PHYSICS
for you

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PHYSICS FOR YOU
wishes all readers a very
Happy New Year
2017

NEET | JEE ESSENTIALS

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Unit 7 — Heat and Thermodynamics

☞ Heat and Temperature

- Heat is a form of energy that is transferred between two bodies or between adjacent part of a body or between a system and its environment at different temperatures.

- When some heat is given to a body and its state does not change, the temperature of the body rises and if heat is taken from a body its temperature falls *i.e.*, temperature can be regarded as the effect of cause, heat.

- Temperature is that property of a body which helps us to decide the degree of its hotness.

- Zeroth law of thermodynamics : If two systems A and B are each in thermal equilibrium with a third system C, then A and B will be in thermal equilibrium with each other.

- Scales of Temperature : Centigrade or Celsius (°C), Fahrenheit (°F) and Kelvin (K), are commonly used temperature scales.

    ➤ Relation between these scales

    $$\frac{T_C}{5} = \frac{T_F - 32}{9} = \frac{T_K - 273}{5}$$

- Although the temperature of a body can be raised without limit, it cannot be lowered without limit and theoretically limiting low temperature is taken to be zero on the Kelvin scale.

Boiling water — 212 °F | 100 °C | 373 K

Freezing water — 32 °F | 0 °C | 273 K

Fahrenheit | Celsius | Kelvin
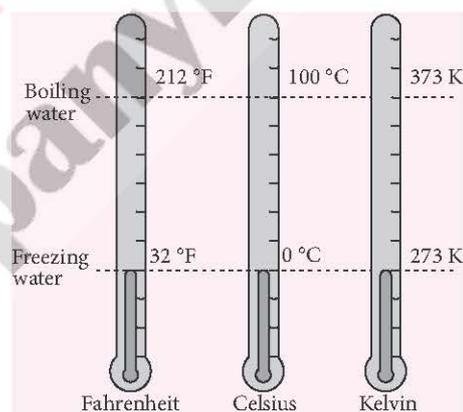
- Mathematically, heat supplied to the body, or heat released by the body.

    $\Delta Q = ms\Delta T$

    Where, m = masss of the body, s = specific heat of the body and ΔT = change in temperature

- The amount of heat required to change the state of a body of mass m from solid to liquid at melting point of the solid or from liquid to gas at boiling point of the liquid is $\Delta Q = mL$, where L is the latent heat of the substance.

- As $\Delta Q = ms\Delta T \Rightarrow s = \dfrac{\Delta Q}{m\Delta T}$, if the substance undergoes the change of state which occurs at constant temperature ($\Delta T = 0$), then $s = \dfrac{\Delta Q}{0} = \infty$.

Thus the specific heat of a substance when it melts or boils at constant temperature is infinite.

- Thermal or heat capacity of a body, $H = ms$
  - ➤ Thermal capacity of the body and its water equivalent are numerically equal.
  - ➤ If thermal capacity of a body is expressed in terms of mass of water, it is called water equivalent of the body.

- Heating curve of a body (state changes from solid to liquid and liquid to gas)
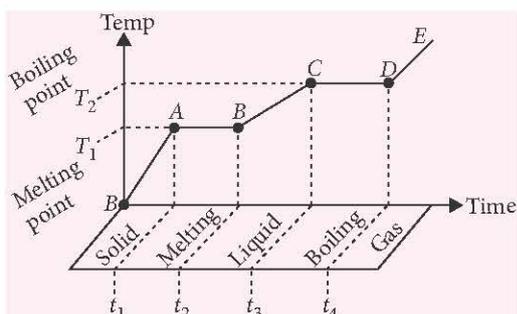
- When two substances at different temperatures are mixed together, heat flows from the substance at higher temperature to the substance at lower temperature till a common temperature is reached. In this process, heat lost by one substance = heat gained by the other substance, i.e.,
  $m_1 \times s_1 (\Delta T_1) = m_2 \times s_2 (\Delta T_2)$

- Joule's mechanical equivalent of heat
  $$J = \frac{\text{mechanical work done } (W)}{\text{heat energy produced } (H)}$$
  $$= 4.2 \text{ J cal}^{-1} = 4.2 \times 10^7 \text{ ergs cal}^{-1}$$
  - ➤ If T is rise in temperature of a body of mass m on falling through a height h, then
    $$J = \frac{mgh}{smT} \text{ or } T = \frac{gh}{Js}$$
  - ➤ When a block of ice of mass m melts on falling through a height h, then
    $$J = \frac{mgh}{mL} \text{ or } h = \frac{JL}{g}$$
  - ➤ When a bullet of mass m moving with velocity v is suddenly stopped, the entire KE of bullet is converted into heat, then
    $$J = \frac{\frac{1}{2}mv^2}{smT} \text{ or } T = \frac{v^2}{2Js}$$

☞ **Thermal Expansion**

- Thermal expansion in solids is of three types
  - ➤ **Coefficient of linear expansion**
    $$\alpha = \frac{\text{Increase in length}}{\text{Original length} \times \text{Rise in temperature}}$$
    $$\alpha = \frac{L_T - L_0}{L_0 \times \Delta T} \text{ or } L_T = L_0 (1 + \alpha \Delta T)$$
  - ➤ **Coefficient of area expansion**
    $$\beta = \frac{\text{Increase in area}}{\text{Original area} \times \text{Rise in temperature}}$$
    $$\beta = \frac{A_T - A_0}{A_0 \times \Delta T} \text{ or } A_T = A_0 (1 + \beta \Delta T)$$
  - ➤ **Coefficient of volume expansion**
    $$\gamma = \frac{\text{Increase in volume}}{\text{Original volume} \times \text{Rise in temperature}}$$
    $$\gamma = \frac{V_T - V_0}{V_0 \times \Delta T} \text{ or } V_T = V_0 (1 + \gamma \Delta T)$$
  - ➤ The three coefficients of thermal expansion are related as $\alpha = \dfrac{\beta}{2} = \dfrac{\gamma}{3}$.

- **Effect of thermal expansion**
  - ➤ With increase in temperature volume of substance increase while mass remains constant, therefore density should decrease.
    $$\rho' = \frac{\rho}{1 + \gamma \cdot \Delta \theta} \approx \rho (1 - \gamma \cdot \Delta \theta) \quad (\text{if } \gamma \cdot \Delta \theta \ll 1)$$
  - ➤ When a solid whose density is less than the density of liquid is floating, then a fraction of it remains immersed. This fraction is $f = \dfrac{\rho_s}{\rho_l}$.
    When temperature is increased, ρ_s and ρ_l both will decrease. Hence, fraction may increase, decrease or remain same. At higher temperature,
    $$f' = f \left(\frac{1 + \gamma_l \Delta \theta}{1 + \gamma_s \Delta \theta} \right); \Delta \theta = \text{increase in temperature}$$
    If $\gamma_l > \gamma_s$ then $f' > f$ or immersed fraction will increase.
  - ➤ When a solid whose density is more than the density of liquid is immersed completely, then upthrust will act on 100% volume of solid and apparent weight appears less than the actual weight.
    $$W_{\text{app}} = W - F_B$$

Here, $F_B = V_s \rho_l g$

With increase in temperature, V_s will increase and ρ_l will decrease, while g will remain unchanged. Therefore upthrust may increase, decrease or remain same.

➢ With increase in temperature, length of pendulum will increase. Therefore time period will increase. A pendulum clock will become slow and it loses the time.
At some higher temperature,

$$T' = T(1 + \alpha \Delta \theta)^{\frac{1}{2}} \text{ or } T' \approx T\left(1 + \frac{1}{2}\alpha\Delta\theta\right) (\text{if } \alpha\Delta\theta \ll 1)$$

$$\Delta T = (T' - T) = \frac{1}{2}T\alpha\Delta\theta$$

Time lost during time interval t is $\Delta t = \frac{\Delta T}{T} \times t$

➢ If temperature of a rod fixed at both ends is increased, then thermal stresses are developed in the rod.
Thermal stress $= Y\alpha\Delta\theta$; Y = Young's modulus of rod

☞ Heat Transfer

• Rate of flow of heat across the material of a solid block between parallel face is

$$\frac{dQ}{dt} = -KA\left(\frac{dT}{dx}\right)$$

Where A is area of cross-section of slab, $\frac{dT}{dx}$ = temperature gradient and K = coefficient of thermal conductivity of the solid.

• In steady state, heat flow $Q = \dfrac{KA(T_1 - T_2)}{x}t$

• Rate of heat flow = Heat current

$$\Rightarrow \frac{dQ}{dt} = \frac{\text{Temperature difference}}{\text{Thermal resistance}}$$

➢ Thermal resistance, $R_{\text{Th}} = \dfrac{T_1 - T_2}{dQ/dt} = \dfrac{x}{KA}$

• Conducting slabs in series
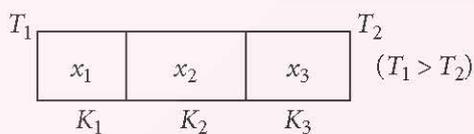
$$Q = \frac{A(T_1 - T_2)t}{\dfrac{x_1}{K_1} + \dfrac{x_2}{K_2} + \dfrac{x_3}{K_3}}$$

➢ Equivalent thermal conductivity (K)

$$\frac{1}{K} = \left(\frac{x_1}{K_1} + \frac{x_2}{K_2} + \frac{x_3}{K_3}\right)\frac{1}{(x_1 + x_2 + x_3)}$$

In general, $K = \dfrac{(\Sigma x_i)}{\left(\Sigma \dfrac{x_i}{K_i}\right)}$

• Conducting slabs in parallel

$$Q = (K_1 A_1 + K_2 A_2 + K_3 A_3)\left(\frac{T_1 - T_2}{x}\right)t$$

➢ Equivalent thermal conductivity

$$K = \frac{K_1 A_1 + K_2 + A_2 + K_3 A_3}{A_1 + A_2 + A_3}$$

In general, $K = \dfrac{(\Sigma K_i A_i)}{\Sigma A_i}$
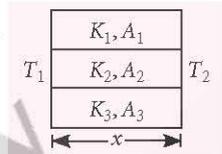
• Time taken in growth of ice layer (from thickness x_1 to x_2) on water surface is $t = \dfrac{\rho L}{2KT}(x_2^2 - x_1^2)$; T = temperature of atmosphere.

➢ The time interval to change the thickness from 0 to x, from x to $2x$ and so on will be in the ratio $\Delta t_1 : \Delta t_2 : \Delta t_3 :: 1 : 3 : 5$.

☞ Radiation

• All objects emit radiations simply because their temperature is above absolute zero, and all objects absorb some of radiations.

• The intensity of radiation is inversely proportional to the square of distance of point of observation form the source (i.e., $I \propto 1/d^2$).

• When a body is heated, all radiations having wavelengths from zero to infinity are emitted.

• Radiations of longer wavelengths are predominant at lower temperature.

• Absorptive power $a = \dfrac{\text{energy absorbed}}{\text{energy incident}}$

– $a < 1$ for ordinary body
– $a = 1$ for perfectly black body

• Spectral absorptive power a_λ = absorptive power of wavelength λ.

– $a_\lambda < 1$ for ordinary body
– $a_\lambda = 1$ for perfectly black body

• Emissive power : Energy radiated per unit area per unit time is called emissive power of a body.

• **Stefan's law :** Emissive power of a body is given by,
$$E = e\sigma T^4$$
Here e = emissivity
$e \le 1$ and $e = 1$ for a perfectly black body

- ➤ Total energy radiated by a body,
  $Q = e\sigma T^4 At$
  Here, A = surface area of body, T = temperature of body, t = time σ = Stefan's constant.
- ➤ Net emissive power of a body
  E = emissive power – absorptive power
  $= e\sigma(T^4 - T_0^4)$
- **Kirchhoff's law :** If different bodies (including a perfectly black body) are kept at same temperature, then emissive power is proportional to the absorptive power.

$$e_\lambda \propto a_\lambda \text{ or } \frac{e_\lambda}{a_\lambda} = \text{constant}$$

$$\text{or } \left(\frac{e_\lambda}{a_\lambda}\right)_{\text{Body-1}} = \left(\frac{e_\lambda}{a_\lambda}\right)_{\text{Body-2}} = \left(\frac{e_\lambda}{a_\lambda}\right)_{\text{Perfectly black body}}$$

- ➤ Good absorbers of a particular wavelength λ are also good emitters of same wavelength.
- ➤ At a given temperature, ratio of e_λ and a_λ for any body is constant. This ratio is equal to e_λ of perfectly black body at that temperature.
- **Wien's Displacement law**
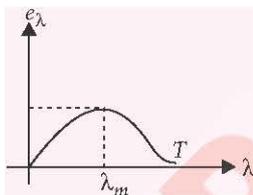
$$\lambda_m \propto \frac{1}{T} \text{ or } \lambda_m T = \text{Constant} = \text{Wien's constant } (b)$$

Here, $b = 2.89 \times 10^{-3}$ mK
Further, area of this graph will give total emissive power which is proportional to T^4.

- **Cooling of a body by radiation**
  - ➤ Rate of cooling
  $$-\frac{dT}{dt} = \frac{eA\sigma}{ms}(T^4 - T_0^4) \text{ or } -\frac{dT}{dt} \propto (T^4 - T_0^4)$$

- ➤ **Newton's law of cooling :** If temperature difference of a body with atmosphere is small, then rate of cooling is proportional to the temperature difference.
- ➤ If body cools by radiation according to this law, then temperature of body decreases exponentially.
  In the figure,
  T_i = initial temperature of body
  T_0 = temperature of surrounding.

  Temperature at any time t can be written as,
  $T = T_0 + (T_i - T_0)e^{-kt}$; k = constant
- ➤ If body is cooling according to this law then to find temperature of a body at any time t, we will have to calculate $e^{-\alpha t}$. To avoid this, you can use a shortcut approximate formula given below
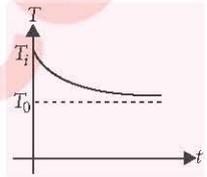
$$\left(\frac{T_1 - T_2}{t}\right) = k\left[\frac{T_1 + T_2}{2} - T_0\right].$$

- ➤ If $\ln(T - T_0)$ be plotted against t, then the equation assumes the form $y = mx + c$; where $m = -k$ and $C = \ln(T_1 - T_0)$
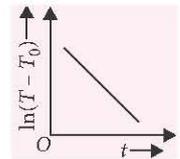
  This is a straight line with negative slope.
- If $Q_{\text{emission}} > Q_{\text{absorption}} \rightarrow$ temperature of body decreases and consequently the body appears colder.
- If $Q_{\text{emission}} < Q_{\text{absorption}} \rightarrow$ temperature of body increases and it appears hotter.
- If $Q_{\text{emission}} = Q_{\text{absorption}} \rightarrow$ temperature of body remains constant (thermal equilibrium).

☞ **Gas Law's**

Name of law	Constant terms	Basic concept	Graph
Boyle's law	(i) Mass of gas (ii) Temperature	PV = constant $V \propto 1/P$ $P_1 V_1 = P_2 V_2$	
Charle's law	(i) Mass of gas (ii) Pressure	V/T = constant; $V \propto T$ $\dfrac{V_1}{T_1} = \dfrac{V_2}{T_2}$ $V_t = V_0(1 + \alpha t)$	 ($\alpha \rightarrow$ volume expansion coefficient = 1/273 °C^{-1})

Gay-Lussac's law	(i) Mass of gas (ii)Volume	P/T = constant; $P \propto T$ $\dfrac{P_1}{T_1} = \dfrac{P_2}{T_2}$ $P_t = P_0(1 + \beta t)$	($\beta \rightarrow$ pressure expansion coefficient = $1/273\ °C^{-1}$)
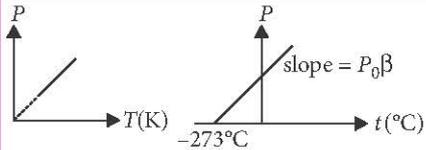
- **Avogadro's law :** At same temperature and pressure equal volumes of all gases contains equal number of molecules.

  $N_1 = N_2$; if P, V and T are same.

- **Dalton's law :** According to this law, the pressure exerted by a mixture of several gases equals the sum of the pressure exerted by each component of gas present in the mixture *i.e.*, $P_{mix} = P_1 + P_2 + P_3 \ldots$

  $$P = \left(\dfrac{RT}{V}\right)n \Rightarrow P_{mix} \propto n$$

  $$\therefore \quad P_{mix} = \dfrac{RT}{V}(n_1 + n_2 + n_3 + \ldots)$$

- **Grahm's law of diffusion :** According to this law, at same temperature and pressure, the rate of diffusion of gases inversely proportional to the square root of the density of gas *i.e.*,

  Rate of diffusion $r_d \propto \dfrac{1}{\sqrt{\rho}}$

  Also, $v_{rms} \propto \dfrac{1}{\sqrt{\rho}}$; so $v_{rms} \propto r_d$

☞ **Kinetic Theory of an Ideal Gas**

- Pressure of an ideal gas inside the container

  $$P = \dfrac{1}{3}\dfrac{mN}{V}v_{rms}^2 = \dfrac{1}{3}\rho v_{rms}^2$$

  Where, m = mass of each molecule, N = total number of molecules, V = volume of container or total volume of gas, ρ = density of gas, v_{rms} = root mean square speed of the gas.

- Various types of speeds of gas molecules

  ➤ Root mean square speed,

  $$v_{rms} = \sqrt{\dfrac{3P}{\rho}} = \sqrt{\dfrac{3RT}{M}} = \sqrt{\dfrac{3kT}{m}}$$

  Here, M = molar mass of the gas

  ➤ Most probable speed

  $$v_{mp} = \sqrt{\dfrac{2P}{\rho}} = \sqrt{\dfrac{2RT}{M}} = \sqrt{\dfrac{2kT}{m}}$$

  ➤ Average speed

  $$v_{av} = \sqrt{\dfrac{8P}{\pi\rho}} = \sqrt{\dfrac{8}{\pi}\dfrac{RT}{M}} = \sqrt{\dfrac{8}{\pi}\dfrac{kT}{m}}$$

- Kinetic energy of gas (internal energy)

  ➤ Translatory kinetic energy

  $$E_T = \dfrac{1}{2}Nmv_{rms}^2 = \dfrac{3}{2}PV$$

  Total internal energy of an ideal gas is kinetic.

  ➤ Energy per unit volume or energy density (E_V)

  $$E_v = \dfrac{\text{Total Energy}}{\text{Volume}} = \dfrac{E}{V} = \dfrac{1}{2}\rho v_{rms}^2 = \dfrac{3}{2}P$$

  ➤ Molar K.E. or Mean Molar K.E. (E) : K.E. of N molecules

  $$E = \dfrac{3}{2}RT = \dfrac{3}{2}NkT \qquad (R = Nk)$$

  ➤ Molecular kinetic energy or mean molecular K.E.($\bar{E}$) : K.E. of a gas molecule

  $$\bar{E} = \dfrac{E}{N} \text{ or } \bar{E} = \dfrac{3}{2}\dfrac{RT}{N} \Rightarrow \bar{E} = \dfrac{3}{2}kT$$

☞ **Degree of Freedom**

- The number of independent ways in which a molecule or an atom can exhibit motion is called its degrees of freedom.

- The number of independent coordinates required to specify the dynamical state of a system is called its degrees of freedom.

- The degrees of freedom are of three types :

  ➤ Translational degree of freedom : Maximum three degree of freedom are there corresponding to translational motion.

  ➤ Rotational degree of freedom : The number of degrees of freedom in this case depends on the structure of the molecule.

  ➤ Vibrational degree of freedom : It exhibits at high temperature.

- Degree of freedom for different gases depends on atomicity of gas.

Atomicity of gas		Translational	Rotational	Vibrational	Total	Graphically
Monatomic e.g., He, Ar, Ne, Ideal gas etc		3	0	0	3	
Diatomic e.g., H_2, O_2, Cl_2, N_2 etc		3	2	0	5	
Triatomic or Polyatomic	(linear) e.g., CO_2, C_2H_2	3	2	2	7	
	Triangular (non-linear) e.g., H_2O, O_3 etc	3	3	0	6	
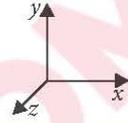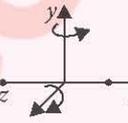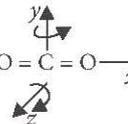
- Relation between degree of freedom and specific heat of gas.
  - Energy of one mole of gas related with each degree of freedom = $RT/2$
  - Energy of one mole of gas related with all degrees of freedom = $fRT/2$
  - Energy of each molecule of gas related with all degrees of freedom = $fkT/2$
  - Internal energy of one mole of ideal gas (total KE) $U = fRT/2$ ($\Delta U = fR\Delta T/2$)

$$C_V = \frac{\Delta U}{\Delta T}; C_V = \frac{f}{2}R = \frac{R}{\gamma - 1}; C_P = C_V + R = \left(\frac{f}{2} + 1\right)R$$

$$C_P = \frac{\gamma R}{\gamma - 1}; \gamma = \frac{C_P}{C_V} = 1 + \frac{2}{f}; \gamma = 1 + \frac{2}{f}$$

☞ **Real Gas Equation**

- Van der waal's equation :

$$\left(P + \frac{n^2 a}{V^2}\right)(V - nb) = nRT \text{ [for } n \text{ - mole]}$$

where a and b are Van der waal's constant

- Critical parameter :(P_c, V_c, T_c)

$$P_c = \frac{a}{27b^2}, T_c = \frac{8}{27}\frac{a}{Rb}, V_c = 3nb$$

- Approximate ideal gas law

$$PV = nRT = NkT$$

Here, n = number of moles, N = number of molecules, $k = R/N_A$

☞ **Mixture of Non Reactive Gases**

- $n = n_1 + n_2$ • $P = P_1 + P_2$
- $U = U_1 + U_2$ • $\Delta U = \Delta U_1 + \Delta U_2$

$$C_V = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2}$$ $$C_P = \frac{n_1 C_{P_1} + n_2 C_{P_2}}{n_1 + n_2} = C_V + R$$

$$\gamma = \frac{C_P}{C_V} \text{ or } \frac{n}{\gamma - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}$$

- $$M = \frac{n_1 M_1 + n_2 M_2}{n_1 + n_2}$$

☞ **Thermodynamic System and Process**

- A thermodynamic system can be described by specifying its pressure (P), volume (V), temperature (T), internal energy (U) and the number of moles (n).
- The relation between the thermodynamic variables (P, V, T) of the system is called equation of state.
  - For n moles of an ideal gas, equation of state is $PV = nRT$
- Thermodynamics system may be of three types :
  - Open system : It exchanges both energy and matter with the surroundings.
  - Closed system : It exchanges only energy (not matter) with the surroundings.

- ➢ Isolated system : It exchanges neither energy nor matter with the surroundings.

☞ Work Done

- • Mathematical method : $\Delta W = P\Delta V$

  $$\Rightarrow \quad W = \int_{V_1}^{V_2} PdV$$

  - ➢ If P constant, $W = P(V_2 - V_1) = nR(T_2 - T_1)$
  - ➢ If V constant, $W = 0$
  - ➢ If T constant, $W = 2.303\, nRT \log_{10}(V_2/V_1)$
    $$= 2.303\, nRT \log_{10}(P_1/P_2)$$
  - ➢ If Q constant, $W = nR(T_1 - T_2)/\gamma - 1$
    $$= (P_1V_1 - P_2V_2)/\gamma - 1$$

- • Graphical method :
  - ➢ Work done = Area enclosed between P - V curve on V axis

  - ➢ **Sign concept for work done :**
    - – If $V\uparrow \Rightarrow dV = +\,ve \Rightarrow$ expansion of gas $\Delta W = (+ve) \Rightarrow$ work done by the system.
    - – If $V\downarrow \Rightarrow dV = -\,ve \Rightarrow$ Compression of gas $\Delta W = (-ve) \Rightarrow$ work done on the system.
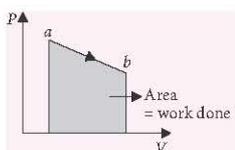
- ➢ **Sign concept for heat :**
  - – If heat given to system or heat absorbed by the system $\Delta Q = +ve$
  - – If heat rejected by system or heat evolved by the system $\Delta Q = -ve$

- ➢ **Sign concept for internal energy :** Obtained by difference of $(\Delta Q - \Delta W)$
  - – If $dU\,(+ve) \Rightarrow$ then $U\uparrow$
  - – If $dU\,(-ve) \Rightarrow$ then $U\downarrow$

- • First law of thermodynamic is based on energy conservation, $\Delta Q = \Delta W + dU$
- • Heat and work both are path dependent so they called unexact differential parameter.
- • Internal energy is a point function or state function, Internal energy only depends on initial and final state of system so it is called exact differential parameter.

- • Different types of thermodynamics processes

Basic Point	Isochoric	Isobaric	Isothermal	Adiabatic
Constant parameter	Volume	Pressure	Temperature	Heat
Equation of state	$P \propto T$	$V \propto T$	PV = constant	PV^{γ} = constant, $TV^{\gamma-1}$ = constant $P^{1-\gamma}T^{\gamma}$ = constant, $P\rho^{-\gamma}$ = constant,
Work done	ΔW = zero	$\Delta W = P\Delta V$ $= nR\Delta T$	$\Delta W = 2.303\, nRT \log_{10} \dfrac{V_2}{V_1}$	$\Delta W = \dfrac{nR(T_1 - T_2)}{\gamma - 1} = \dfrac{P_1V_1 - P_2V_2}{\gamma - 1}$
Applied to first law of thermodynamics	$\Delta Q = dU$ $= nC_V\Delta T$	$\Delta Q \neq 0,$ $\Delta W \neq 0,$ $\Delta U \neq 0$ $\Delta Q = \mu C_P\Delta T,$ $dU = \mu C_V\Delta T$	$\Delta U = 0, \Delta Q = \Delta W$	$\Delta Q = 0, \Delta W = -\Delta U = -nC_V\Delta T$
P-V diagram				
Slope of PV graph	Slope = $dP/dV = \infty$	Slope = $dP/dV = 0$	Slope = $dP/dV = -P/V$	Slope = $dP/dV = -\gamma P/V$
Specific condition	Closed container	Constant pressure (Open container)	(i) Conductive vessel (ii) slowly process	(i) Non-conductive vessel/insulated vessel (ii) Rapidly/fast/sudden process

☞ Second Law of Thermodynamics

- • **Kelvin Plank's statement :** It state that in cyclic process total heat can not be converted into mechanical work.
- • **Claussius statement :** It is impossible to have net heat flow from a low temperature body to a high temperature body.
- • **Carnot's theorem :** This theorem states that

efficiency of any irreversible heat engine can not be greater than or equal to efficiency of a reversible heat engine provided both work between same heat source and sink.

- **Heat engine :** Main elements of heat engine
  - Heat source at temperature T_1 = Heat reservoir
  - Working substance
  - Sink at low temperature T_2 = Cold reservoir

Heat source at higher temperature T_1 K

$Q_1 \propto T_1$ (Heat Given)

$(T_1 > T_2)$ — working substance — W (By system) output

$Q_2 \propto T_2$ (Heat Rejected)

Sink at lower temperature T_2 K
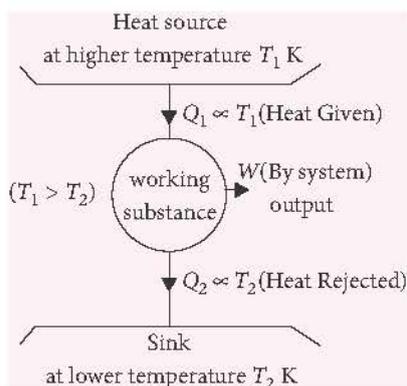
$$Q_1 = Q_2 + W$$

Efficiency $\eta = \dfrac{W}{Q_1} = \dfrac{Q_1 - Q_2}{Q_1} = \dfrac{T_1 - T_2}{T_1}$

$$\eta = \left(1 - \frac{Q_2}{Q_1}\right) \times 100\% = \left(1 - \frac{T_2}{T_1}\right) \times 100\%$$

- **Refrigerator :** It is just opposite to a heat engine.
  - In refrigerator, heat is absorbed from a cold body and some external work is to be done on refrigerant and the total heat is given out at higher temperature source.
  - The coefficient of performance (C.O.P.) is reciprocal of efficiency and for refrigerator it is better to work out with its C.O.P.

Heat source at higher temperature T_1 K

Q_1 (Heat Rejected)

working substance ← W (on system)

$(T_1 > T_2)$ Q_2 (heat absorbed)

Cold body sink at lower temperature T_2 K — Refrigerator
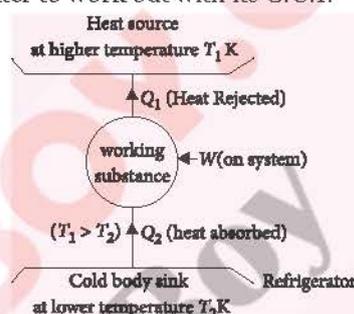
$$\text{C.O.P.}(\beta) = \frac{Q_2 \text{ (Heat absorbed from cold body)}}{W \text{ (Mechanical work on refrigerator)}}$$

$$\text{C.O.P.} = \frac{Q_2}{Q_1 - Q_2} = \frac{T_2}{T_1 - T_2}$$

SPEED PRACTICE

1. A cylindrical metallic rod in thermal contact with two reservoirs of heat at its two ends conducts an amount of heat Q in time t. The metallic rod is melted and the material is formed into a rod of half the radius of the original rod. What is the amount of heat conducted by the new rod, when placed in the same thermal contact with the two reservoir in time t ?

   (a) $\dfrac{Q}{4}$ (b) $\dfrac{Q}{16}$ (c) $2Q$ (d) $\dfrac{Q}{2}$

2. A piece of blue glass heated to a high temperature and a piece of red glass at room temperature, are taken inside a dimly lit room, then
   (a) the blue piece will look blue and red will look as usual
   (b) red looks brighter red and blue looks ordinary blue
   (c) blue shines like brighter red compared to the red piece
   (d) both the pieces will look equally red.

3. A steel scale measures the length of a copper rod as 80 cm when both are at 20°C, the calibration temperature for the scale. The scale read for the length of the rod when both are at 40°C is (Given, α for steel $= 11 \times 10^{-6}$ °C^{-1} and α for copper $= 17 \times 10^{-6}$ °C^{-1}.)
   (a) 97.096 cm   (b) 80.0096 cm
   (c) 89.0096 cm   (d) 92.23 cm

4. If the ratio of specific heat of a gas at constant pressure to that at constant volume is γ, the change in internal energy of a mass of the gas, when the volume changes from V to $2V$ at constant pressure P, is
   (a) $\dfrac{PV}{(\gamma-1)}$ (b) PV (c) $\dfrac{R}{(\gamma-1)}$ (d) $\dfrac{\gamma PV}{(\gamma-1)}$

5. A reversible engine converts $(1/6)^{\text{th}}$ of the heat into work. When the temperature of the sink is reduced by 62 K, the efficiency of the engine is doubled. The temperatures of the source and sink respectively are
   (a) 95°C and 47°C   (b) 80°C and 37°C
   (c) 99°C and 37°C   (d) 90°C and 47°C

6. A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats γ. It is moving with speed v and is suddenly brought to rest. Assuming no heat is lost to the surroundings, its temperature increases by
   (a) $\dfrac{(\gamma-1)}{2(\gamma+2)R}Mv^2$ (b) $\dfrac{(\gamma-1)}{2\gamma R}Mv^2$
   (c) $\dfrac{\gamma Mv^2}{2R}$ (d) $\dfrac{(\gamma-1)}{2R}Mv^2$

7. The average translational energy and the rms speed of molecules in a sample of oxygen gas at 300 K are 6.21×10^{-21} J and 484 m s^{-1} respectively. The corresponding values at 600 K are nearly
   (a) 12.42×10^{-21} J and 968 m s^{-1}
   (b) 8.78×10^{-21} J and 684 m s^{-1}
   (c) 6.21×10^{-21} J and 968 m s^{-1}
   (d) 12.42×10^{-21} J and 684 m s^{-1}

8. The Wien's displacement law express relation between
   (a) wavelength corresponding to maximum energy and temperature
   (b) radiation energy and wavelength
   (c) temperature and wavelength
   (d) colour of light and temperature.

9. At what temperature is $(1/2)k_B T$ equal to minimum rotational energy permitted for a hydrogen molecule?
   (a) 87 K   (b) 92 K   (c) 82 K   (d) 98 K

10. A polyatomic gas with n degrees of freedom has a mean energy per molecule given by
    (a) $\dfrac{nkT}{N}$ (b) $\dfrac{nkT}{2N}$ (c) $\dfrac{nkT}{2}$ (d) $\dfrac{3kT}{2}$

11. 10 g of ice cubes at 0°C are released in a tumbler (water equivalent 55 g) at 40°C. Assuming that negligible heat is taken from the surroundings, the temperature of water in the tumbler becomes nearly ($L = 80$ cal g^{-1})
    (a) 31°C   (b) 22°C   (c) 19°C   (d) 15°C

12. A piece of lead is dropped from an aeroplane at a height of 200 m. If 60% of the striking energy is converted into heat, then the rise in temperature is (specific heat for lead is 127.7 J kg^{-1} K^{-1})
    (a) 9.2 K   (b) 9.8 K   (c) 12.7 K   (d) 11.4 K

13. The molecules of a given mass of a gas have rms velocity of 200 m s^{-1} at 27°C and 1.0×10^5 N m^{-2} pressure. When the temperature and pressure of the

gas are respectively, 127°C and 0.05×10^5 N m^{-2}, the rms velocity of its molecules in m s^{-1} is

(a) $\dfrac{100\sqrt{2}}{3}$ (b) $\dfrac{100}{3}$ (c) $100\sqrt{2}$ (d) $\dfrac{400}{\sqrt{3}}$

[NEET Phase I 2016]

14. Two identical bodies are made of a material for which the heat capacity increases with temperature. One of these is at 100 °C, while the other one is at 0 °C. If the two bodies are brought into contact, then, assuming no heat loss, the final common temperature is
    (a) 50 °C   (b) more than 50 °C
    (c) less than 50 °C but greater than 0 °C
    (d) 0 °C   **[NEET Phase II 2016]**

15. A body cools from a temperature $3T$ to $2T$ in 10 minutes. The room temperature is T. Assume that Newton's law of cooling is applicable. The temperature of the body at the end of next 10 minutes will be
    (a) $\dfrac{7}{4}T$ (b) $\dfrac{3}{2}T$ (c) $\dfrac{4}{3}T$ (d) T
    [NEET Phase II 2016]

16. The temperature inside a refrigerator is t_2 °C and the room temperature is t_1 °C. The amount of heat delivered to the room for each joule of electrical energy consumed ideally will be
    (a) $\dfrac{t_1}{t_1 - t_2}$ (b) $\dfrac{t_1 + 273}{t_1 - t_2}$
    (c) $\dfrac{t_2 + 273}{t_1 - t_2}$ (d) $\dfrac{t_1 + t_2}{t_1 + 273}$
    [NEET Phase II 2016]

17. A pendulum clock loses 12 s a day if the temperature is 40°C and gains 4 s a day if the temperature is 20°C. The temperature at which the clock will show correct time, and the co-efficient of linear expansion (α) of the metal of the pendulum shaft are respectively
    (a) 25°C; $\alpha = 1.85 \times 10^{-5}$ °C^{-1}
    (b) 60°C; $\alpha = 1.85 \times 10^{-4}$ °C^{-1}
    (c) 30°C; $\alpha = 1.85 \times 10^{-3}$ °C^{-1}
    (d) 55°C; $\alpha = 1.85 \times 10^{-2}$ °C^{-1}
    [JEE Main Offline 2016]

18. An ideal gas undergoes a quasi static, reversible process in which its molar heat capacity C remains constant. If during this process the relation of pressure P and volume V is given by PV^n = constant, then n is given by (Here C_P and C_V are molar specific

heat at constant pressure and constant volume, respectively)

(a) $n = \dfrac{C_P}{C_V}$ (b) $n = \dfrac{C - C_P}{C - C_V}$
(c) $n = \dfrac{C_P - C}{C - C_V}$ (d) $n = \dfrac{C - C_V}{C - C_P}$

[JEE Main Offline 2016]

19. 'n' moles of an ideal gas undergoes a process $A \to B$ as shown in the figure. The maximum temperature of the gas during the process will be
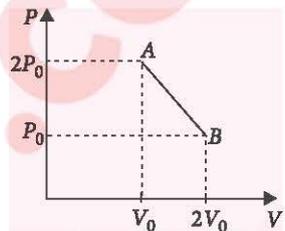
(a) $\dfrac{9P_0V_0}{4nR}$ (b) $\dfrac{3P_0V_0}{2nR}$ (c) $\dfrac{9P_0V_0}{2nR}$ (d) $\dfrac{9P_0V_0}{nR}$

[JEE Main Offline 2016]

20. 200 g water is heated from 40° C to 60°C. Ignoring the slight expansion of water, the change in its internal energy is close to (Given specific heat of water = 4184 J kg^{-1} K^{-1})
    (a) 167.4 kJ (b) 8.4 kJ (c) 4.2 kJ (d) 16.7 kJ
    [JEE Main Online 2016]

SOLUTIONS

1. **(b) :** $A' = A/4$ and $l' = 4l$
   $\dfrac{Q'}{Q} = \dfrac{A'}{A} \cdot \dfrac{l}{l'} = \dfrac{1}{4} \cdot \dfrac{1}{4} = \dfrac{1}{16}$, $\therefore Q' = \dfrac{Q}{16}$

2. **(c) :** According to Stefan's law, $E \propto T^4$
   As the temperature of blue glass is more than that of red glass, so it will appear brighter than red glass.

3. **(b) :** 1 cm length of steel scale at 40°C
   $= 1$ cm $+ 1 \times (11 \times 10^{-6})(40 - 20)$ cm $= 1.00022$ cm
   Length of copper rod at 40°C
   $= 80$ cm $+ (80 \times 17 \times 10^{-6})(40 - 20)$ cm $= 80.0272$ cm
   Number of division on the steel scale
   $= \dfrac{80.0272 \text{ cm}}{1.00022 \text{ cm}} \approx 80.0096$
   Length of the rod = 80.0096 cm

4. **(a) :** As $\dfrac{C_P}{C_V} = \gamma$ $\therefore$ $\dfrac{C_P - C_V}{C_V} = \gamma - 1$
   or $C_V = \dfrac{C_P - C_V}{\gamma - 1} = \dfrac{R}{\gamma - 1}$
   Now $\Delta U = nC_V\Delta T = \dfrac{nR\Delta T}{\gamma - 1}$ ($\because P\Delta V = nR\Delta T$)
   $\Rightarrow \Delta U = \dfrac{P\Delta V}{\gamma - 1} = \dfrac{P(2V - V)}{\gamma - 1} = \dfrac{PV}{\gamma - 1}$

5. **(c)** : As $\eta = 1 - \dfrac{T_2}{T_1}$, $\dfrac{1}{6} = 1 - \dfrac{T_2}{T_1}$; $T_1 = \dfrac{6}{5}T_2$

Also, $\eta' = 2 \times \dfrac{1}{6} = \dfrac{1}{3} = 1 - \dfrac{T_2'}{T_1'}$

or $\dfrac{1}{3} = 1 - \dfrac{(T_2 - 62)}{T_1}$ or $\dfrac{1}{3} = 1 - \dfrac{(T_2 - 62)}{(6/5)T_2}$

Hence, $T_2 = 310\ K = 37°C$

Also, $T_1 = \dfrac{6}{5}T_2 = \dfrac{6}{5} \times 310\ K = 372\ K = 99°C$

6. **(d)** : Loss in KE of the gas $= \dfrac{1}{2}mv^2$

Heat gained by gas $= nC_V\Delta T$

$\therefore \quad \dfrac{1}{2}mv^2 = nC_V\Delta T = \dfrac{m}{M}\cdot\dfrac{R}{\gamma - 1}\Delta T$

or $\Delta T = \dfrac{(\gamma - 1)}{2R}Mv^2$

7. **(d)** : Average translational energy $\propto$ temperature. Thus, when temperature is doubled, average translational energy becomes double, i.e., 12.42×10^{-21} J.

Further, as $v_{rms} \propto \sqrt{T}$; when T becomes 2 times of its previous value, v_{rms} becomes $\sqrt{2}$ times of its previous value, i.e.,

$\sqrt{2} \times 484\ m\ s^{-1} = 684\ m\ s^{-1}$

8. **(a)** : Wien's displacement law states that the product of absolute temperature and the wavelength at which the emissive power is maximum is constant i.e. $\lambda_{max}\ T = $ constant. Therefore it expresses relation between wavelength corresponding to maximum energy and temperature.

9. **(a)** : Kinetic energy of rotation $= \dfrac{1}{2}I\omega^2$

$= \dfrac{(I\omega)^2}{2I} = \dfrac{L^2}{2I}$ (as $L = I\omega$)

For $\dfrac{1}{2}k_BT$ to be equal to minimum rotational energy,

$\dfrac{1}{2}k_BT = \dfrac{L_{min}^2}{2I}$ or $T = \dfrac{L_{min}^2}{k_BI}$ ...(i)

From quantum mechanics,

$L_{min}^2 = \dfrac{h}{2\pi} \approx 10^{-34}\ kg\ m^2\ s^{-1}$ ...(ii)

In case of hydrogen molecule
$I = 2mR^2$, $m = 1.67 \times 10^{-27}$ kg, $R \approx 5 \times 10^{-11}$ m
Thus, $I = 2(1.67 \times 10^{-27})(5 \times 10^{-11})^2\ kg\ m^2$
$= 8.3 \times 10^{-48}\ kg\ m^2$ ...(iii)
From eqns. (i), (ii) and (iii),

$T = \dfrac{(10^{-34})^2}{(1.38 \times 10^{-23})(8.3 \times 10^{-48})} \approx 87\ K$

10. **(c)** : According to law of equipartition of energy, the energy per degree of freedom is $\dfrac{1}{2}kT$. For a polyatomic gas with n degrees of freedom, the mean energy per molecule $= \dfrac{1}{2}nkT$.

11. **(b)** : Let the final temperature be T.
Heat required by ice $= mL + m \times s \times (T - 0)$
$= 10 \times 80 + 10 \times 1 \times T$
Heat lost by water $= 55 \times (40 - T)$
By using law of calorimetry,
heat gained = heat lost; $800 + 10T = 55 \times (40 - T)$
$\Rightarrow \quad T = 21.54°C = 22°C$

12. **(a)** : We are given, $h = 200$ m, $c = 127.7$ J kg^{-1} K^{-1}
Let m be the mass of the piece of lead.
Potential energy of the lead piece $= mgh$
Since 60% of the potential energy is converted into heat, heat produced $= (60/100) \times mgh = 0.6\ mgh$.
If ΔT is the rise in temperature, then
heat gained by the piece $= mc\Delta T$
Assuming that there is no loss of heat,
heat gained = heat produced
or $mc\Delta T = 0.6\ mgh$

or $\Delta T = \dfrac{0.6\ gh}{c} = \dfrac{0.6 \times 9.8 \times 200}{127.7} = 9.2\ K$

13. **(d)**

14. **(b)** : Since, heat capacity of material increases with increase in temperature so, body at 100 °C has more heat capacity than body at 0 °C. Hence, final common temperature of the system will be closer to 100 °C.

$\therefore \quad T_c > 50$ °C

15. **(b)** : According to Newton's law of cooling,

$\dfrac{dT}{dt} = K(T - T_s)$

For two cases, $\dfrac{dT_1}{dt} = K(T_1 - T_s)$ and $\dfrac{dT_2}{dt} = K(T_2 - T_s)$

Here, $T_s = T$, $T_1 = \dfrac{3T + 2T}{2} = 2.5\ T$

and $\dfrac{dT_1}{dt} = \dfrac{3T - 2T}{10} = \dfrac{T}{10}$

$T_2 = \dfrac{2T + T'}{2}$ and $\dfrac{dT_2}{dt} = \dfrac{2T - T'}{10}$

So, $\dfrac{T}{10} = K(2.5\ T - T)$ ...(i)

$\dfrac{2T - T'}{10} = K\left(\dfrac{2T + T'}{2} - T\right)$ ...(ii)

Dividing eqn. (i) by eqn. (ii), we get

$$\frac{T}{2T - T'} = \frac{(2.5T - T)}{\left(\dfrac{2T + T'}{2} - T\right)}$$

$$T' = 3(2T - T') \text{ or, } 4T' = 6T \quad \therefore \ T' = \frac{3}{2}T$$

16. (b): Temperature inside refrigerator = t_2 °C
Room temperature = t_1 °C
For refrigerator,

$$\frac{\text{Heat given to higher temperature } (Q_1)}{\text{Heat taken from lower temperature } (Q_2)} = \frac{T_1}{T_2}$$

$$\frac{Q_1}{Q_2} = \frac{t_1 + 273}{t_2 + 273}$$

$$\Rightarrow \quad \frac{Q_1}{Q_1 - W} = \frac{t_1 + 273}{t_2 + 273} \text{ or, } \frac{W}{Q_1} = \frac{t_1 - t_2}{t_1 + 273}$$

The amount of heat delivered to the room for each joule of electrical energy ($W = 1$ J)

$$Q_1 = \frac{t_1 + 273}{t_1 - t_2}$$

17. (a): Time period of the pendulum clock at temperature θ is given by

$$T_\theta = 2\pi \sqrt{\frac{l_\theta}{g}} = 2\pi \sqrt{\frac{l_0(1 + \alpha\theta)}{g}} = 2\pi \sqrt{\frac{l_0}{g}}(1 + \alpha\theta)^{\frac{1}{2}}$$

$$T_\theta \approx T_0\left(1 + \frac{1}{2}\alpha\theta\right) \qquad \qquad \text{...(i)}$$

Assume pendulum clock gives correct time at temperature θ_0

$$\therefore \quad T_{\theta_0} = T_0\left(1 + \frac{1}{2}\alpha\theta_0\right) \qquad \qquad \text{...(ii)}$$

At θ = 40°C > θ_0 as clock loses time.

$$T_{40} = T_0\left(1 + \frac{1}{2}\alpha \times 40\right) \qquad \qquad \text{...(iii)}$$

At θ = 20°C < θ_0 as clock gains time.

$$T_{20} = T_0\left(1 + \frac{1}{2}\alpha \times 20\right) \qquad \qquad \text{...(iv)}$$

From equations (ii) and (iii), we get

$$\frac{T_{40} - T_{\theta_0}}{T_0} = \frac{1}{2}\alpha(40 - \theta_0)$$

or $12 \text{ s} = \alpha(40 - \theta_0) (12 \text{ h})$ ...(v)

From equations (ii) and (iv), we get

$$\frac{T_{\theta_0} - T_{20}}{T_0} = \frac{1}{2}\alpha(\theta_0 - 20)$$

or, $4 \text{ s} = \alpha(\theta_0 - 20)(12 \text{ h})$ ...(vi)

From equations (v) and (vi), we get

$$3(\theta_0 - 20) = (40 - \theta_0)$$

$$3\theta_0 + \theta_0 = 40 + 60$$

$$\theta_0 = \frac{100}{4} = 25°\text{C}$$

From equation (vi), $4 \text{ s} = \alpha(25 - 20)(12 \times 3600 \text{ s})$

$$\alpha = \frac{4}{5 \times 12 \times 3600} = 1.85 \times 10^{-5} \, °\text{C}^{-1}$$

18. (b): Here, $PV^n = $ constant
or, $PnV^{n-1} dV + V^n dP = 0$
or, $nPdV = -V\, dP$
Also, from ideal gas equation $PV = nRT$
$PdV + VdP = nR\, dT$ or $PdV - nPdV = nRdT$

or, $PdV = \frac{nRdT}{(1-n)}$

Also, $dQ = dU + dW \Rightarrow nC\, dT = nC_V dT + PdV$

$$nCdT = nC_V dT + \frac{nRdT}{(1-n)}$$

or, $C = C_V + \frac{R}{(1-n)}$ or, $(1-n) = \frac{R}{C - C_V}$

or, $n = 1 - \frac{R}{C - C_V} = \frac{C - (C_V + R)}{C - C_V} = \frac{C - C_P}{C - C_V}$

19. (a): Equation of line AB is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$P - P_0 = \frac{2P_0 - P_0}{V_0 - 2V_0}(V - 2V_0)$$

or $P = -\frac{P_0}{V_0}V + 3P_0$ or, $PV = -\frac{P_0}{V_0}V^2 + 3P_0 V$

or $nRT = -\frac{P_0}{V_0}V^2 + 3P_0 V$

or $T = \frac{1}{nR}\left(-\frac{P_0}{V_0}V^2 + 3P_0 V\right)$ ...(i)

For maximum value of T, $\dfrac{dT}{dV} = 0$

or $-\frac{P_0}{V_0}(2V) + 3P_0 = 0 \quad \therefore \ V = \frac{3}{2}V_0$

So, from equation (i)

$$T_{\max} = \frac{1}{nR}\left(-\frac{P_0}{V_0} \times \frac{9}{4}V_0^2 + \frac{9}{2}P_0 V_0\right) = \frac{9}{4}\frac{P_0 V_0}{nR}$$

20. (d): For isochoric process, $\Delta U = Q = ms\,\Delta T$
Here, $m = 200$ g $= 0.2$ kg, $s = 4184$ J kg^{-1} K^{-1}
$\Delta T = 60°\text{C} - 40°\text{C} = 20\,°\text{C} = 20$ K
$\therefore \ \Delta U = 0.2 \times 4184 \times 20 = 16736$ J $= 16.7$ kJ

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ACE YOUR WAY CBSE

Oscillations and Waves

Time Allowed : 3 hours
Maximum Marks : 70

GENERAL INSTRUCTIONS

(i)   All questions are compulsory.

(ii) Q. no. 1 to 5 are very short answer questions and carry 1 mark each.

(iii) Q. no. 6 to 10 are short answer questions and carry 2 marks each.

(iv) Q. no. 11 to 22 are also short answer questions and carry 3 marks each.

(v)   Q. no. 23 is a value based question and carries 4 marks.

(vi) Q. no. 24 to 26 are long answer questions and carry 5 marks each.

(vii) Use log tables if necessary, use of calculator is not allowed.

SECTION-A

1. What change in mass is required to double the frequency of a harmonic oscillator?

2. Why is a loud sound heard at resonance?

3. If an explosion takes place at the bottom of a lake, will the shock waves in water be longitudinal or transverse?

4. How does the frequency of a tuning fork change, when the temperature is increased?

5. The length of a string tied to two rigid supports is 40 cm. What is the maximum wavelength of the stationary wave produced in it?

SECTION-B

6. Show that the motion of a particle represented by $y = \sin \omega t - \cos \omega t$ is simple harmonic with a time period of $2\pi/\omega$.

7. The length of a second's pendulum on the surface of earth is 1 m. What will be the length of a second's pendulum on the moon?

8. A wire stretched between two rigid supports vibrates in its fundamental mode with a frequency of 45 Hz. The mass of the wire is 3.5×10^{-2} kg and its linear mass density is 4.0×10^{-2} kg m^{-1}. What is (a) the speed of a transverse wave on the string, and (b) the tension in the string?

9. What are the differences between stationary waves and progressive waves?

OR

Why is a tuning fork used as a standard oscillator? On what factors does the pitch of a tuning fork depend?

10. A body of mass m is situated in a potential field $U(x) = U_0 (1 - \cos \alpha x)$ where U_0 and α are constants. Find the time period of small oscillations.

SECTION-C

11. Figure (a) shows a spring of force constant k clamped rigidly at one end and a mass m attached to its free end. A force F applied at the free end stretches the spring. Figure (b) shows the same spring with both ends free and attached to mass m at either end. Each end of the spring in figure (b) is stretched by the same force F.
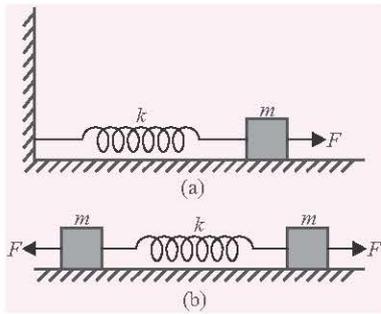
(i) What is the maximum extension of the spring in the two cases?

(ii) If the mass in figure (a) and the two masses in figure (b) are released, what is the period of oscillation in each case?

12. A pipe 20 cm long is closed at one end. Which harmonic mode of the pipe is resonantly excited by a 430 Hz source? Will the same source be in resonance with the pipe if both ends are open? (Speed of sound in air is 340 m s^{-1}).

13. A train, standing at the outer signal of a railway station blows a whistle of frequency 400 Hz in still air. The speed of sound in still air can be taken as 340 m s^{-1}.

(i) What is the frequency of the whistle for a platform observer when the train

(a) approaches the platform with a speed of 10 m s^{-1}?

(b) recedes from the platform with a speed of 10 m s^{-1}?

(ii) What is the speed of sound in each case?

14. Two simple harmonic motions are represented by the equations:

$y_1 = 10 \sin \dfrac{\pi}{4} (12t + 1)$, $y_2 = 5(\sin 3\pi t + \sqrt{3} \cos 3\pi t)$

Here y_1 and y_2 are in cm and t is in second.
Find the ratio of their amplitudes. What are time periods of the two motions?

15. A simple pendulum is hung in a stationary lift and its periodic time is T. What will be the effect on its periodic time T if

(i) the lift goes up with uniform velocity v,

(ii) the lift goes up with uniform acceleration a, and

(iii) the lift comes down with uniform acceleration a?

16. Use the formula $v = \sqrt{\dfrac{\gamma P}{\rho}}$ to explain why the speed of sound in air

(i) is independent of pressure,

(ii) increases with temperature,

(iii) increases with humidity.

17. State the principle of superposition of waves. Distinguish between conditions for the production of stationary waves and beats.

18. Show that for small oscillations the motion of a simple pendulum is simple harmonic. Derive an expression for its time period. Does it depend on the mass of the bob?

19. On a quiet day, two persons A and B, each sounding a note of frequency 580 Hz, are standing a few metres apart. Calculate the number of beats heard by each in one second when A moves towards B with a velocity of 4 m s^{-1}. (Speed of sound in air = 330 m s^{-1}.)

20. A spring balance has a scale that reads from 0 to 50 kg. The length of the scale is 20 cm. A body suspended from this balance, when displaced and released, oscillates with a period of 0.6 s. What is the weight of the body?

21. The patterns of standing waves formed on a stretched string at two instants of time are shown in figure. The velocity of two waves superimposing to form stationary waves is 360 m s^{-1} and their frequencies are 256 Hz.
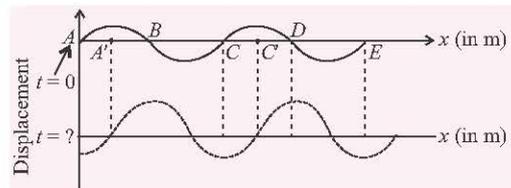
(i) Calculate the time at which the second curve is plotted.

(ii) Mark nodes and antinodes on the curve.

(iii) Calculate the distance between A' and C'.

OR

A horizontal spring block system of mass M executes simple harmonic motion. When the block is passing through its equilibrium position, an object of mass m is put on it and the two move together. Find the new amplitude and frequency of vibration.

22. Discuss first three modes of vibration of a closed organ pipe.

23. Rohit was a good football player. But since last few days he was getting pain in his stomach. His parents took him to a doctor who examined him and asked him to get an ultrasound done to detect the exact cause. Rohit was afraid of ultrasound scanner and refused to get it done. His parents made him understand that the scanner uses ultrasonic rays which go inside and detect any problem inside the body. So he got it done and the scanner showed that he has small tumour in his stomach and that has to be operated as early as possible. Doctor operated him off the tumour and after a month he became fine again.

Answer the following questions based on above information :

(i) What are the values shown by Rohit's parents?

(ii) On which principle does the ultrasonic scanner work?

(iii) If the ultrasound uses the operating frequency of 4.2 MHz, the speed of sound in the tissue is 1.7 km s^{-1}. What is the wavelength of the sound in tissue?

24. A cylindrical piece of cork of base area A and height h floats in a liquid of density ρ_1. The cork is depressed slightly and then released. Show that the cork oscillates up and down simple harmonically with a period

$$T = 2\pi \sqrt{\frac{h\rho}{\rho_1 g}}$$

where ρ is the density of cork. (Ignore damping due to viscosity of the liquid).

OR

A mass attached to a spring is free to oscillate, with angular velocity ω, in a horizontal plane without friction or damping. It is pulled to a distance x_0 and pushed towards the centre with a velocity v_0 at time $t = 0$. Determine the amplitude of the resulting oscillations in terms of the parameters ω, x_0 and v_0.

25. Find the total energy of the particle executing SHM and show graphically the variation of potential energy and kinetic energy with displacement in SHM.

OR

Explain the formation of beats analytically. Prove that the beat frequency is equal to the difference in frequencies of the two superposing waves.

26. Derive Newton's formula for the speed of sound in a gas. Why and what correction was applied by Laplace in this formula?

OR

Explain why (or how) :

(i) in a sound wave, a displacement node is a pressure antinode and vice versa,

(ii) bats can ascertain distances, directions, nature and sizes of the obstacles without any eyes,

(iii) a violin note and sitar note may have the same frequency, yet we can distinguish between the two notes,

(iv) solids can support both longitudinal and transverse waves, but only longitudinal waves can propagate in gases, and

(v) the shape of a pulse gets distorted during propagation in a dispersive medium.

1. The time period (T) of a harmonic oscillator (a mass attached to a spring) is given by $T = 2\pi\sqrt{m/k}$, where k is the force constant of the spring. If υ is the frequency of the harmonic oscillator,

$$\upsilon = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \text{or} \quad \upsilon \propto 1/\sqrt{m}$$

Thus, to double the frequency of the oscillator, the mass should be reduced to (1/4)th of its original value.

2. At resonance, a compression falls on a compression and a rarefaction falls on a rarefaction. On account of this, the amplitude of the vibrating particles increases. Since the intensity of sound is directly proportional to the square of the amplitude of the vibrating particles, hence maximum sound is heard at resonance position.

3. An explosion in a lake generates shock waves in water thereby resulting in a tremendous increase in pressure in the medium (water). A shock wave is thus a longitudinal wave travelling at a speed which is greater than that of a longitudinal wave of ordinary intensity.

4. As the temperature increases, the length of the prong of the tuning fork increases. This increases the wavelength of the stationary waves set up in the tuning fork. As frequency, $\upsilon \propto 1/\lambda$, so frequency of the tuning fork decreases.

5. When the string vibrates in one segment, $L = \dfrac{\lambda}{2}$

$\therefore \quad \lambda = 2L = 2 \times 40 \text{ cm} = 80 \text{ cm}$

6. Given, $y = \sin \omega t - \cos \omega t$

$$= \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin\omega t - \frac{1}{\sqrt{2}}\cos\omega t\right)$$

$$= \sqrt{2}\left(\cos\frac{\pi}{4}\sin\omega t - \sin\frac{\pi}{4}\cos\omega t\right)$$

$$\therefore \ y = \sqrt{2}\sin\left(\omega t - \frac{\pi}{4}\right)$$

Hence, $(\sin \omega t - \cos \omega t)$ represents SHM.
Again,

$$y = \sqrt{2}\sin\left(\omega t - \frac{\pi}{4}\right) = \sqrt{2}\sin\left(\omega t - \frac{\pi}{4} + 2\pi\right)$$

$$= \sqrt{2}\sin\left[\omega\left(t + \frac{2\pi}{\omega}\right) - \frac{\pi}{4}\right]$$

Hence, time period of SHM is $2\pi/\omega$.

7. For second's pendulum, $T = 2$ s

$$\because \ T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow 2 = 2\pi\sqrt{\frac{l}{g}} \Rightarrow \frac{l}{g} = \frac{1}{\pi^2} = \text{constant}$$

$$\therefore \ \frac{l_{\text{moon}}}{g_{\text{moon}}} = \frac{l_{\text{earth}}}{g_{\text{earth}}}$$

$$\Rightarrow l_{\text{moon}} = l_{\text{earth}}\left(\frac{g_{\text{moon}}}{g_{\text{earth}}}\right) = 1 \times \frac{1}{6} = \frac{1}{6} \text{ m}$$

8. Here, $\upsilon = 45$ Hz, $M = 3.5 \times 10^{-2}$ kg ;
mass/length $= \mu = 4.0 \times 10^{-2}$ kg m^{-1}

$$\therefore \ l = \frac{M}{\mu} = \frac{3.5 \times 10^{-2}}{4.0 \times 10^{-2}} = \frac{7}{8}\text{ m}$$

As wire vibrates in its fundamental mode

$$\frac{\lambda}{2} = l = \frac{7}{8} \quad \therefore \ \lambda = \frac{7}{4}\text{ m} = 1.75\text{ m}$$

The speed of the transverse wave, $v = \upsilon\lambda$

$$= 45 \times 1.75 = 78.75 \text{ m s}^{-1}$$

(b) As, $v = \sqrt{\dfrac{T}{\mu}}$

$$T = v^2 \times \mu = (78.75)^2 \times 4.0 \times 10^{-2} = 248.06\text{ N}$$

9. Difference between stationary and progressive waves

	Stationary waves	Progressive waves
(i)	The disturbance remains confined to a particular region, and there is no onward motion.	The disturbance travels forward, being handed over from one particle to the neighbouring particles.
(ii)	There is no transfer of energy in the medium.	Energy is transferred in the medium along the waves.
(iii)	The amplitude of vibration of particles varies from zero at nodes to maximum at antinodes.	The amplitude of vibration of each particle is same.

OR

When a tuning fork is struck lightly against a rubber pad, it produces only fundamental tone. If it is struck forcefully, it produces overtones which soon dies out. So a tuning fork can be used as a source of standard frequency.

Factors on which the pitch of a tuning fork depends :
(i) It is inversely proportional to the square of the length of its prongs.
(ii) It is directly proportional to the thickness of the fork.
(iii) It is directly proportional to the square root of the Young's modulus of elasticity of its material.
(iv) It is inversely proportional to the square root of the density of its material.

Hence low frequency tuning forks are long and thin while high frequency tuning forks are short and thick.

10. Given, $U(x) = U_0(1 - \cos \alpha x)$
Differentiating both sides with respect to x

$$\frac{dU(x)}{dx} = U_0(0 + \alpha\sin\alpha x) = U_0\alpha\sin\alpha x$$

$$\therefore \ F = -\frac{dU(x)}{dx} = -U_0\alpha\sin\alpha x$$

For small oscillations, $\sin\theta \approx \theta$
$\Rightarrow \quad \sin \alpha x \approx \alpha x$

$\therefore \qquad\qquad F = -U_0\alpha(\alpha x) = -U_0\alpha^2 x \qquad \text{...(i)}$

Also, $\qquad\qquad F = -kx \qquad\qquad\qquad \text{...(ii)}$

From equations (i) and (ii)

$$k = U_0\alpha^2$$

Thus, $\qquad\qquad T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{U_0\alpha^2}}$

11. (i) Maximum extension of the spring :
(a) Suppose the maximum extension produced in the spring is y. Then,

$$F = k\,y \qquad\qquad \text{(in magnitude)}$$

or $\qquad\qquad y = F/k$

(b) In this case, force F on each mass acts as the force of reaction developed due to force F on

the other mass. Therefore, in this case also, maximum extension is given by $y = F/k$

(ii) Period of oscillation:

If T_1 is the time period in case (a), then

$$T_1 = 2\pi\sqrt{\frac{m}{k}}$$

In case (b), the time period of oscillation of a two body oscillator (two bodies of mass m_1 and m_2 connected at the ends of a spring of spring constant k) is given by

$$T_2 = 2\pi\sqrt{\frac{\mu}{k}}$$

where μ is called the reduced mass of the system defined as

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

In the present case, $m_1 = m_2 = m$. So,

$$\mu = \frac{m \times m}{m + m} = \frac{m}{2}$$

Thus, $T_2 = 2\pi\sqrt{\frac{m/2}{k}} = 2\pi\sqrt{\frac{m}{2k}}$

12. Here, $L = 20$ cm $= 0.2$ m, $\upsilon_n = 430$ Hz, $v = 340$ m s^{-1}

The frequency of nth normal mode of vibration of closed pipe is

$$\upsilon_n = (2n-1)\frac{v}{4L} \quad \therefore \quad 430 = (2n-1)\frac{340}{4 \times 0.2}$$

or $\quad 2n-1 = \frac{430 \times 4 \times 0.2}{340} = 1.01$

$\quad 2n = 2.01$ or $n \approx 1$

Hence, it will excite the 1st normal mode of vibration. In a pipe, open at both ends, we have

$$\upsilon_n = n \times \frac{v}{2L} \Rightarrow \frac{n \times 340}{2 \times 0.2} = 430 \therefore n = \frac{430 \times 2 \times 0.2}{340} = 0.5$$

As n has to be an integer, therefore, open organ pipe cannot be in resonance with the source.

13. Here, frequency of source of sound, $\upsilon = 400$ Hz; speed of sound, $v = 340$ m s^{-1} speed of source, $v_s = 10$ m s^{-1}

(i) (a) When the train approaches the platform, the apparent frequency as heard by the observer on the platform will be

$$\upsilon' = \frac{v}{v - v_s}\upsilon = \frac{340}{340 - 10} \times 400$$

$$= \frac{340}{330} \times 400 = 412.1 \text{ Hz}$$

(b) When the train recedes from the platform, the apparent frequency as heard by the observer will be :

$$\upsilon' = \frac{v}{v + v_s}\upsilon = \frac{340}{340 + 10} \times 400 = \frac{340}{350} \times 400$$

$$= 388.6 \text{ Hz} \simeq 389 \text{ Hz}$$

(ii) The speed of sound in each case remains same i.e., 340 m s^{-1}.

14. $y_1 = 10\sin\frac{\pi}{4}(12t+1) = 10\sin\left(3\pi t + \frac{\pi}{4}\right)$ ...(i)

$y_2 = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$

$= 10\left(\sin 3\pi t \times \frac{1}{2} + \cos 3\pi t \times \frac{\sqrt{3}}{2}\right)$

$= 10\left(\sin 3\pi t \cos\frac{\pi}{3} + \cos 3\pi t \sin\frac{\pi}{3}\right)$

or $\quad y_2 = 10\sin\left(3\pi t + \frac{\pi}{3}\right)$ ...(ii)

The general equation for SHM is

$$y = A\sin(\omega t + \phi_0) = A\sin\left(\frac{2\pi}{T}t + \phi_0\right) \quad \text{...(iii)}$$

Comparing equations (i) and (ii) with (iii), we get

$A_1 = 10$ cm, $A_2 = 10$ cm, $\frac{2\pi}{T_1} = \frac{2\pi}{T_2} = 3\pi$

$\therefore \quad \frac{A_1}{A_2} = 1; T_1 = T_2 = \frac{2}{3}$ s

15. (i) When the lift goes up figure (a) with uniform velocity v, tension in the string, $T' = mg$. The value of g remains unaffected.
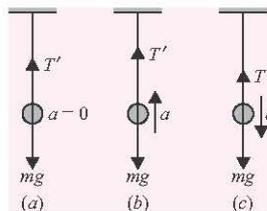
The period T remains same as that in stationary lift,

i.e., $T = 2\pi\sqrt{\frac{l}{g}}$

(ii) When the lift goes up with acceleration a as shown in figure (b), the net upward force on the bob is $T' - mg = ma$ $\therefore$ $T' = m(g + a)$

The effective value of g is $(g + a)$ and the time period is

$$T_1 = 2\pi\sqrt{\frac{l}{g + a}}$$

Clearly, $T_1 < T$, i.e., time period decreases.

(iii) When lift comes down with acceleration a figure (c), the net downward force on the bob is

$\quad mg - T' = ma \quad \therefore \quad T' = m(g - a)$

The effective value of g becomes $(g - a)$ and the time period is

$$T_2 = 2\pi\sqrt{\frac{l}{g - a}}$$

Clearly, $T_2 > T$, i.e., time period increases.

16. (i) Effect of pressure: The speed of sound in a gas is given by, $v = \sqrt{\dfrac{\gamma P}{\rho}}$

At constant temperature, $PV = $ constant;

$\dfrac{Pm}{\rho} = $ constant

Since m is constant, so $\dfrac{P}{\rho} = $ constant

i.e., when pressure changes, density also changes in the same ratio so that the factor $\dfrac{P}{\rho}$ remains unchanged. Hence the pressure has no effect on the speed of sound in a gas for a given temperature.

(ii) Effect of temperature : We know that

$$PV = nRT \quad \text{or} \quad P = \dfrac{nRT}{V}$$

Also $v = \sqrt{\dfrac{\gamma P}{\rho}} = \sqrt{\dfrac{\gamma nRT}{\rho V}} = \sqrt{\dfrac{\gamma RT}{M}}$

where $M = $ molecular weight of the gas
As γ, R and M are constants, so $v \propto \sqrt{T}$,
i.e., velocity of sound in a gas is directly proportional to the square root of its temperature, hence we conclude that the velocity of sound in air increases with increase in temperature.

(iii) Effect of humidity: As $v = \sqrt{\dfrac{\gamma P}{\rho}}$, i.e., $v \propto \dfrac{1}{\sqrt{\rho}}$

The density of water vapours is less than that of dry air. Since the speed of sound is inversely proportional to the square root of density, so sound travels faster in moist air than in dry air.

17. Principle of superposition of waves states that when a number of waves travel through a medium simultaneously, the resultant displacement of any particle of the medium at any given time is equal to the algebraic sum of the displacements due to the individual waves. Mathematically,

$$y = y_1 + y_2 + y_3 + \ldots + y_n$$

(i) When two waves of same frequency moving with the same speed in the opposite directions in a medium superpose on each other, they produce stationary waves.

(ii) When two waves of slightly different frequencies moving with the same speed in the same direction in a medium superpose on each other they produce beats.

18. Suppose at any instant during oscillation, the bob of a simple pendulum lies at position A when its displacement is $OA = x$ and the thread makes angle θ with the vertical. The forces acting on the bob are

(i) Weight mg of the bob acting vertically downwards.
(ii) Tension T along the string.

Thus, the restoring force is

$F = -mg \sin \theta$

$= -mg\left(\theta - \dfrac{\theta^3}{3!} + \dfrac{\theta^5}{5!} - \ldots \right)$

However, if θ is so small that its higher powers can be neglected, then, $F = -mg\,\theta$

If l is the length of the simple pendulum, then

$$\theta(\text{rad}) = \dfrac{\text{arc}}{\text{radius}} = \dfrac{x}{l}$$

$\therefore \qquad F = -mg\dfrac{x}{l}$

or $\quad ma = -\dfrac{mg}{l}x$ or $a = -\dfrac{g}{l}x = -\omega^2 x$

Hence for small oscillations, the motion of the bob is simple harmonic. Its time period is

$$T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{\sqrt{g/l}} \quad \text{or} \quad T = 2\pi\sqrt{\dfrac{l}{g}}$$

Obviously, the time period of a simple pendulum depends on its length l and acceleration due to gravity g but is independent of the mass m of the bob.

19. In one case, A can be regarded as a source of sound moving towards observer B.

$$v_s = + 4 \text{ m s}^{-1}, v_o = 0$$

$\therefore \qquad \upsilon' = \dfrac{v - v_o}{v - v_s} \times \upsilon = \dfrac{330 - 0}{330 - 4} \times 580$

$$= \dfrac{330}{326} \times 580 = 587 \text{ Hz}$$

In another case, A can be regarded as observer moving towards stationary source B.

$$v_o = -4 \text{ m s}^{-1}, v_s = 0$$

$\therefore \qquad \upsilon'' = \dfrac{v - v_o}{v - v_s} \times \upsilon = \dfrac{330 + 4}{330 - 0} \times 580$
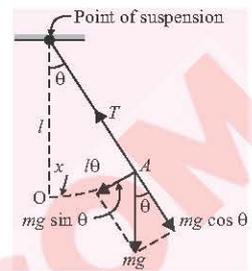
$$= \dfrac{334}{330} \times 580 = 587 \text{ Hz}$$

Number of beats heard per second by A
$= \upsilon'' - \upsilon = 587 - 580 = 7$

Number of beats heard per second by B
$= \upsilon' - \upsilon = 587 - 580 = 7$

20. Here, Maximum mass, $m = 50$ kg,
Maximum extension, $y = 20 - 0 = 20$ cm $= 0.2$ m
Maximum force, $F = mg = 50 \times 9.8 = 490$ N

∴ Spring constant,
$$k = \frac{F}{y} = \frac{490}{0.2} = 2450 \text{ N m}^{-1}$$

When a body of mass M is suspended from the spring balance, it oscillates with a period of 0.6 s.

∴ Time period, $T = 2\pi\sqrt{\dfrac{M}{k}}$ or $T^2 = 4\pi^2 \dfrac{M}{k}$

∴ $M = \dfrac{T^2 k}{4\pi^2} = \dfrac{(0.6)^2 \times 2450}{4 \times (3.14)^2}$
$M = 22.36$ kg

∴ Weight of the body,
$W = Mg = 22.36 \times 9.8 = 219.1$ N

21. (i) Here, $v = 360$ m s^{-1}, $\upsilon = 256$ Hz
$$\lambda = \frac{v}{\upsilon} = \frac{360}{256} = 1.406 \text{ m}$$
$$AA' = \frac{\lambda}{4} = \frac{1.406}{4} = 0.3516 \text{ m}$$
Time (t) at which the second curve is plotted
$$\frac{AA'}{v} = \frac{0.3516}{360} = 9.8 \times 10^{-4} \text{ s}$$
(ii) Nodes : A, B, C, D, E
Antinodes : A', C'
(iii) Distance between A' and C'
$= \lambda = 1.406$ m $= 1.41$ m

OR

Original frequency,
$$\upsilon = \frac{1}{2\pi}\sqrt{\frac{k}{M}}$$

Let A = Initial amplitude of oscillation
v = Velocity of mass M when passing through mean position.
Maximum kinetic energy = Total energy
or $\dfrac{1}{2}Mv^2 = \dfrac{1}{2}kA^2$

∴ $v = \sqrt{\dfrac{k}{M}}\, A$

When mass m is put on the system,
total mass $= (M + m)$. If v' is the velocity of the combination in equilibrium position, then by the conservation of linear momentum,
$$Mv = (M + m)v' \quad \text{or} \quad v' = \frac{Mv}{M + m}$$
If A' is the new amplitude, then
$$\frac{1}{2}(M + m)v'^2 = \frac{1}{2}kA'^2$$

or $A' = \sqrt{\dfrac{M + m}{k}} \times v' = \sqrt{\dfrac{M + m}{k}} \times \dfrac{Mv}{M + m}$

$\quad = \sqrt{\dfrac{M + m}{k}} \times \dfrac{M}{M + m} \times \sqrt{\dfrac{k}{M}}\, A = \sqrt{\dfrac{M}{M + m}}\, A$

New frequency, $\upsilon' = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{M + m}}$

22. (i) **First mode of vibration :** In this mode of vibration, there is only one node at the closed end and one antinode at the open end. If L is the length of the organ pipe, then

(a)
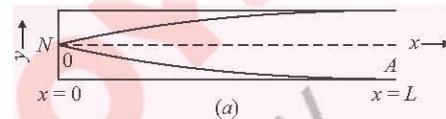
$$L = \frac{\lambda_1}{4} \quad \text{or} \quad \lambda_1 = 4L$$

Frequency,
$$\upsilon_1 = \frac{v}{\lambda_1} = \frac{v}{4L} = \upsilon \text{ (say)}$$

This frequency is called first harmonic or fundamental frequency.

(ii) **Second mode of vibration :** In this mode of vibration, there is one node and one antinode between a node at the closed end and an antinode at the open end.

(b)
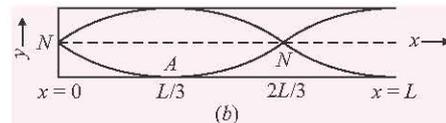
$$L = \frac{3\lambda_2}{4} \quad \text{or} \quad \lambda_2 = \frac{4L}{3}$$

Frequency,
$$\upsilon_2 = \frac{v}{\lambda_2} = \frac{3v}{4L} = 3\upsilon$$

This frequency is called first overtone or third harmonic.

(iii) **Third mode of vibration :** In this mode of vibration, there are two nodes and two antinodes between a node at the closed end and an antinode at the open end.

(c)

∴ $L = \dfrac{5\lambda_3}{4}$ or $\lambda_3 = \dfrac{4L}{5}$

Frequency, $\upsilon_3 = \dfrac{v}{\lambda_3} = \dfrac{5v}{4L} = 5\,\upsilon$

Hence different frequencies produced in a closed organ pipe are in the ratio $1:3:5:7:\ldots$ i.e., only odd harmonics are present in a closed organ pipe.
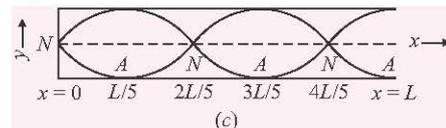
23. (i) Sense of responsibility, concern for their son and awareness about technology.
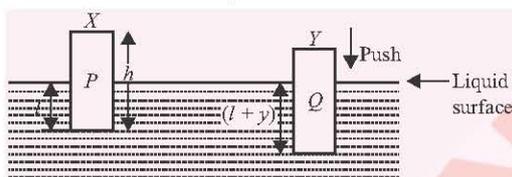
(ii) An ultrasonic scanner works on the principle of reflection of ultrasonic waves from a region where there is a change in tissue density.

(iii) Wavelength, $\lambda = \dfrac{v}{\upsilon} = \dfrac{1.7\,\text{km s}^{-1}}{4.2\,\text{MHz}} = \dfrac{1.7\times10^3\,\text{m s}^{-1}}{4.2\times10^6\,\text{s}^{-1}}$

$= 404.76\,\mu\text{m}$

24. Let X be the equilibrium position of a cylinder floating in a given liquid. Assume

V = volume of cork = Ah

m = mass of cork = $Ah\rho$.

l = length of the cylindrical piece of cork dipped in the liquid upto point P in position X.

W = weight of the cylindrical cork.

W_1 = weight of the liquid displaced by the cork.

$W = mg = (Ah\rho)g$

∴ W_1 = Area of cross-section of cork × length of cylinder dipped in liquid × density of liquid × g

$= Al\rho_1 g$

∴   According to the law of flotation,

$W = W_1$ or $Ah\rho g = Al\rho_1 g$

or $h = \dfrac{l\rho_1}{\rho}$

Let the cylinder be pushed into the liquid through a small distance y from equilibrium. Since $W = W_1$ then the restoring force acting on the cylinder is given by

$F = -$ weight of the liquid displaced by the length y of the cylindrical cork

$= -(Ay)\rho_1 \times g = -(A\rho_1 g)y = -ky$

where $k = A\rho_1 g$ is the force constant.

If a be the acceleration produced in the cylindrical piece of the cork, then

$a = \dfrac{F}{\text{mass of cork}} = \dfrac{F}{m} = \dfrac{-A\rho_1 g y}{Ah\rho}$

or $a = -\dfrac{\rho_1 g}{h\rho}y$

Now as the acceleration of the cylindrical cork is directly proportional to its displacement from equilibrium position and acts towards the equilibrium position, so the motion of the bob is simple harmonic having time period (T) given by

$$T = 2\pi\sqrt{\dfrac{\text{displacement}}{\text{acceleration}}} = 2\pi\dfrac{\sqrt{y}}{\sqrt{\dfrac{\rho_1 g}{h\rho}y}}$$

or $$T = 2\pi\sqrt{\dfrac{h\rho}{\rho_1 g}}$$

OR

Let the displacement of the particle at any time t be represented by

$x = A\cos(\omega t + \phi_0)$ ...(i)

where A = amplitude, ϕ_0 = initial phase

If v be the velocity of the particle at time t, Then

$v = \dfrac{dx}{dt} = \dfrac{d}{dt}\big[A\cos(\omega t + \phi_0)\big]$

$= -A\omega\sin(\omega t + \phi_0)$ ...(ii)

At $t = 0$, $x = x_0$ and $v = v_0$

By putting $t = 0$ in equations (i) and (ii), we get

$x_0 = A\cos\phi_0$, $v_0 = -A\omega\sin\phi_0$

or $v_0 = -\omega\sqrt{(A\sin\phi_0)^2} = -\omega\sqrt{A^2(1-\cos^2\phi_0)}$

or $= -\omega\sqrt{A^2 - x_0^2}$ ...(iii)

Equation (iii) shows that initial velocity is negative. Squaring on both sides of equation (iii), we get

$v_0^2 = \omega^2(A^2 - x_0^2); \quad A^2 - x_0^2 = \dfrac{v_0^2}{\omega^2}$

$A^2 = x_0^2 + \dfrac{v_0^2}{\omega^2}; \quad A = \sqrt{\left(x_0^2 + \dfrac{v_0^2}{\omega^2}\right)}$

25. The energy of a harmonic oscillator is partly kinetic and partly potential. When a body is displaced from its equilibrium position by doing work upon it, it acquires potential energy. When the body is released, it begins to move back to equilibrium position, thus acquires kinetic energy.

At any instant, the displacement of a particle executing SHM is given by

$x = A\cos(\omega t + \phi_0)$

∴   Velocity, $v = \dfrac{dx}{dt} = -\omega A\sin(\omega t + \phi_0)$

Hence, kinetic energy of the particle at any time t is given by

$K = \dfrac{1}{2}mv^2 = \dfrac{1}{2}m\omega^2 A^2\sin^2(\omega t + \phi_0)$

But $A^2 \sin^2(\omega t + \phi_0) = A^2[1 - \cos^2(\omega t + \phi_0)]$

$$= A^2 - A^2 \cos^2(\omega t + \phi_0) = A^2 - x^2$$

or $\qquad K = \dfrac{1}{2} m\omega^2 (A^2 - x^2) = \dfrac{1}{2} k(A^2 - x^2)$

When the displacement of a particle from its equilibrium position is x, the restoring force acting on it is

$$F = -kx$$

If we displace the particle further through a small distance dx, then work done against the restoring force is given by

$$dW = -Fdx = +kxdx$$

The total work done in moving the particle from mean position ($x = 0$) to displacement x is given by

$$W = \int dW = \int_0^x kxdx = k\left[\dfrac{x^2}{2}\right]_0^x = \dfrac{1}{2}kx^2$$

The work done against the restoring force is stored as the potential energy of the particle. Hence potential energy of a particle at displacement x is given by

$$U = \dfrac{1}{2}kx^2 = \dfrac{1}{2}m\omega^2 x^2 = \dfrac{1}{2}m\omega^2 A^2 \cos^2(\omega t + \phi_0)$$

At any displacement x, the total energy of a harmonic oscillator is given by

$$E = K + U = \dfrac{1}{2}k(A^2 - x^2) + \dfrac{1}{2}kx^2$$

or $\quad E = \dfrac{1}{2}kA^2 = \dfrac{1}{2}m\omega^2 A^2 = 2\pi^2 m\upsilon^2 A^2$

$$(\because \omega = 2\pi\upsilon)$$

Thus the total mechanical energy of a harmonic oscillator is independent of time or displacement.

At the mean position, $x = 0$

Kinetic energy, $K = \dfrac{1}{2}k(A^2 - 0^2) = \dfrac{1}{2}kA^2$

Potential energy, $U = \dfrac{1}{2}k(0^2) = 0$

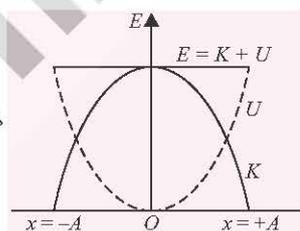Hence at the mean position, particle has only kinetic energy.

At the extreme positions, $x = \pm A$

Kinetic energy,

$K = \dfrac{1}{2}k(A^2 - A^2) = 0$

Potential energy, $U = \dfrac{1}{2}kA^2$

Hence at the two extreme positions particle has only potential energy.

OR

Consider two harmonic waves of frequencies υ_1 and υ_2 (υ_1 being slightly greater than υ_2) and each of amplitude A travelling in a medium in the same direction. The displacements due to the two waves at a given observation point may be represented by

$$y_1 = A \sin\omega_1 t = A \sin 2\pi \upsilon_1 t$$
$$y_2 = A \sin\omega_2 t = A \sin 2\pi \upsilon_2 t$$

By the principle of superposition, the resultant displacement at the given point will be

$$y = y_1 + y_2 = A \sin 2\pi \upsilon_1 t + A \sin 2\pi \upsilon_2 t$$

$$= 2A \cos 2\pi\left(\dfrac{\upsilon_1 - \upsilon_2}{2}\right)t \cdot \sin 2\pi\left(\dfrac{\upsilon_1 + \upsilon_2}{2}\right)t$$

If we write

$$\upsilon_{mod} = \dfrac{\upsilon_1 - \upsilon_2}{2} \quad \text{and} \quad \upsilon_{av} = \dfrac{\upsilon_1 + \upsilon_2}{2}$$

then $\qquad y = 2A \cos(2\pi \upsilon_{mod} t) \sin(2\pi \upsilon_{av} t)$

or $\qquad y = R \sin(2\pi \upsilon_{av} t)$

where $R = 2A \cos(2\pi \upsilon_{mod} t)$ is the amplitude of the resultant wave.

The amplitude R of the resultant wave will be maximum, when

$\qquad \cos 2\pi \upsilon_{mod} t = \pm 1$

or $\qquad 2\pi \upsilon_{mod} t = n\pi \qquad$ where $n = 0, 1, 2, \dots$

or $\qquad \pi(\upsilon_1 - \upsilon_2)t = n\pi$

or $\qquad t = \dfrac{n}{\upsilon_1 - \upsilon_2} = 0, \dfrac{1}{\upsilon_1 - \upsilon_2}, \dfrac{2}{\upsilon_1 - \upsilon_2}, \dots$

$\therefore \quad$ Time interval between two successive maxima

$$= \dfrac{1}{\upsilon_1 - \upsilon_2}$$

Similarly, the amplitude R will be minimum, when

$\qquad \cos 2\pi \upsilon_{mod} t = 0$

or $\qquad 2\pi \upsilon_{mod} t = (2n + 1)\pi/2$ where $n = 0, 1, 2, \dots$

or $\qquad \pi(\upsilon_1 - \upsilon_2)t = (2n + 1)\pi/2$

or $t = \dfrac{(2n + 1)}{2(\upsilon_1 - \upsilon_2)} = \dfrac{1}{\upsilon_1 - \upsilon_2}, \dfrac{3}{2(\upsilon_1 - \upsilon_2)}, \dfrac{5}{(2\upsilon_1 - \upsilon_2)}, \dots$

$\therefore \quad$ The time interval between successive minima

$$= \dfrac{1}{\upsilon_1 - \upsilon_2}$$

Clearly, both maxima and minima of intensity occur alternately. Hence the time interval between two successive beats

$$t_{beat} = \dfrac{1}{\upsilon_1 - \upsilon_2}$$

The number of beats produced per second is called beat frequency.

$$\upsilon_{beat} = \frac{1}{t_{beat}} \quad \text{or} \quad \upsilon_{beat} = \upsilon_1 - \upsilon_2$$

26. Newton assumed that sound waves travel through a gas under isothermal conditions. Thus the temperature of gas remains constant. If B_{iso} is the bulk modulus of the gas at constant temperature, then the speed of sound in the gas will be

$$v = \sqrt{\frac{B_{iso}}{\rho}}$$

For an isothermal change,

$$PV = \text{constant} \qquad \text{(Boyle's law)}$$

Differentiating both sides, we get

$$PdV + VdP = 0$$

or $\quad P = -\dfrac{VdP}{dV} = -\dfrac{dP}{dV/V}$

$$= \frac{\text{Change in pressure } (dP)}{\text{Change in volume } (dV)/\text{Original volume } (V)} = B_{iso}$$

Hence the Newton's formula for the speed of sound in a gas is

$$v = \sqrt{\frac{P}{\rho}}$$

Speed of sound in air at STP,

$$v = \sqrt{\frac{1.013 \times 10^5}{1.293}} \simeq 280 \text{ m s}^{-1}$$

This value is about 16% less than the experimental value (331 m s^{-1}) of the speed of sound in air at STP. Hence Newton's formula is not acceptable.

The French scientist Laplace pointed out that sound travels through a gas under adiabatic conditions not under isothermal conditions.

So, when sound travels through a gas, the temperature does not remain constant. The pressure and volume variations are adiabatic. If B_{adia} is the adiabatic bulk modulus of the gas, then the formula for the speed of sound in the gas would be

$$v = \sqrt{\frac{B_{adia}}{\rho}}$$

For an adiabatic change, $PV^{\gamma} = \text{constant}$

Differentiating both sides, we get

$$P\gamma V^{\gamma-1} dV + V^{\gamma} dP = 0$$

or $\qquad P\gamma V^{\gamma-1} dV = -V^{\gamma} dP$

$$\gamma P = \frac{-dP}{dV/V} = B_{adia}$$

where $\gamma = C_P/C_V$, is the ratio of two specific heats. Hence the Laplace formula for the speed of sound in a gas is

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

This modification of Newton's formula is known as Laplace correction.

For air $\gamma = 7/5$, so speed of sound in air at STP will be

$$v = \sqrt{\gamma}\sqrt{\frac{P}{\rho}} = \sqrt{\frac{7}{5}} \times 280 \text{ m s}^{-1} = 331.3 \text{ m s}^{-1}$$

OR

(i) In a sound wave, a node is a point where the displacement is zero as here a compression and a rarefaction meet and the pressure is maximum, so it is also called pressure antinode.

While an antinode is a point where the amplitude displacement is maximum but pressure is minimum. So this point is also called pressure node.

Hence displacement node is a pressure antinode and displacement antinode is pressure node.

(ii) Bats emit ultrasonic waves of large frequencies (small wavelength) when they fly. These ultrasonic waves are received by them after reflection from the obstacle. Their ears are so sensitive and trained that they can not only get the information of the distance of the obstacle but also the nature of the reflecting surface.

(iii) The quality of the sound produced by an instrument depends upon the number of overtones. Since the number of overtones is different in the cases of sounds produced by violin and sitar therefore we can distinguish through them.

(iv) Solids possess both the volume elasticity and the shear elasticity. Therefore they can support both longitudinal and transverse waves.
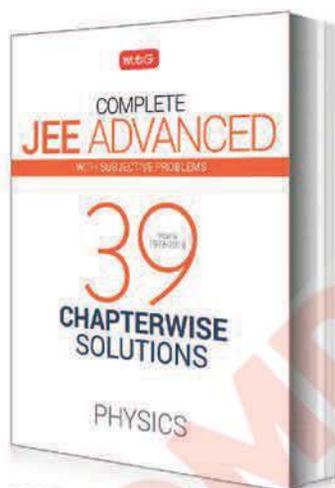
On the other hand, gases have only the volume elasticity and no shear elasticity, so only longitudinal waves can propagate in gases.

(v) A sound pulse is a combination of waves of different wavelengths. In a dispersive medium, the waves of different wavelengths travel with different speeds in different directions, *i.e.*, with different velocities. So the shape of the pulse gets distorted, *i.e.*, a plane wavefront in a non-dispersive medium does not remain a plane wavefront in a dispersive medium.

◆◆

MPP-7 | MONTHLY Practice Problems | Class XI

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

## Thermodynamics
Kinetic Theory of Gases

Total Marks : 120   Time Taken : 60 min

1. Let $\bar{v}$, v_{rms} and v_p respectively denote the mean speed, the root mean square speed and the most probable speed of the molecules in an ideal monatomic gas at absolute temperature T. The mass of a molecule is m. Then,
   (a) No molecules can have speed greater than v_{rms}.
   (b) No molecule can have speed less than $\dfrac{v_p}{\sqrt{2}}$.
   (c) $v_p < \bar{v} > v_{rms}$.
   (d) The average kinetic energy of a molecule is $\dfrac{3}{4}mv_p^2$.

2. Figure shows graph of pressure versus density for an ideal gas at two temperatures T_1 and T_2.

   (a) $T_1 > T_2$ (b) $T_1 = T_2$
   (c) $T_1 < T_2$ (d) None of these
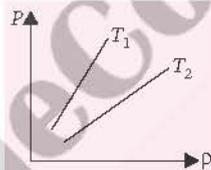
3. A cyclic process is shown on the P–T diagram.
   Which of the curves show the same process on a V–T diagram?

   (a)   (b)

   (c)   (d)
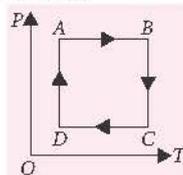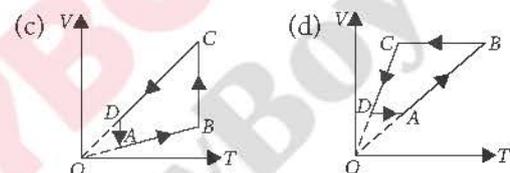
4. A Carnot engine is designed to operate between 480 K and 300 K. If the engine actually produces 1.2 J of mechanical energy per calorie of heat absorbed, then the ratio of actual efficiency to theoretical efficiency is
   (a) 16/21   (b) 21/16   (c) 5/16   (d) 16/5

5. A gas at pressure P is adiabatically compressed so that its density becomes twice that of initial value. Given that $\gamma = C_P/C_V = 7/5$, what will be the final pressure of the gas?
   (a) $2P$ (b) $1.4P$ (c) $2.64P$ (d) P

6. The root mean square velocity of the molecules in a sample of helium is (5/7)th that of the molecules in a sample of hydrogen. If the temperature of the hydrogen gas is 0 °C, that of helium sample is about
   (a) 0 °C   (b) 4 °C   (c) 273 °C   (d) 100 °C

7. One mole of an ideal gas requires 207 J heat to raise the temperature by 10 K when heated at constant pressure. If the same gas is heated at constant volume to raise the temperature by the same 10 K, the heat required is
   (Given the gas constant $R = 8.3$ J mol^{-1} K^{-1})
   (a) 198.7 J (b) 290 J   (c) 215.3 J (d) 124 J

8. At what frequency would the wavelength of sound be of the order of the mean free path in nitrogen at 1.02 atm pressure and 18.0 °C? Take the diameter of the nitrogen molecule to be 315 pm and speed of sound to be 343 m s^{-1}.
   (a) 5.36×10^9 Hz (b) 7.38×10^8 Hz
   (c) 2.88×10^8 Hz (d) 3.88×10^9 Hz

9. A thermodynamic process is shown in the figure. The pressure and volumes corresponding to some points in the figure are
   $P_A = 3 \times 10^4$ Pa; $V_A = 2 \times 10^{-3}$ m^3;
   $P_B = 8 \times 10^4$ Pa; $V_D = 5 \times 10^{-3}$ m^3.
   In the process AB, 600 J of heat is added to the system and in process BC, 200 J of heat is added to the system. The change in internal energy of the system in process AC would be
   (a) 560 J   (b) 800 J   (c) 600 J   (d) 640 J
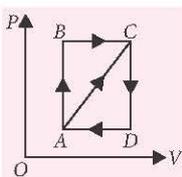
10. The internal energy of a monatomic ideal gas is 1.5 nRT. One mole of helium is kept in a cylinder of cross section 8.5 cm^2. The cylinder is closed by a light frictionless piston. The gas is heated slowly in a process during which a total of 42 J heat is given to the gas. If the temperature rises through 2 °C, find the distance moved by the piston. Atmospheric pressure = 100 kPa.
    (a) 10 cm   (b) 20 cm   (c) 30 cm   (d) 40 cm

11. A gas is expanded from volume V_0 to $2V_0$ under three different processes is shown in figure. Process 1 is isobaric, process 2 is isothermal and process 3 is adiabatic. Let ΔU_1, ΔU_2 and ΔU_3 be the change in internal energy of the gas in these three processes. Then
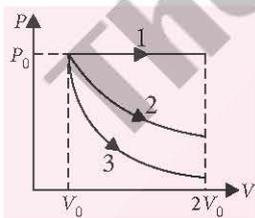
    (a) $\Delta U_1 > \Delta U_2 > \Delta U_3$ (b) $\Delta U_1 < \Delta U_2 < \Delta U_3$
    (c) $\Delta U_2 < \Delta U_1 < \Delta U_3$ (d) $\Delta U_2 < \Delta U_3 < \Delta U_1$

12. If the pressure of n mole of an ideal gas varies according to the law, $P = P_0 - aV^2$, where P_0 and a are constant, the highest temperature of the gas attained is

(a) $\dfrac{P_0}{nR}\left(\dfrac{P_0}{3a}\right)^{1/2}$ (b) $\dfrac{2P_0}{nR}\left(\dfrac{P_0}{3a}\right)^{1/2}$

(c) $\dfrac{2P_0}{3nR}\left(\dfrac{P_0}{3a}\right)^{1/2}$ (d) $\dfrac{P_0}{3nR}\left(\dfrac{P_0}{3a}\right)^{1/2}$

Assertion & Reason Type

Directions : In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as :
(a) If both assertion and reason are true and reason is the correct explanation of assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of assertion.
(c) If assertion is true but reason is false.
(d) If both assertion and reason are false.

13. **Assertion :** All molecular motion ceases at –273°C.
    Reason : Temperature below –273°C cannot be attained.

14. **Assertion :** A refrigerator transfers heat from lower temperature to higher temperature.
    Reason : Heat cannot be transferred from lower temperature to higher temperature normally.

15. **Assertion :** The total translational kinetic energy of all the molecules of a given mass of an ideal gas is 1.5 times the product of its pressure and its volume.
    Reason : The molecules of a gas collide with each other and the velocities of the molecules change due to collision.

JEE MAIN / JEE ADVANCED / PETs
Only One Option Correct Type

16. Two identical containers A and B have frictionless pistons. They contain the same volume of an ideal gas at the same temperature. The mass of the gas in A is m_A and that in B is m_B. The gas in each cylinder is now allowed to expand isothermally to double the initial volume. If the changes in the pressure in A and B are found to be ΔP and $1.5\,\Delta P$ respectively then
    (a) $4m_A = 9m_B$ (b) $2m_A = 3m_B$
    (c) $3m_A = 2m_B$ (d) $9m_A = 4m_B$

17. A smooth vertical tube having two different cross sections is open from both the ends but closed by two sliding pistons as shown in figure and tied with an inextensible string. One mole of an ideal gas is enclosed between the piston.
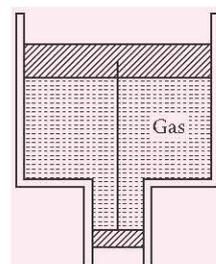
The difference in cross-sectional areas of the two pistons is given ΔS. The masses of pistons are m_1 and m_2 for larger and smaller one, respectively. Find the temperature by which the gas is raised so that the pistons will be displaced by a distance l. Take atmospheric pressure equal to P_0.

(a) $[P_0\Delta S + (m_1 + m_2)g]\dfrac{l}{R}$

(b) $[P_0\Delta S + (m_1 - m_2)g]\dfrac{l}{R}$

(c) $\dfrac{P_0\Delta S l}{R}$ (d) $\dfrac{(m_1 - m_2)gl}{R}$

18. Figure shows a process $ABCA$ performed on an ideal gas. The net heat given to the system during the process will be
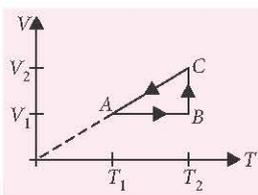
(a) $nR\left(T_2 \ln \dfrac{V_2}{V_1} - (T_2 - T_1)\right)$

(b) $nR\left(T_2 \ln \dfrac{V_2}{V_1} - (T_1 + T_2)\right)$

(c) $nR\left(T_1 \ln \dfrac{V_1}{V_2} + (T_1 - T_2)\right)$

(d) $nR\left(T_1 \ln \dfrac{V_1}{V_2} - (T_2 - T_1)\right)$

19. Assume that the temperature remains essentially constant in the upper part of the atmosphere. The mean molecular weight of air is M. An expression for the variation in pressure in the upper atmosphere with height h is (P_0 is the pressure at $h = 0$)

(a) $P = 2P_0\, e^{-Mgh/RT}$ (b) $P = P_0\, e^{-Mgh/RT}$

(c) $P = P_0\, e^{-Mgh/2RT}$ (d) $P = P_0\, e^{-2Mgh/RT}$

More than One Options Correct Type

20. A gas may expand either adiabatically or isothermally. A number of P–V curves are drawn for the two processes over different ranges of pressure and volume at different temperatures. It will be found that
(a) two adiabatic curves do not intersect
(b) two isothermal curves do not intersect
(c) an adiabatic curve and an isothermal curve may intersect
(d) the magnitude of the slope of an adiabatic curve is greater than the magnitude of the slope of an isothermal curve for the same value of pressure and volume.

21. An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass m. The piston and the cylinder have equal cross-sectional area A. When the piston is in equilibrium, the volume of the gas is V_0 and its pressure is P_0. The piston is slightly displaced from the equilibrium position and released. Then (Assuming that the system is completely isolated from its surrounding.)
(a) Piston will execute SHM.
(b) Motion of the piston will be periodic only.

(c) Frequency of motion is $\dfrac{2A\gamma}{\pi}\sqrt{\dfrac{P_0}{mV_0}}$.

(d) Frequency of motion is $\dfrac{A}{2\pi}\sqrt{\dfrac{\gamma P_0}{mV_0}}$.

22. An ideal gas whose adiabatic exponent equals γ is expanded according to the law $P = \alpha V$, where α is a constant. The initial volume of the gas is equal to V_0. As a result of expansion the volume of the gas increases η times. Then
(a) the increment of the internal energy of the gas

is $\alpha V_0^2 \dfrac{(\eta^2 - 1)}{(\gamma - 1)}$.

(b) the work performed by the gas is $0.5\,\alpha V_0^2(\eta^2 - 1)$.
(c) the molar heat capacity of the gas in the process

$R\dfrac{(\gamma + 1)}{(\gamma - 1)}$.

(d) Both (b) and (c).

23. The speeds of ten particles in m s^{-1} are 0, 1.0, 2.0, 3.0, 3.0, 3.0, 4.0, 4.0, 5.0 and 6.0.
(a) The average speed is 3.1 m s^{-1}.
(b) The root mean square speed is 3.75 m s^{-1}.
(c) The most probable speed of these particle is 3 m s^{-1}.
(d) Mean square speed is 17 m^2 s^{-2}.

Integer Answer Type

24. A vessel has 6 g of hydrogen at pressure P and temperature 500 K. A small hole is made in it so that hydrogen leaks out. If the final pressure is $P/2$ and the temperature falls to 300 K, the mass of hydrogen (in g) that leaks out is

25. A vessel contains a mixture of 1 mole of oxygen and 2 moles of nitrogen at 300 K. Find the ratio of average rotational kinetic energies per O_2 molecule to per N_2 molecule.

26. n moles of a gas in a cylinder under a piston are transferred infinitely slowly from a state with a volume of V_0 and a pressure $3P_0$ to another state with $3V_0$ and a pressure P_0 as shown in figure.
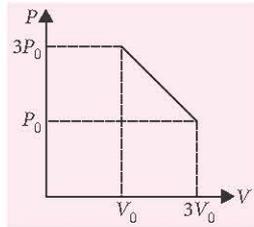
If the maximum temperature that the gas will reach in this process is $\dfrac{x P_0 V_0}{nR}$, what is the value of x?

Comprehension Type

A container of volume $4V_0$ made of a perfectly non-conducting material is divided into two equal parts by a fixed rigid wall whose lower half is non-conducting and upper half is purely conducting. The right side of the wall is divided into equal parts (initially) by means of a massless non-conducting piston free to move as shown. Section A contains 2 mol of a gas while the section B and C contain 1 mol each of the same gas ($\gamma = 1.5$) at pressure P_0. The heater in left part is switched on till the final pressure in section C becomes $125/27\, P_0$.
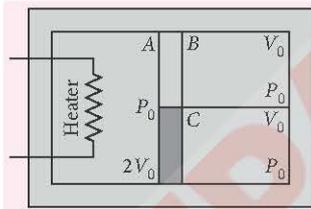
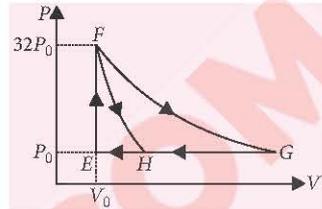**27.** Final temperature in part C is

(a) $\dfrac{P_0 V_0}{R}$ (b) $\dfrac{5 P_0 V_0}{3R}$

(c) $\dfrac{P_0 V_0}{3R}$ (d) $\dfrac{5 P_0 V_0}{R}$

28. The heat supplied by the heater is

(a) $\dfrac{368}{9} P_0 V_0$ (b) $\dfrac{113}{5} P_0 V_0$

(c) $\dfrac{316}{9} P_0 V_0$ (d) $\dfrac{405}{8} P_0 V_0$

Matrix Match Type

29. One mole of a monatomic ideal gas is taken along two cyclic processes $E \to F \to G \to E$ and $E \to F \to H \to E$ as shown in the PV diagram. The processes involved are purely isochoric, isobaric, isothermal or adiabatic. Match the paths in column I with the magnitudes of the work done in column II.
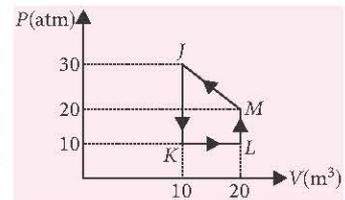
Column I	Column II
(A) $G \to E$	(P) $160\, P_0 V_0 \ln 2$
(B) $G \to H$	(Q) $36\, P_0 V_0$
(C) $F \to H$	(R) $24\, P_0 V_0$
(D) $F \to G$	(S) $31\, P_0 V_0$

	A	B	C	D
(a)	Q	R	S	P
(b)	P	Q	S	R
(c)	S	Q	P	R
(d)	S	R	Q	P

30. Heat given to process is positive, match the following option of column I with the corresponding option of column II.

Column I	Column II
(A) JK	(P) $\Delta W > 0$
(B) KL	(Q) $\Delta Q < 0$
(C) LM	(R) $\Delta W < 0$
(D) MJ	(S) $\Delta Q > 0$

	A	B	C	D
(a)	Q, S	S	P	Q
(b)	P	Q, R	Q	P, R
(c)	Q	P, S	S	Q, R
(d)	P	Q	P, S	R

❖❖

Keys are published in this issue. Search now! ☺

Newton's Law of Gravitation

Gravitational force (F) between two bodies is directly proportional to product of masses and inversely proportional to square of the distance between them.

$$\vec{F} = -\frac{Gm_1 m_2}{r^2} \cdot \hat{r}$$

Acceleration due to gravity

- For a body falling freely under gravity, the acceleration in the body is called acceleration due to gravity.
- Relationship between g and G

$$g = \frac{GM_e}{R_e^2} = \frac{4}{3}\pi G R_e \rho$$

where G = gravitational constant

ρ = density of earth

M_e and R_e be the mass and radius of earth

Characteristics of gravitational force

- It is always attractive.
- It is independent of the medium.
- It is a conservative and central force.
- It holds good over a wide range of distance.

Gravitational potential

Work done in bringing a unit mass from infinity to a point in the gravitational field.

$$V = \frac{-GM}{r}$$

Kepler's Laws of Planetary Motion

Law of orbits : Every planet revolves around the sun in an elliptical orbit and the sun is situated at one of its foci.

Law of areas : The areal velocity of the planet around the sun is constant

$$i.e., \quad \frac{dA}{dt} = \text{a constant}$$

Law of periods : The square of the time period of revolution of a planet is directly proportional to the cube of semi major axis of the elliptical orbit.

$$T^2 \propto a^3$$

Gravitational Potential Energy

Work done in bringing the given body from infinity to a point in the gravitational field.

$$U = -GMm/r$$

Escape speed

The minimum speed of projection of a body from surface of earth so that it just crosses the gravitational field of earth.

$$v_e = \sqrt{\frac{2GM}{R}}$$

Variation of acceleration due to gravity (g)

Due to altitude (h)

$$g_h = g\left(1 - \frac{2h}{R_e}\right)$$

The value of g goes on decreasing with height.

Due to depth (d)

$$g_d = g\left(1 - \frac{d}{R_e}\right)$$

The value of g decreases with depth.

Due to rotation of earth

$$g_\lambda = g - R_e \omega^2 \cos^2 \lambda$$

At equator, $\lambda = 0°$

$$g_{\lambda_{min}} = g - R_e \omega^2$$

At poles, $\lambda = 90°$

$$g_{\lambda_{max}} = g_p = g$$

Types of Satellite

Polar satellite

- Time period : 100 min
- Revolves in polar orbit around the earth.
- Height : 500-800 km.
- Uses : Weather forecasting, military spying

Geostationary satellite

- Time period : 24 hours
- Same angular speed in same direction with earth.
- Height : 36000 km.
- Uses : GPS, satellite communication (TV)

Earth's Satellite

Orbital speed of satellite

The minimum speed required to put the satellite into a given orbit.

$$v_0 = R_e \sqrt{\frac{g}{R_e + h}}$$

For satellite orbiting close to the earth's surface

$$v_0 = \sqrt{gR_e}$$

Time period of satellite

$$T = \frac{2\pi}{R_e}\sqrt{\frac{(R_e + h)^3}{g}}$$

For satellite orbiting close to the earth's surface
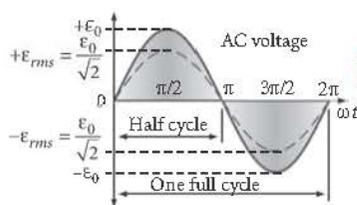
$$T = 2\pi\sqrt{\frac{R_e}{g}} = 84.6 \text{ min}$$

Energy of satellite

- Kinetic energy $K = \dfrac{GM_e m}{2(R_e + h)}$

- Potential energy $U = \dfrac{-GM_e m}{R_e + h}$

- Total energy

$$E = K + U = -\frac{GM_e m}{2(R_e + h)}$$

AC voltage

$+\varepsilon_0$
$+\varepsilon_{rms} = \dfrac{\varepsilon_0}{\sqrt{2}}$

$-\varepsilon_{rms} = \dfrac{\varepsilon_0}{\sqrt{2}}$
$-\varepsilon_0$

Half cycle — One full cycle

Alternating Current
Current which changes continuously in magnitude and periodically in direction.

Transformer
- Transformer ratios
$$\frac{\varepsilon_S}{\varepsilon_P} = \frac{I_P}{I_S} = \frac{N_S}{N_P} = k$$
- Efficiency of a transformer,
$$\eta = \frac{\text{output power}}{\text{input power}} = \frac{\varepsilon_S I_S}{\varepsilon_P I_P}.$$

Step-up transformer, $\varepsilon_S > \varepsilon_P$, $I_S < I_P$ and $N_S > N_P$.

Step-down transformer, $\varepsilon_S < \varepsilon_P$, $I_S > I_P$ and $N_S < N_P$.

Alternating voltage
$\varepsilon = \varepsilon_0 \sin \omega t$

Applied across capacitor

Applied across resistor

Applied across inductor

Purely capacitive circuit
Current leads the voltage by a phase angle of $\pi/2$.
$I = I_0 \sin(\omega t + \pi/2)$; $I_0 = \dfrac{\varepsilon_0}{X_C} = \omega C \varepsilon_0$
where $X_C = 1/\omega C$

Purely resistive circuit
Alternating voltage is in phase with current.
$I = \varepsilon/R = I_0 \sin\omega t$

Purely inductive circuit
Current lags behind the voltage by a phase angle of $\pi/2$.
$I = I_0 \sin(\omega t - \pi/2)$; $I_0 = \varepsilon_0/X_L = \varepsilon_0/\omega L$
where $X_L = \omega L$

Combining LCR in series

Power in ac circuit
Average power (P_{av})
$$P_{av} = \varepsilon_{rms} I_{rms} \cos\phi$$
$$= \frac{\varepsilon_0 I_0}{2} \cos\phi$$

Series LCR circuit
- $\varepsilon = \varepsilon_0 \sin\omega t$, $I = I_0 \sin(\omega t - \phi)$
- Impedance of the circuit: $Z = \sqrt{R^2 + (X_L - X_C)^2}$
- Phase difference between current and voltage is ϕ
$$\tan\phi = \frac{X_L - X_C}{R}$$
- For $X_L > X_C$, ϕ is +ve. (Predominantly inductive)
- For $X_L < X_C$, ϕ is −ve. (Predominantly capacitive)

Resonant series LCR circuit
When $X_L = X_C$, $Z = R$, current becomes maximum.
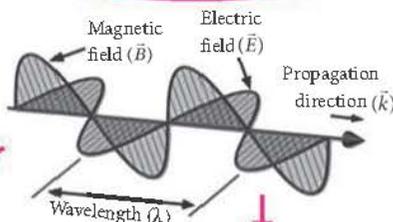Resonant frequency $\omega_r = \dfrac{1}{\sqrt{LC}}$

Quality factor
It is a measure of sharpness of resonance.
$$\therefore \quad Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Power factor
- **Power factor:** $\cos\phi = \dfrac{R}{Z}$
- In pure resistive circuit, $\phi = 0°$; $\cos\phi = 1$
- In purely inductive or capacitive circuit
$\phi = \pm\dfrac{\pi}{2}$; $\cos\phi = 0$
- In series LCR circuit,
At resonance, $X_L = X_C$
$\therefore Z = R$ and $\phi = 0°$, $\cos\phi = 1$

Electromagnetic Waves
Waves having sinusoidal variation of electric and magnetic field at right angles to each other and perpendicular to direction of waves propagation.

Displacement current
Displacement current arises wherever the electric flux is changing with time.
$I_D = \varepsilon_0 \, d\phi_E/dt$

Magnetic field $(\vec{B})$ — Electric field $(\vec{E})$ — Propagation direction $(\vec{k})$ — Wavelength (λ)

Maxwell's equations
$$\oint \vec{E} \cdot d\vec{S} = \frac{q}{\varepsilon_0} \quad \text{(Gauss's law for electrostatics)}$$
$$\oint \vec{B} \cdot d\vec{S} = 0 \quad \text{(Gauss's law for magnetism)}$$
$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt} \quad \text{(Faraday's law of electromagnetic induction)}$$
$$\oint \vec{B} \cdot d\vec{l} = \mu_0\left(I + \varepsilon_0 \frac{d\phi_E}{dt}\right) \text{(Maxwell-Ampere's circuital law)}$$

Energy density of electromagnetic waves
Average energy density
$$<u> = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{1}{2}\frac{B_0^2}{\mu_0}$$
Intensity of electromagnetic wave $= \dfrac{1}{2}\varepsilon_0 E_0^2 c$

Production of electromagnetic waves
- Through accelerating charge
- By harmonically oscillating electric charges.
- Through oscillating electric dipoles.

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Unit 7

ELECTRONIC DEVICES COMMUNICATION SYSTEMS

☞ **Energy Bands in Solids**

- Range of energy possessed by electron in a solid is known as energy bands.
- There are two types of energy band

  ➢ **Valence band** : Range of energy possessed by valence electron is known as valence band. These electrons are bounded and not responsible for flow of current.

  ➢ **Conduction band** : Range of energy possessed by free electron is known as conduction band. These electrons are responsible for flow of current.

- Forbidden energy gap (E_g) : It is the energy gap between the bottom of the conduction band and top of the valence band. No electron exist in this gap.
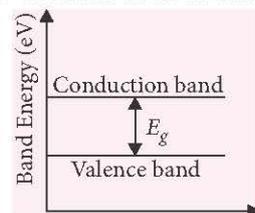
  ➢ Width of forbidden energy gap depends upon the nature of substance.

  ➢ If width is more then valence electrons are strongly attached with nucleus.

  ➢ As temperature increases, forbidden energy gap decreases (very slightly).

- According to energy band theory there are three types of solids.

	Property	Conductors	Semiconductors	Insulators
1.	Electrical conductivity and its value	Very high, $10^2 – 10^8$ S m^{-1}	Between those of conductors and insulators $10^5 – 10^{-6}$ S m^{-1}	Negligible, $10^{-11} – 10^{-19}$ S m^{-1}
2.	Resistivity and its value	Negligible $10^{-2} – 10^{-8}$ Ω m	Between those of conductors and insulators $10^{-5} – 10^6$ Ω m	Very high, $10^{11} – 10^{19}$ Ω m
3.	Energy gap and its value	Zero or very small	More than that in conductors but less than that in insulators For Ge, $E_g = 0.72$ eV; for Si, $E_g = 1.1$ eV; for GaAs, $E_g = 1.3$ eV	Very large For diamond, $E_g = 7$ eV
4.	Current carriers	Free electrons	Free electrons and holes	Free electrons