometrical and logarithmic functions can't be derived using this method. Formulae containing more than one term which are added or subtracted like $s = ut + \frac{1}{2}at^2$ also can't be derived.
  - The relation derived from this method gives no information about the dimensionless constants.
  - If dimensions are given, physical quantity may not be unique as many physical quantities have the same dimensions.
  - It gives no information whether a physical quantity is a scalar or a vector.

**SI PREFIXES**

The magnitudes of physical quantities vary over a wide range. The CGPM recommended standard prefixes for magnitude too large or too small to be expressed more compactly for certain powers of 10.

## Prefixes used for different powers of 10

| Power of 10 | Prefix | Symbol | Power of 10 | Prefix | Symbol |
|---|---|---|---|---|---|
| $10^{18}$ | exa | E | $10^{-1}$ | deci | d |
| $10^{15}$ | peta | P | $10^{-2}$ | centi | c |
| $10^{12}$ | tera | T | $10^{-3}$ | milli | m |
| $10^{9}$ | giga | G | $10^{-6}$ | micro | μ |
| $10^{6}$ | mega | M | $10^{-9}$ | nano | n |
| $10^{3}$ | kilo | k | $10^{-12}$ | pico | p |
| $10^{2}$ | hecto | h | $10^{-15}$ | femto | f |
| $10^{1}$ | deca | da | $10^{-18}$ | atto | a |

## Units of important Physical Quantities

| Physical quantity | Unit | Physical quantity | Unit |
|---|---|---|---|
| Angular acceleration | rad $s^{-2}$ | Frequency | hertz |
| Moment of inertia | kg – $m^2$ | Resistance | kg $m^2$ $A^{-2}$ $s^{-3}$ |
| Self inductance | Henry | Surface tension | newton/m |
| Magnetic flux | Weber | Universal gas constant | joule $K^{-1}$ $mol^{-1}$ |
| Pole strength | A–m | Dipole moment | Coulomb–meter |
| Viscosity | Poise | Stefan constant | watt $m^{-2}$ $K^{-4}$ |
| Reactance | Ohm | Permittivity of free space ($\varepsilon_0$) | Coulomb$^2$/N–$m^2$ |
| Specific heat | J/kg°C | Permeability of free space ($\mu_0$) | Weber/A-m |
| Strength of magnetic field | newton $A^{-1}$ $m^{-1}$ | Planck's constant | joule–sec |
| Astronomical distance | Parsec | Entropy | J/K |

## Dimensions of Important Physical Quantities

| Physical quantity | Dimensions | Physical quantity | Dimensions |
|---|---|---|---|
| Momentum | $M^1 L^1 T^{-1}$ | Capacitance | $M^{-1} L^{-2} T^4 A^2$ |
| Calorie | $M^1 L^2 T^{-2}$ | Modulus of rigidity | $M^1 L^{-1} T^{-2}$ |
| Latent heat capacity | $M^0 L^2 T^{-2}$ | Magnetic permeability | $M^1 L^1 T^{-2} A^{-2}$ |
| Self inductance | $M^1 L^2 T^{-2} A^{-2}$ | Pressure | $M^1 L^{-1} T^{-2}$ |
| Coefficient of thermal conductivity | $M^1 L^1 T^{-3} K^{-1}$ | Planck's constant | $M^1 L^2 T^{-1}$ |
| Power | $M^1 L^2 T^{-3}$ | Solar constant | $M^1 L^0 T^{-3}$ |
| Impulse | $M^1 L^1 T^{-1}$ | Magnetic flux | $M^1 L^2 T^{-2} A^{-1}$ |
| Hole mobility in a semi conductor | $M^{-1} L^0 T^2 A^1$ | Current density | $M^0 L^{-2} T^0 A^1$ |
| Bulk modulus of elasticity | $M^1 L^{-1} T^{-2}$ | Young modulus | $M^1 L^{-1} T^{-2}$ |
| Potential energy | $M^1 L^2 T^{-2}$ | Magnetic field intensity | $M^0 L^{-1} T^0 A^1$ |
| Gravitational constant | $M^{-1} L^3 T^{-2}$ | Magnetic Induction | $M^1 T^{-2} A^{-1}$ |
| Light year | $M^0 L^1 T^0$ | Permittivity | $M^{-1} L^{-3} T^4 A^2$ |
| Thermal resistance | $M^{-1} L^{-2} T^3 K$ | Electric Field | $M^1 L^1 T^{-3} A^{-1}$ |
| Coefficient of viscosity | $M^1 L^{-1} T^{-1}$ | Resistance | $M L^2 T^{-3} A^{-2}$ |

## Sets of Quantities having same dimensions

| S.N. | Quantities | Dimensions |
|---|---|---|
| 1. | Strain, refractive index, relative density, angle, solid angle, phase, distance gradient, relative permeability, relative permittivity, angle of contact, Reynolds number, coefficient of friction, mechanical equivalent of heat, electric susceptibility, etc. | $[M^0 L^0 T^0]$ |
| 2. | Mass and inertia | $[M^1 L^0 T^0]$ |
| 3. | Momentum and impulse. | $[M^1 L^1 T^{-1}]$ |
| 4. | Thrust, force, weight, tension, energy gradient. | $[M^1 L^1 T^{-2}]$ |
| 5. | Pressure, stress, Young's modulus, bulk modulus, shear modulus, modulus of rigidity, energy density. | $[M^1 L^{-1} T^{-2}]$ |
| 6. | Angular momentum and Planck's constant (h). | $[M^1 L^2 T^{-1}]$ |
| 7. | Acceleration, g and gravitational field intensity. | $[M^0 L^1 T^{-2}]$ |
| 8. | Surface tension, free surface energy (energy per unit area), force gradient, spring constant. | $[M^1 L^0 T^{-2}]$ |
| 9. | Latent heat capacity and gravitational potential. | $[M^0 L^2 T^{-2}]$ |
| 10. | Thermal capacity, Boltzmann constant, entropy. | $[ML^2T^{-2}K^{-1}]$ |
| 11. | Work, torque, internal energy, potential energy, kinetic energy, moment of force, $(q^2/C)$, $(LI^2)$, $(qV)$, $(V^2C)$, $(I^2Rt)$, $\frac{V^2}{R}t$, $(VIt)$, $(PV)$, $(RT)$, $(mL)$, $(mc\,\Delta T)$ | $[M^1 L^2 T^{-2}]$ |
| 12. | Frequency, angular frequency, angular velocity, velocity gradient, radioactivity $\frac{R}{L}, \frac{1}{RC}, \frac{1}{\sqrt{LC}}$ | $[M^0 L^0 T^{-1}]$ |
| 13. | $\left(\frac{\ell}{g}\right)^{1/2}, \left(\frac{m}{k}\right)^{1/2}, \left(\frac{L}{R}\right), (RC), (\sqrt{LC})$, time | $[M^0 L^0 T^1]$ |
| 14. | $(VI)$, $(I^2R)$, $(V^2/R)$, Power | $[M L^2 T^{-3}]$ |

## Some Fundamental Constants

| | |
|---|---|
| Gravitational constant (G) | $6.67 \times 10^{-11}$ N m$^2$ kg$^{-2}$ |
| Speed of light in vacuum (c) | $3 \times 10^8$ ms$^{-1}$ |
| Permeability of vacuum ($\mu_0$) | $4\pi \times 10^{-7}$ H m$^{-1}$ |
| Permittivity of vacuum ($\varepsilon_0$) | $8.85 \times 10^{-12}$ F m$^{-1}$ |
| Planck constant (h) | $6.63 \times 10^{-34}$ Js |
| Atomic mass unit (amu) | $1.66 \times 10^{-27}$ kg |
| Energy equivalent of 1 amu | 931.5 MeV |
| Electron rest mass ($m_e$) | $9.1 \times 10^{-31}$ kg $\equiv$ 0.511 MeV |
| Avogadro constant ($N_A$) | $6.02 \times 10^{23}$ mol$^{-1}$ |
| Faraday constant (F) | $9.648 \times 10^4$ C mol$^{-1}$ |
| Stefan–Boltzmann constant ($\sigma$) | $5.67 \times 10^{-8}$ W m$^{-2}$ K$^{-4}$ |
| Wien constant (b) | $2.89 \times 10^{-3}$ mK |
| Rydberg constant ($R_\infty$) | $1.097 \times 10^7$ m$^{-1}$ |
| Triple point for water | 273.16 K (0.01°C) |
| Molar volume of ideal gas (NTP) | 22.4 L = $22.4 \times 10^{-3}$ m$^3$ mol$^{-1}$ |

## KEY POINTS

- Trigonometric functions $\sin\theta$, $\cos\theta$, $\tan\theta$ etc and their arrangements $\theta$ are dimensionless.
- Dimensions of differential coefficients $\left[\frac{d^n y}{dx^n}\right] = \left[\frac{y}{x^n}\right]$
- Dimensions of integrals $\left[\int y\,dx\right] = [yx]$
- We can't add or subtract two physical quantities of different dimensions.
- Independent quantities may be taken as fundamental quantities in a new system of units.

## PRACTICAL PHYSICS

- **Rules for Counting Significant Figures**

  **For a number greater than 1**
  - All non-zero digits are significant.
  - All zeros between two non-zero digits are significant. Location of decimal does not matter.
  - If the numbe is without decimal part, then the terminal or trailing zeros are not significant.
  - Trailing zeros in the decimal part are significant.

- **For a Number Less Than 1**

  Any zero to the right of a non-zero digit is significant. All zeros between decimal point and first non-zero digit are not significant.

- **Significant Figures**

  All accurately known digits in measurement plus the first uncertain digit together form significant figure.

  **Ex.** $0.108 \rightarrow 3SF$, $40.000 \rightarrow 5SF$,
  $1.23 \times 10^{-19} \rightarrow 3SF$, $0.0018 \rightarrow 2SF$

- **Rounding off**

  $6.87 \rightarrow 6.9$, $6.84 \rightarrow 6.8$, $6.85 \rightarrow 6.8$,
  $6.75 \rightarrow 6.8$, $6.65 \rightarrow 6.6$, $6.95 \rightarrow 7.0$

- **Order of magnitude :**

  Power of 10 required to represent a quantity

  $49 = 4.9 \times 10^1 \approx 10^1 \Rightarrow$ order of magnitude $=1$

  $51 = 5.1 \times 10^1 \approx 10^2 \Rightarrow$ order of magnitude $= 2$

  $0.051 = 5.1 \times 10^{-2} \approx 10^{-1} \Rightarrow$ order of magnitude $= -1$

- **Propagation of combination of errors**

  **Error in Summation and Difference :** $x = a + b$ then $\Delta x = \pm (\Delta a + \Delta b)$

- **Error in Product and Division** A physical quantity X depend upon Y & Z as $X = Y^a Z^b$ then maximum possible fractional error in X.

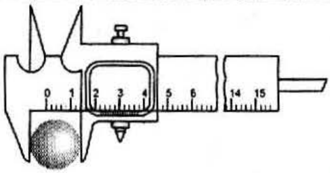
$$\frac{\Delta X}{X} = |a| \frac{\Delta Y}{Y} + |b| \frac{\Delta Z}{Z}$$

- **Error in Power of a Quantity** : $x = \frac{a^m}{b^n}$ then $\frac{\Delta x}{x} = \pm \left[ m\left(\frac{\Delta a}{a}\right) + n\left(\frac{\Delta b}{b}\right) \right]$

- **Least count** : The smallest value of a physical quantity which can be measured accurately with an instrument is called the least count of the measuring instrument.

- **Vernier Callipers** Least count = 1MSD – 1 VSD

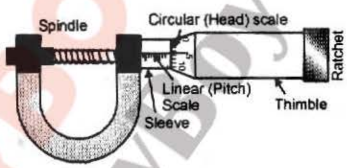
  (MSD $\rightarrow$ main scale division, VSD $\rightarrow$ Vernier scale division)




**Ex.** A vernier scale has 10 parts, which are equal to 9 parts of main scale having each path equal to 1 mm then least count = $1 \text{ mm} - \frac{9}{10} \text{ mm} = 0.1 \text{ mm}$
[$\because$ 9 MSD = 10 VSD]

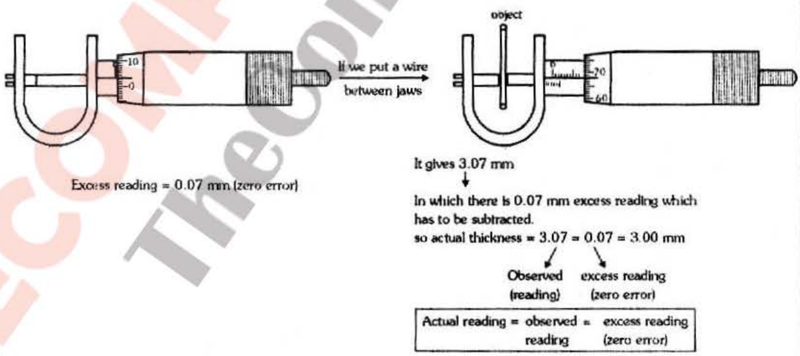
- **Screw Gauge** :

$$\text{Least count} = \frac{\text{pitch}}{\text{total no. of divisions on circular scale}}$$




**Zero Error :**

If there is no object between the jaws (i.e. jaws are in contact), the screwgauge should give zero reading. But due to extra material on jaws, even if there is no object, it gives some excess reading. This excess reading is called Zero error.




Excess reading = 0.07 mm (zero error)

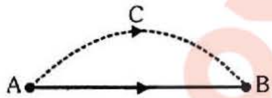
**Ex.** The distance moved by spindle of a screw gauge for each turn of head is 1mm. The edge of the humble is provided with a angular scale carrying 100 equal divisions. The least count = $\frac{1\text{mm}}{100} = 0.01 \text{ mm}$

# Important Notes

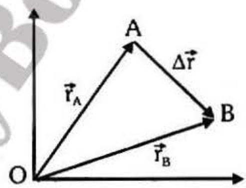
# 4 Kinematics

- **Distance and Displacement**

  Total length of path (ACB) covered by the particle, in definite time interval is called distance. Displacement vector or displacement is the minimum distance (AB) and directed from initial position to final position.




- **Displacement in terms of position vector**




  From $\Delta OAB$ $\quad \Delta\vec{r} = \vec{r}_B - \vec{r}_A$

  $\vec{r}_B = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$ and $\vec{r}_A = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

  $\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

- **Average velocity** $= \dfrac{\text{Displacement}}{\text{Time interval}} = \vec{v}_{av} = \dfrac{\Delta\vec{r}}{\Delta t}$

- **Average speed** $= \dfrac{\text{Distance travelled}}{\text{Time interval}}$

- **For uniform motion**

  Average speed $= |\text{average velocity}| = |\text{instataneous velocity}|$

- **Velocity** $\vec{v} = \dfrac{d\vec{r}}{dt} = \dfrac{d}{dt}\left(x\hat{i} + y\hat{j} + z\hat{k}\right) = \dfrac{dx}{dt}\hat{i} + \dfrac{dy}{dt}\hat{j} + \dfrac{dz}{dt}\hat{k} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$

- **Average Acceleration** $= \dfrac{\text{total change in velocity}}{\text{total time taken}} = \vec{a}_{av} = \dfrac{\Delta\vec{v}}{\Delta t}$

- **Acceleration**

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(v_x\hat{i} + v_y\hat{j} + v_z\hat{k}\right) = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

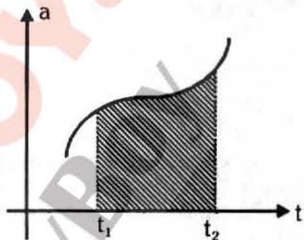
**Important points about 1D motion**

- Distance $\geq |\text{displacement}|$ and Average speed $\geq |\text{average velocity}|$
- If distance $> |\text{displacement}|$ this implies

  (a) atleast at one point in path, velocity is zero.

(b) The body must have retarded during the motion

- Acceleration positive indicates velocity increases and speed may increase or decrease
- Speed increase if acceleration and velocity both are positive or negative (i.e. both have same sign)
- **In 1-D motion** $a = \frac{dv}{dt} = v\frac{dv}{dx}$
- **Graphical integration in Motion analysis**

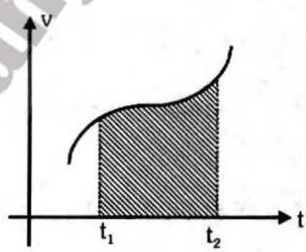



**shaded area = change in velocity**

- $a = \frac{dv}{dt} \Rightarrow \int_{v_1}^{v_2} dv = \int_{t_1}^{t_2} a\,dt \Rightarrow v_2 - v_1 = \int_{t_1}^{t_2} a\,dt$

  $\Rightarrow$ Change in velocity

  = Area between acceleration curve and time axis, from $t_1$ to $t_2$.




**shaded area = displacement**

- $v = \frac{dx}{dt} \Rightarrow \int_{x_1}^{x_2} dx = \int_{t_1}^{t_2} v\,dt \Rightarrow x_2 - x_1 = \int_{t_1}^{t_2} v\,dt$

  $\Rightarrow$ Change in position = displacement

  = area between velocity curve and time axis, from $t_1$ to $t_2$.

**Important point about graphical analysis of motion**

- Instantaneous velocity is the slope of position time curve. $\left(v = \frac{dx}{dt}\right)$
- Slope of velocity-time curve = instantaneous acceleration $\left(a = \frac{dv}{dt}\right)$
- v-t curve area gives displacement. $\left[\Delta x = \int v\,dt\right]$
- a-t curve area gives change in velocity. $\left[\Delta v = \int a\,dt\right]$

$$
\begin{tikzcd}
\boxed{\textbf{Displacement}} \arrow[r, shift left, "\text{Differentiation}"] & \boxed{\textbf{Velocity}} \arrow[l, shift left, "\text{Integration}"] \arrow[r, shift left, "\text{Differentiation}"] & \boxed{\textbf{Acceleration}} \arrow[l, shift left, "\text{Integration}"]
\end{tikzcd}
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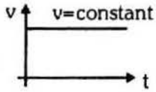
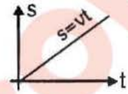
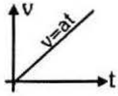
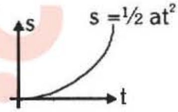
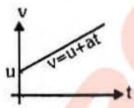
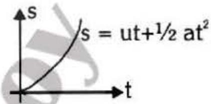
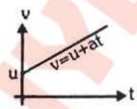
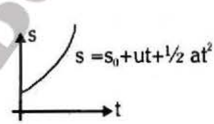
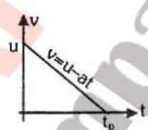
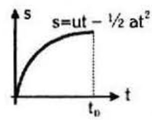
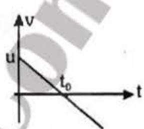
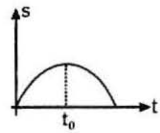
| | Different Cases | v–t graph | s–t graph |
|---|---|---|---|
| 1. | Uniform motion |   |   |
| 2. | Uniformly accelerated motion with u = 0 at t = 0 |   |   |
| 3. | Uniformly accelerated with $u \neq 0$ at t = 0 |   |   |
| 4. | Uniformly accelerated motion with $u \neq 0$ and $s = s_0$ at t = 0 |   |   |
| 5. | Uniformly retarded motion till velocity becomes zero |   |   |
| 6. | Uniformly retarded then accelerated in opposite direction |   |   |

**Motion with constant acceleration : Equations of motion**

- *In vector form :*

$\vec{v} = \vec{u} + \vec{a}t$ $\quad\quad \Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = \vec{s} = \left(\dfrac{\vec{u}+\vec{v}}{2}\right)t = \vec{u}t + \dfrac{1}{2}\vec{a}t^2 = \vec{v}t - \dfrac{1}{2}\vec{a}t^2$

$v^2 = u^2 + 2\vec{a}.\vec{s}$ $\quad\quad \vec{s}_{n^{th}} = \vec{u} + \dfrac{\vec{a}}{2}(2n-1)$

[$S_{n^{th}} \rightarrow$ displacement in $n^{th}$ second]

- *In scalar form (for one dimensional motion) :*

$v = u + at$ $\quad\quad s = \left(\dfrac{u+v}{2}\right)t = ut + \dfrac{1}{2}at^2 = vt - \dfrac{1}{2}at^2$

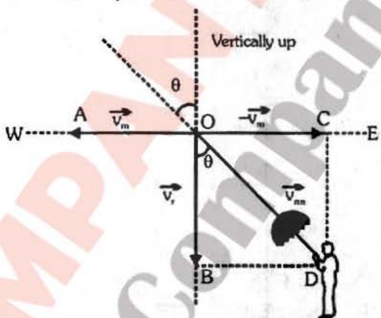
$v^2 = u^2 + 2as$ $\quad\quad s_n = u + \dfrac{a}{2}(2n-1)$

- **RELATIVE MOTION**

There is no meaning of motion without reference or observer. If reference is not mentioned then we take the ground as a reference of motion. Generally velocity or displacement of the particle w.r.t. ground is called actual velocity or actual displacement of the body. If we describe the motion of a particle w.r.t. and object which is also moving w.r.t. ground then velocity of particle w.r.t. ground is its actual velocity $(\vec{v}_{act})$ and velocity of particle w.r.t. moving object is its relative velocity $(\vec{v}_{rel.})$ and the velocity of moving object (w.r.t. ground) is the reference velocity $(\vec{v}_{ref.})$ then $\vec{v}_{rel} = \vec{v}_{act} - \vec{v}_{ref}$

$$\boxed{\vec{v}_{actual} = \vec{v}_{relative} + \vec{v}_{reference}}$$

- **Relative velocity of Rain w.r.t. the Moving Man :** A man walking west with velocity $\vec{v}_m$, represented by $\overrightarrow{OA}$. Let the rain be falling vertically downwards with velocity $\vec{v}_r$, represented by $\overrightarrow{OB}$ as shown in figure.




- The relative velocity of rain w.r.t. man $\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$ will be represented by diagonal $\overrightarrow{OD}$ of rectangle OBDC.

$$\therefore v_{rm} = \sqrt{v_r^2 + v_m^2 + 2v_r v_m \cos 90°} = \sqrt{v_r^2 + v_m^2}$$

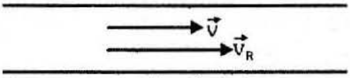
If $\theta$ is the angle which $\vec{v}_{rm}$ makes with the vertical direction then

$$\tan\theta = \frac{BD}{OB} = \frac{v_m}{v_r} \Rightarrow \theta = \tan^{-1}\left(\frac{v_m}{v_r}\right)$$

- **Swimming into the River**

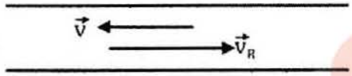
A man can swim with velocity $\vec{v}$, i.e. it is the velocity of man w.r.t. still water. If water is also flowing with velocity $\vec{v}_R$ then velocity of man relative to ground $\vec{v}_m = \vec{v} + \vec{v}_R$

- If the swimming is in the direction of flow of water or along the downstream

then 

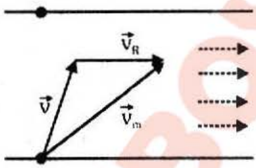
$v_m = v + v_R$

- If the swimming is in the direction opposite to the flow of water or along

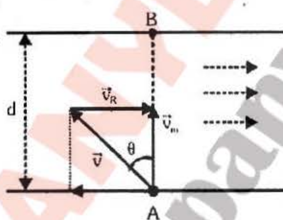
the upstream then 

$v_m = v - v_R$

- If man is crossing the river as shown in the figure i.e. $\vec{v}$ and $\vec{v}_R$ not collinear then use the vector algebra $\vec{v}_m = \vec{v} + \vec{v}_R$ **(assuming $v > v_R$)**



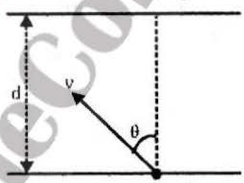

- **For shortest path :**



For minimum displacement

To reach at B, $v\sin\theta = v_R \Rightarrow \sin\theta = \frac{v_R}{v}$

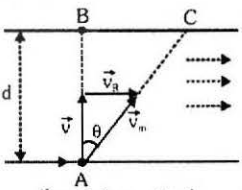
- **Time of crossing**



$t = \frac{d}{v\cos\theta}$

**Note** : If $v_R > v$ then for minimum drifting $\sin\theta = \frac{v}{v_R}$

- **For minimum time**

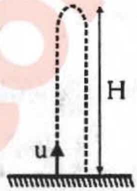


(for minimum time)

then $\boxed{t_{min} = \frac{d}{v}}$

## MOTION UNDER GRAVITY

If a body is thrown vertically up with a velocity u in the uniform gravitational field (neglecting air resistance) then




(i) Maximum height attained $H = \dfrac{u^2}{2g}$

(ii) Time of ascent = time of descent = $\dfrac{u}{g}$

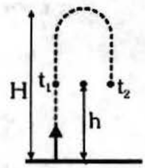
(iii) Total time of flight = $\dfrac{2u}{g}$

(iv) Velocity of fall at the point of projection = u (downwards)

(v) **Gallileo's law of odd numbers** : For a freely falling body ratio of successive distance covered in equal time internval 't'

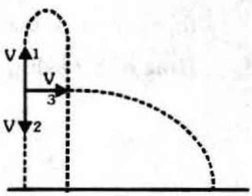
$$S_1 : S_2 : S_3 : ....S_n = 1 : 3 : 5 : .... : 2n-1$$

- At any point on its path the body will have same speed for upward journey and downward journey.
- If a body thrown upwards crosses a point in time $t_1$ & $t_2$ respectively then height of point $h = \frac{1}{2} g t_1 t_2$

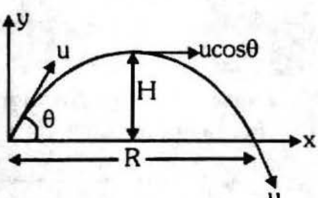



Maximum height $H = \dfrac{1}{8} g(t_1 + t_2)^2$

- A body is thrown upward, downward & horizontally with same speed takes time $t_1$, $t_2$ & $t_3$ respectively to reach the ground then $t_3 = \sqrt{t_1 t_2}$ & height from where the particle was throw is $H = \dfrac{1}{2} g t_1 t_2$




- **PROJECTILE MOTION**




▫ **Horizontal Motion**

$u\cos\theta = u_x$

$a_x = 0$

$x = u_x t = (u\cos\theta)t$

▫ **Vertical Motion :**

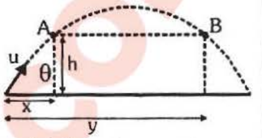
$v_y = u_y - gt$ where $u_y = u\sin\theta$; $y = u_y t - \dfrac{1}{2}gt^2 = u\sin\theta t - \dfrac{1}{2}gt^2$

Net acceleration = $\vec{a} = a_x\hat{i} + a_y\hat{j} = -g\hat{j}$

▫ **At any instant** : $v_x = u\cos\theta$, $v_y = u\sin\theta - gt$

- **For projectile motion :**

- A body crosses two points at same height in time $t_1$ and $t_2$ the points are at distance x and y from starting point then
  (a) $x + y = R$
  (b) $t_1 + t_2 = T$
  (c) $h = \frac{1}{2} g t_1 t_2$
  (d) Average velocity from A to B is $u\cos\theta$




- If a person can throw a ball to a maximum distance 'x' then the maximum height to which he can throw the ball will be $(x/2)$

- **Velocity of particle at time t** :

$$\vec{v} = v_x\hat{i} + v_y\hat{j} = u_x\hat{i} + (u_y - gt)\hat{j} = u\cos\theta\hat{i} + (u\sin\theta - gt)\hat{j}$$

If angle of velocity $\vec{v}$ from horizontal is $\alpha$, then

$$\tan\alpha = \frac{v_y}{v_x} = \frac{u_y - gt}{u_x} = \frac{u\sin\theta - gt}{u\cos\theta} = \tan\theta - \frac{gt}{u\cos\theta}$$

- At highest point : $v_y = 0,\ v_x = u\cos\theta$

- Time of flight : $T = \frac{2u_y}{g} = \frac{2u\sin\theta}{g}$

- Horizontal range : $R = (u\cos\theta)T = \frac{2u^2\sin\theta\cos\theta}{g} = \frac{u^2\sin 2\theta}{g} = \frac{2u_x u_y}{g}$

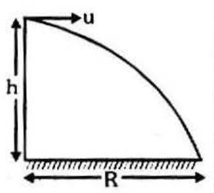
  It is same for $\theta$ and $(90° - \theta)$ and maximum for $\theta = 45°$

- Maximum height $H = \frac{u_y^2}{2g} = \frac{u^2\sin^2\theta}{2g} = \frac{1}{8}gT^2$

- $\frac{H}{R} = \frac{1}{4}\tan\theta$

- Equation of trajectory $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta} = x\tan\theta\left(1 - \frac{x}{R}\right)$

- **Horizontal projection from some height**

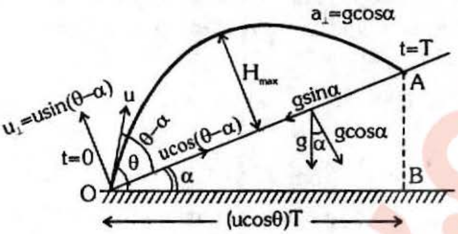



- Time of flight $T = \sqrt{\frac{2h}{g}}$

- Horizontal range $R = uT = u\sqrt{\frac{2h}{g}}$

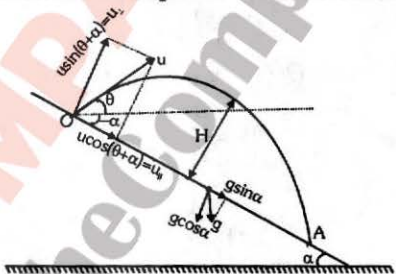
- Angle of velocity at any instant with horizontal $\theta = \tan^{-1}\left(\frac{gt}{u}\right)$

- **Projectile motion on inclined plane- up motion**




- Time of flight $T = \dfrac{2u_{\perp}}{g_{\perp}} = \dfrac{2u\sin(\theta-\alpha)}{g\cos\alpha}$
- Maximum height $H_{max} = \dfrac{u_{\perp}^2}{2g_{\perp}} = \dfrac{u^2\sin^2(\theta-\alpha)}{2g\cos\alpha}$
- Range on inclined plane $R = OA = \dfrac{2u^2\sin(\theta-\alpha)\cos\theta}{g\cos^2\alpha}$
- Maximum range $R_{max} = \dfrac{u^2}{g(1+\sin\alpha)}$ at angle $\theta = \dfrac{\pi}{4} + \dfrac{\alpha}{2}$

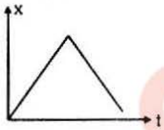
- **Projectile motion on inclined plane – down motion (put α=–α in above)**




- Time of flight $T = 2t_H = \dfrac{2u_{\perp}}{a_{\perp}} = \dfrac{2u\sin(\theta+\alpha)}{g\cos\alpha}$
- Maximum height $H = \dfrac{u_{\perp}^2}{2a_{\perp}} = \dfrac{u^2\sin^2(\theta+\alpha)}{2g\cos\alpha}$
- Range on inclined plane $R = OA = \dfrac{2u^2\cos\theta\sin(\theta+\alpha)}{g\cos^2\alpha}$
- Maximum range $R_{max} = \dfrac{u^2}{g(1-\sin\alpha)}$ at angle $\theta = \dfrac{\pi}{4} - \dfrac{\alpha}{2}$

**KEY POINTS :**

- A positive acceleration can be associated with a "slowing down" of the body because the origin and the positive direction of motion are a matter of choice.
- The x-t graph for a particle undergoing rectilinear motion, cannot be as shown in figure because infinitesimal changes in velocity are physically possible only in infinitesimal time.




- In oblique projection of a projectile the speed gradually decreases up to the highest point and then increases because the tangential acceleration opposes the motion till the particle reaches the highest point, and then it favours the motion of the particle.
- In free fall, the initial velocity of a body may not be zero.
- A body can have acceleration even if its velocity is zero at an instant.
- Average velocity of a body may be equal to its instantaneous velocity.
- The trajectory of an object moving under constant acceleration can be straight line or parabola.
- The path of one projectile as seen from another projectile is a straight line as relative acceleration of one projectile w.r.t. another projectile is zero.

# Important Notes

# 5 Laws of Motion and Friction

- **Force**
  A push or pull that one object exerts on another.

- **Forces in nature**
  There are four fundamental forces in nature :
  1. Gravitational force
  2. Electromagnetic force
  3. Strong nuclear force
  4. Weak force

- **Types of forces on macroscopic objects**
  **(a) Field Forces or Range Forces :**
  These are the forces in which contact between two objects is not necessary.
  **Ex.** (i) Gravitational force between two bodies.
  (ii) Electrostatic force between two charges.

  **(b) Contact Forces :**
  Contact forces exist only as long as the objects are touching each other.
  **Ex.** (i) Normal force. (ii) Frictional force

  **(c) Attachment to Another Body :**
  Tension (T) in a string and spring force (F = kx) comes in this group.

- **Newton's first law of motion (or Galileo's law of Inertia)**
  Every body continues in its state of rest or uniform motion in a straight line unless compelled by an external unbalanced force to change that state.
  **Inertia :** Inertia is the property of the body due to which body opposes the change of it's state. Inertia of a body is measured by mass of the body.

$$\boxed{\text{inertia} \propto \text{mass}}$$

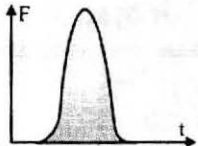
- **Newton's second law**

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt} \quad \text{(Linear momentum } \vec{p} = m\vec{v}\text{)}$$

□ For constant mass system $\vec{F} = m\vec{a}$

- **Momentum** : It is the product of the mass and velocity of a body i.e. momentum $\vec{p} = m\vec{v}$
  **SI Unit** : kg m $s^{-1}$ **Dimensions** : $[M L T^{-1}]$

- **Impulse** : Impulse = product of force with time.




impulse = area under curve

For a finite interval of time from $t_1$ to $t_2$ then the impulse = $\int_{t_1}^{t_2} \vec{F}dt$

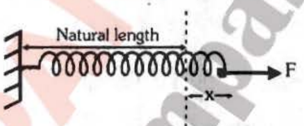
If constant force acts for an interval $\Delta t$ then : Impulse = $\vec{F}\Delta t$

**Impulse – Momentum theorem**

Impulse of a force is equal to the change of momentum $\boxed{\vec{F}\Delta t = \Delta\vec{p}}$

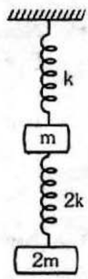
- **Newton's third law of motion** : Whenever a particle A exerts a force on another particle B, B simultaneously exerts a force on A with the same magnitude in the opposite direction.
- **Spring Force (According to Hooke's law) :**

In equilibrium F=kx (k is spring constant)




**Note** : Spring force is non impulsive in nature.

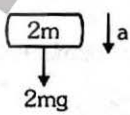
**Ex.** If the lower spring is cut, find acceleration of the blocks, immediately after cutting the spring.




**Sol.** Initial stretches $x_{upper} = \frac{3mg}{k}$ and $x_{lower} = \frac{mg}{k}$

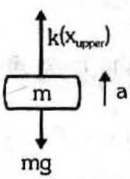
On cutting the lower spring, by virtue of non–impulsive nature of spring the stretch in upper spring remains same immediately after cutting the spring. Thus,

**Lower block** :




$2mg = 2ma \Rightarrow a = g$

**Upper block** :




$k\left(\frac{3mg}{k}\right) - mg = ma \Rightarrow a = 2g$

- **Motion of bodies in contact**

When two bodies of masses $m_1$ and $m_2$ are kept on the frictionless surface and a force F is applied on one body, then the force with which one body presses the other at the point of contact is called force of contact. These two bodies will move with same acceleration a.

(i) When the force F acts on the body with mass $m_1$ as shown in fig.(i)

$$F = (m_1 + m_2)a$$

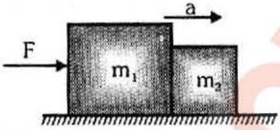



Fig.(1) : When the force F acts on mass $m_1$

If the force exerted by $m_2$ on $m_1$ is $f_1$ (force of contact) then for body $m_1$: $(F - f_1) = m_1 a$

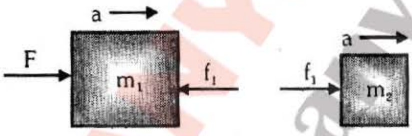



Fig. 1(a) : F.B.D. representation of action and reaction forces.

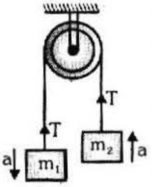
For body $m_2$ : $f_1 = m_2 a \Rightarrow$ action of $m_1$ on $m_2$: $f_1 = \frac{m_2 F}{m_1 + m_2}$

- **Pulley system**

A single fixed pulley changes the direction of force only and in general, assumed to be massless and frictionless.

## SOME CASES OF PULLEY

**Case – I**




Let $m_1 > m_2$

now for mass $m_1$, $m_1 g - T = m_1 a$

for mass $m_2$, $T - m_2 g = m_2 a$

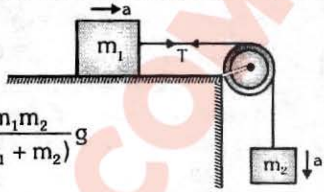
$$\text{Acceleration} = a = \frac{(m_1 - m_2)}{(m_1 + m_2)} g = \frac{\text{net pulling force}}{\text{total mass to be pulled}}$$

$$\text{Tension} = T = \frac{2m_1 m_2}{(m_1 + m_2)} g = \frac{2 \times \text{Product of masses}}{\text{Sum of two masses}} g$$

Reaction at the suspension of pulley $R = 2T = \frac{4 m_1 m_2 g}{(m_1 + m_2)}$

**Case – II**

For mass $m_1$ : $T = m_1 a$
For mass $m_2$ : $m_2 g - T = m_2 a$




Acceleration $a = \frac{m_2 g}{(m_1 + m_2)}$ and $T = \frac{m_1 m_2}{(m_1 + m_2)} g$

## FRAME OF REFERENCE

- **Inertial frames of reference :** A reference frame which is either at rest or in uniform motion along the straight line. A non–accelerating frame of reference is called an inertial frame of reference.

  All the fundamental laws of physics have been formulated in respect of inertial frame of reference.

- **Non–inertial frame of reference :** An accelerating frame of reference is called a non–inertial frame of reference. Newton's laws of motion are not directly applicable in such frames, before application we must add pseudo force.

- **Pseudo force**: The force on a body due to acceleration of non–inertial frame is called fictitious or apparent or pseudo force and is given by $\vec{F} = -m\vec{a}_0$, where $\vec{a}_0$ is acceleration of non–inertial frame with respect to an inertial frame and m is mass of the particle or body. The direction of pseudo force must be opposite to the direction of acceleration of the non–inertial frame.

- When we draw the free body diagram of a mass, with respect to an **inertial frame of reference** we apply only the real forces (forces which are actually acting on the mass). But when the free body diagram is drawn from a non–inertial frame of reference a pseudo force (in addition to all real forces) has to be applied to make the equation $\vec{F} = m\vec{a}$ to be valid in this frame also.

- **Man in a Lift**

  **(a)** If the lift moving with constant velocity v upwards or downwards. In this case there is no accelerated motion hence no pseudo force experienced by observer inside the lift.

  So apparent weight $W' = Mg$ = Actual weight.

  **(b)** If the lift is accelerated upward with constant acceleration a. Then forces acting on the man w.r.t. observed inside the lift are

  (i) Weight $W = Mg$ downward

  (ii) Fictitious force $F_0 = Ma$ downward.

  So apparent weight $W' = W + F_0 = Mg + Ma = M(g+a)$

**(c)** If the lift is accelerated downward with acceleration a<g.

Then w.r.t. observer inside the lift fictitious force $F_0=Ma$ acts upward while weight of man $W = Mg$ always acts downward.

So apparent weight $W' = W - F_0 = Mg - Ma = M(g-a)$

**Special Case :**

If $a=g$ then $W'=0$ (condition of weightlessness).

Thus, in a freely falling lift the man will experience weightlessness.

**(d)** If lift accelerates downward with acceleration $a > g$ . Then as in Case (c). Apparent weight $W' = M(g-a)$ is negative, i.e., the man will be accelerated upward and will stay at the ceiling of the lift.

## FRICTION

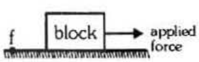
Friction is the force of two surfaces in contact, or the force of a medium acting on a moving object. (i.e. air on aircraft.)

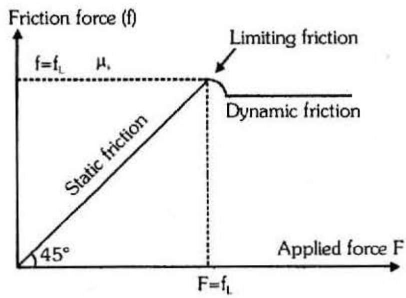
Frictional forces arise due to molecular interactions. In some cases friction acts as a supporting force and in some cases it acts as opposing force.

- **Cause of Friction:** Friction is arises on account of strong atomic or molecular forces of attraction between the two surfaces at the point of actual contact.
- **Types of friction**

Friction
- Static friction (No relative motion between objects)
- Dynamic friction (Kinetic friction) (There is relative motion between objects)

- **Graph between applied force and force of friction**



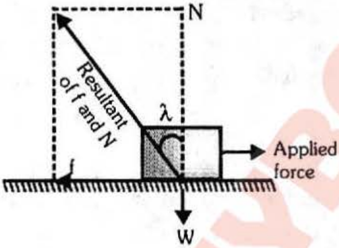



- **Static friction coefficient** $\mu_s = \frac{(f_s)_{max}}{N}$, $0 \le f_s \le \mu_s N$, $\vec{f}_s = -\vec{F}_{applied}$

  $(f_s)_{max} = \mu_s N =$ limiting friction

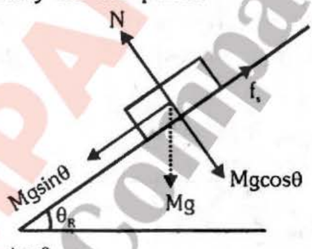
- **Sliding friction coefficient** $\mu_k = \frac{f_k}{N}$, $\vec{f}_k = -(\mu_k N)\hat{v}_{relative}$

- **Angle of Friction ($\lambda$)**




$\tan\lambda = \frac{f_S}{N} = \frac{\mu_s N}{N} = \mu_s$

- **Angle of repose** : The maximum angle of an inclined plane for which a block remains stationary on the plane.




$\tan\theta_R = \mu_s$

- For smooth surface $\theta_R = 0$

- **Dependent Motion of Connected Bodies**

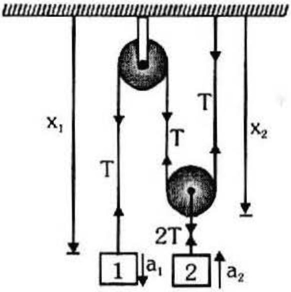
  **Method I** : Method of constraint equations

$$\boxed{\Sigma x_i = \text{constant} \Rightarrow \Sigma \dot{x}_i = 0 \Rightarrow \Sigma \ddot{x}_i = 0}$$

- For n moving bodies we have $x_1, x_2, \ldots x_n$
- No. of constraint equations = no. of strings

**Method II : Method of virtual work :** The sum of scalar products of forces applied by connecting links of constant length and displacement of corresponding contact points equal to zero.

$$\boxed{\sum \vec{F}_i \cdot \delta\vec{r}_i = 0 \Rightarrow \sum \vec{F}_i \cdot \vec{v}_i = 0 \Rightarrow \sum \vec{F} \cdot \vec{a}_i = 0}$$




Here $2a_2 = a_1$

**KEY POINTS**

- Aeroplanes always fly at low altitudes because according to Newton's III law of motion as aeroplane displaces air & at low altitude density of air is high.
- Rockets move by pushing the exhaust gases out so they can fly at low & high altitude.
- Pulling a lawn roller is easier than pushing it because pushing increases the apparent weight and hence friction.
- A moongphaliwala sells his moongphali using a weighing machine in an elevator. He gain more profit if the elevator is accelerating up because the apparent weight of an object increases in an elevator while accelerating upward.
- Pulling (figure I) is easier than pushing (figure II) on a rough horizontal surface because normal reaction is less in pulling than in pushing.

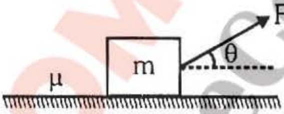



Fig. I ;

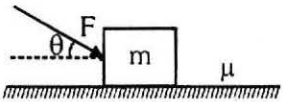



Fig. II

- While walking on ice, one should take small steps to avoid slipping. This is because smaller step increases the normal reaction and that ensure smaller friction.
- A man in a closed cabin (lift) falling freely does not experience gravity as inertial and gravitational mass have equivalence.

# Important Notes

# 6 Work, Energy and Power

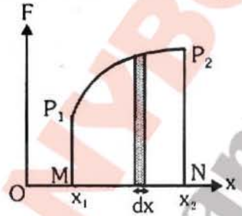
- **Work done** $W = \int dW = \int \vec{F}.d\vec{r} = \int F dr \cos\theta$

  [where $\theta$ is the angle between $\vec{F}$ & $d\vec{r}$ ]

  - For constant force $W = \vec{F}.\vec{d} = F d \cos\theta$
  - For Unidirectional force

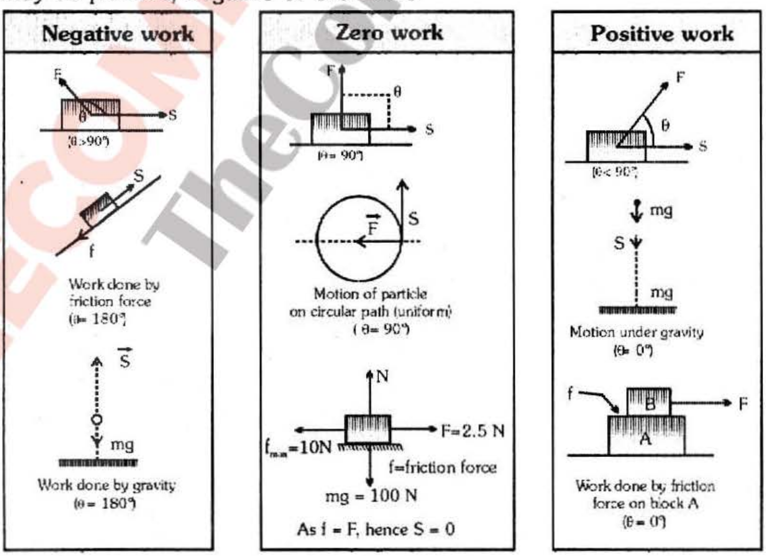
    $W = \int dW = \int F dx =$ Area between F–x curve and x-axis.

- **Calculation of work done from force–displacement graph :**




Total work done, $W = \sum_{x_1}^{x_2} dW = \sum_{x_1}^{x_2} F dx =$ Area of $P_1P_2NM = \int_{x_1}^{x_2} F dx$

- **Nature of work done** : Although work done is a scalar quantity, yet its value may be positive, negative or even zero



**Conservative Forces**

- Work done does not depend upon path.
- Work done in a round trip is zero.
- Central forces, spring forces etc. are conservative forces
- When only a conservative force acts within a system, the kinetic energy and potential energy can change into each other. However, their sum, the mechanical energy of the system, doesn't change.
- Work done is completely recoverable.
- If $\vec{F}$ is a conservative force then $\vec{\nabla} \times \vec{F} = \vec{0}$ (i.e. curl of $\vec{F}$ is zero)

**Non-conservative Forces**

- Work done depends upon path.
- Work done in a round trip is not zero.
- Force are velocity-dependent & retarding in nature e.g. friction, viscous force etc.
- Work done against a non-conservative force may be dissipated as heat energy.
- Work done is not recoverable.

**Kinetic energy**

- The energy possessed by a body by virtue of its motion is called kinetic energy.

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m(\vec{v}.\vec{v})$$

- Kinetic energy is a frame dependent quantity because velocity is a frame depends.

**Potential energy**

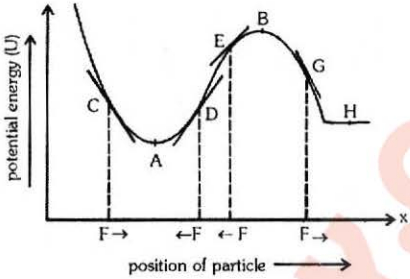
- The energy which a body has by virtue of its position or configuration in a conservative force field.
- Potential energy is a relative quantity.
- Potential energy is defined only for conservative force field.
- Relationship between conservative force field and potential energy :

$$\vec{F} = -\nabla U = -\text{grad}(U) = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} - \frac{\partial U}{\partial z}\hat{k}$$

- If force varies only with one dimension (along x-axis) then

$$F = -\frac{dU}{dx} \Rightarrow U = -\int_{x_1}^{x_2} F dx$$

- **Potential energy curve and equilibrium**




It is a curve which shows change in potential energy with postion of a particle.

- **Stable Equilibrium :**

When a particle is slightly displaced from equilibrium position and it tends to come back towards equilibrium then it is said to be in stable equilibrium

At point **C** : slope $\frac{dU}{dx}$ is negative so F is positive

At point **D** : slope $\frac{dU}{dx}$ is positive so F is negative

At point **A** : It is the point of stable equilibrium.

$$U = U_{min}, \ \frac{dU}{dx} = 0 \quad \text{and} \quad \frac{d^2U}{dx^2} = \text{positive}$$

- **Unstable equilibrium :**

When a particle is slightly displaced from equilibrium and it tends to move away from equilibrium position then it is said to be in unstable equilibrium

At point **E** : slope $\frac{dU}{dx}$ is positive so F is negative

At point **G** : slope $\frac{dU}{dx}$ is negative so F is postive

At point **B** : It is the point of unstable equilibrium.

$$U = U_{max}, \ \frac{dU}{dx} = 0 \text{ and } \frac{d^2U}{dx^2} = \text{negative}$$

- **Neutral equilibrium :**

  When a particle is slightly displaced from equilibrium position and no force acts on it then equilbirum is said to be neutral equilibrium. Point H is at neutral equilibrium $\Rightarrow$ U = constant ; $\frac{dU}{dx}=0, \frac{d^2U}{dx^2}=0$

- **Work energy theorem** : $W = \Delta KE$

  Change in kinetic energy = work done by all force

- **For conservative force** $F(x) = -\frac{dU}{dx}$

  change in potential energy $\Delta U = -\int F(x)dx$

- **Law of conservation of Mechanical energy**

  Total mechanical (kinetic + potential) energy of a system remains constant if only conservative forces are acting on the system of particles or the work done by all other forces is zero. From work energy theorem $W = \Delta KE$

  **Proof :** For internal conservative forces $W_{int} = -\Delta U$

  So $W = W_{ext} + W_{int} = 0 + W_{int} = -\Delta U \Rightarrow -\Delta U = \Delta KE$

  $\Rightarrow \Delta(KE+U) = 0 \Rightarrow KE + U = \text{constant}$

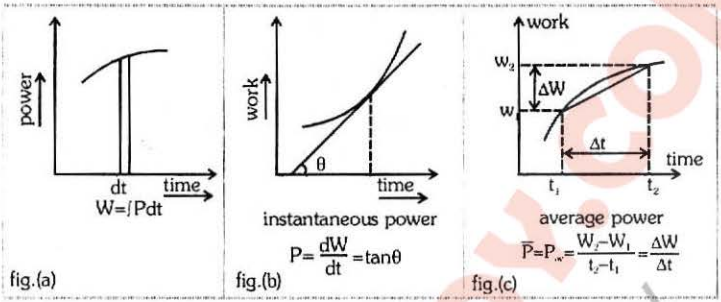
- Spring force $F=-kx$, Elastic potential energy stored in spring $U(x) = \frac{1}{2}kx^2$

- Mass and energy are equivalent and are related by $E = mc^2$

- **Power**
  - Power is a scalar quantity with dimension $M^1L^2T^{-3}$
  - SI unit of power is J/s or watt
  - 1 horsepower = 746 watt = 550 ft–lb/sec.

- **Average power** $P_{av} = W/t$

- **Instantaneous power** $P = \frac{dW}{dt} = \frac{\vec{F}.d\vec{r}}{dt} = \vec{F}.\vec{v}$




- For a system of varying mass $\vec{F} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$
- If $\vec{v}$ = constant then $\vec{F} = \vec{v}\frac{dm}{dt}$ then $P = \vec{F}.\vec{v} = v^2\frac{dm}{dt}$
- In rotatory motion : $P = \tau\frac{d\theta}{dt} = \tau\omega$

## KEY POINTS

- A body may gain kinetic energy and potential energy simultaneously because principle of conservation of mechanical energy may not be valid every time.
- Comets move around the sun in elliptical orbits. The gravitational force on the comet due to sun is not normal to the comet's velocity but the work done by the gravitational force is zero in complete round trip because gravitational force is a conservative force.
- Work done by static friction may be positive because static friction may acts along the direction of motion of an object.

# Important Notes

# 7 Circular Motion

- **Definition of Circular Motion**

When a particle moves in a plane such that its distance from a fixed (or moving) point remains constant then its motion is called as circular motion with respect to that fixed point. That fixed point is called centre and the distance is called radius of circular path.

**Radius Vector :**

The vector joining the centre of the circle and the center of the particle performing circular motion is called radius vector. It has constant magnitude and variable direction. It is directed outward

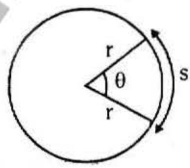
**Frequency (n) :**

No. of revolutions described by particle per sec. is its frequency. Its unit is revolutions per second (r.p.s.) or revolutions per minute (r.p.m.)

**Time Period (T) :**

It is time taken by particle to complete one revolution. $T = \frac{1}{n}$

- Angle $\theta = \frac{\text{arc length}}{\text{radius}} = \frac{s}{r}$




- Average angular velocity $\omega = \frac{\Delta\theta}{\Delta t}$ (a scalar quantity)
- Instantaneous angular velocity $\omega = \frac{d\theta}{dt}$ (a vector quantity)
- For uniform angular velocity $\omega = \frac{2\pi}{T} = 2\pi f$ or $2\pi n$

Angular displacement $\theta = \omega t$

$\omega \rightarrow$ Angular frequency n or f = frequency

- Relation between ω and v $\quad \omega = \frac{v}{r}$

  - In vector form velocity $\quad \vec{v} = \vec{\omega} \times \vec{r}$

- Acceleration $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(\vec{\omega} \times \vec{r}) = \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v} = \vec{a}_t + \vec{a}_C$

- Tangential acceleration: $a_t = \frac{dv}{dt} = \alpha r$

$$\left[\vec{a}_t = \text{component of } \vec{a} \text{ along } \vec{v} = (\vec{a} \cdot \hat{v})\hat{v} = \left(\frac{dv}{dt}\right)\hat{v}\right]$$

- Centripetal acceleration : $a_C = \omega v = \frac{v^2}{r} = \omega^2 r$ or $\vec{a}_C = \omega^2 r(-\hat{r})$

- Magnitude of net acceleration : $a = \sqrt{a_C^2 + a_t^2} = \sqrt{\left(\frac{v^2}{r}\right)^2 + \left(\frac{dv}{dt}\right)^2}$

- Maximum speed of in circular motion.

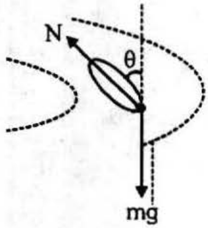
  - On unbanked road : $v_{max} = \sqrt{\mu_s Rg}$

  - On banked road : $v_{max} = \sqrt{\left(\frac{\mu_s + \tan\theta}{1 - \mu_s \tan\theta}\right)Rg} = \sqrt{\tan(\theta + \phi)Rg}$

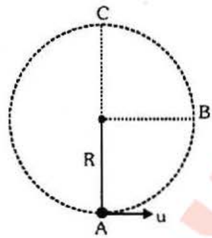
$$v_{min} = \sqrt{Rg\tan(\theta - \phi)}; \; v_{min} \le v_{car} \le v_{max}$$

where $\phi$ = angle of friction = $\tan^{-1}\mu_s$; $\theta$ = angle of banking

- Bending of cyclist : $\tan\theta = \frac{v^2}{rg}$



- **Circular motion in vertical plane**




**A. Condition to complete vertical circle** $u \geq \sqrt{5gR}$

- If $u = \sqrt{5gR}$ then Tension at C is equal to 0

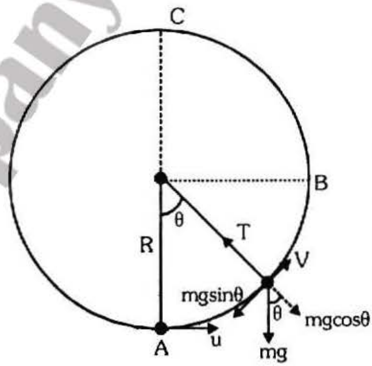
  and tension at A is equal to 6mg

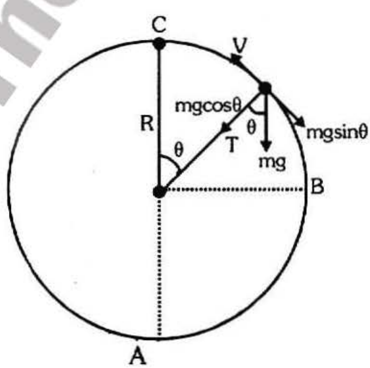
  Velocity at B: $v_B = \sqrt{3gR}$

  Velocity at C: $v_C = \sqrt{gR}$

  From A to B : $T = mg\cos\theta + \dfrac{mv^2}{R}$

  From B to C : $T = \dfrac{mv^2}{R} - mg\cos\theta$



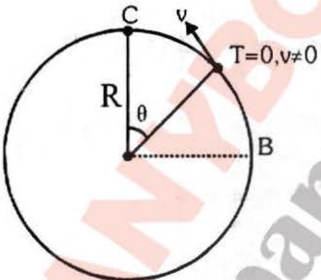
**B. Condition for pendulum motion (oscillating condition)**

$u \le \sqrt{2gR}$ (in between A to B)

Velocity can be zero but T never be zero between A & B.

Because T is given by $T = mg\cos\theta + \dfrac{mv^2}{R}$

**C. Condition for leaving path** : $\sqrt{2gR} < u < \sqrt{5gR}$




Particle crosses the point B but not complete the vertical circle.

Tension will be zero in between B to C & the angle where T = 0

$$\cos\theta = \frac{u^2 - 2gR}{3gR}$$

θ is from vertical line

**Note :** After leaving the circle, the particle will follow a parabolic path.

**KEY POINTS**

- Average angular velocity is a scalar physical quanity whereas instantaneous angular velocity is a vector physical quanity.
- Small Angular displacement $d\vec{\theta}$ is a vector quantity, but large angular displacement θ is scalar quantity.

$$d\vec{\theta}_1 + d\vec{\theta}_2 = d\vec{\theta}_2 + d\vec{\theta}_1 \quad \text{But} \quad \vec{\theta}_1 + \vec{\theta}_2 \ne \vec{\theta}_2 + \vec{\theta}_1$$