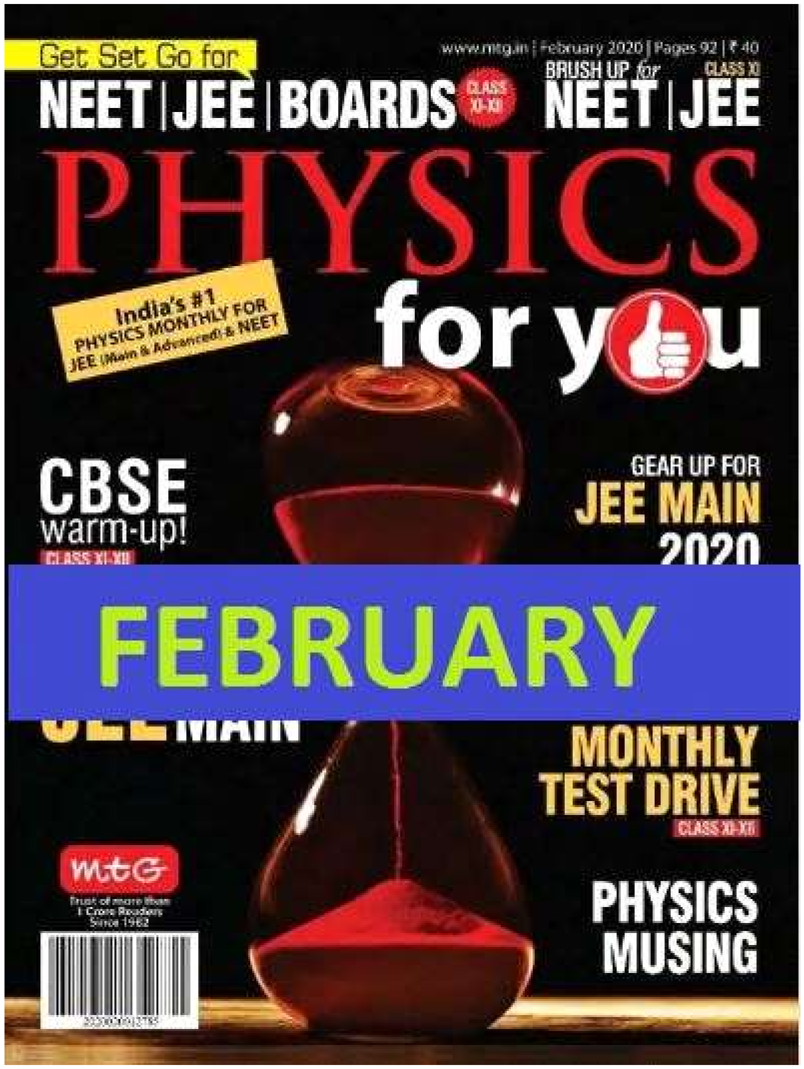

MTG PHYSICS FOR YOU FEBRUARY 2020

18. (d): This is a problem based on superposition of fields due to more than one conductor.

B_1, field due to round part $= \frac{\mu_0 I}{2R} \times \frac{90°}{360°} = \frac{\mu_0 I}{8R}$
(along negative direction of x-axis)

B_2, field due to linear part 1 in the x-y plane

$$= \frac{1}{2}\frac{\mu_0 I}{2\pi R}$$

(halved because field at the end due to a long conductor is half of the field at the middle part.)

$$= \frac{\mu_0 I}{4\pi R} \quad \text{(along negative } z\text{-axis)}$$

B_3, field due to linear part 2 in yz-plane

$$= \frac{\mu_0 I}{4\pi(2R)} \quad \text{(along negative } x\text{-axis)}$$

$\therefore$ Vector component of B along z-axis $= \frac{-\mu_0 I}{4\pi R}\hat{k}$
Vector component of B along x-axis

$$= -\left(\frac{\mu_0 I}{8R} + \frac{\mu_0 I}{8\pi R}\right)\hat{i}$$

Vector component of B along y-axis = 0
Hence the resultant of B is in the xz plane and

$$\tan\theta_x = \frac{\frac{\mu_0 I}{8R}\left(1+\frac{1}{\pi}\right)}{\frac{\mu_0 I}{4\pi R}} = \frac{1}{2}(1+\pi) \text{ or } \theta_x = 64°, 244°$$

Since both base and perpendicular are negative
$\therefore$ $\theta_x = 244°$ acceptable

$$\text{Modulus of the field} = \frac{\mu_0 I}{4R}\sqrt{\left(\frac{1}{\pi}\right)^2 + \frac{1}{4}\left(1+\frac{1}{\pi}\right)^2}$$

$$= \frac{\mu_0 I}{8\pi R}\sqrt{\pi^2 + 2\pi + 5}$$

$$\text{Here } B = \frac{4\pi \times 10^{-7} \times 10}{8\pi \times 0.1}\sqrt{\pi^2 + 2\pi + 5} = 23\,\mu\text{T}$$

19. (b): Mass of nucleus, $M = 20$ u
$E_\gamma = 6$ MeV $= 6 \times 1.6 \times 10^{-13}$ J
Using momentum conservation principle,

$$0 = Mv + \frac{E_\gamma}{c},$$

$$v = -\frac{E_\gamma}{Mc} = -\frac{6 \times 1.6 \times 10^{-13}}{20 \times 1.6 \times 10^{-27} \times 3 \times 10^8} = -10^5 \text{ m s}^{-1}$$

K.E. of nucleus, $K = \frac{1}{2}Mv^2$

$$= \frac{1}{2} \times 20 \times 1.6 \times 10^{-27} \times (-10^5)^2$$

$$= 16 \times 10^{-17}\text{ J} = \frac{16 \times 10^{-17}}{1.6 \times 10^{-16}}\text{ keV} = 1\text{ keV}$$

20. (a): Considering the free-body diagram of the piece of ice and its motion along and perpendicular to the plane $mg\sin\theta - f_{lim} = ma$
Where a is the acceleration down the plane
$mg\cos\theta = N$
But $f_{lim} = \mu N = \mu\, mg\cos\theta$
$\therefore$ $mg\sin\theta - \mu\, mg\cos\theta = ma$

$$\text{or } a = g\sin\theta - \mu g\cos\theta = g\left(\frac{1}{\sqrt{2}} - \frac{\mu}{\sqrt{2}}\right) = \frac{g}{\sqrt{2}}(1-\mu)$$

When the plane is frictionless, $\mu = 0$

$$\text{Thus, } a = g\sin\theta = g\sin 45° = \frac{g}{\sqrt{2}}$$

Let t_1 be the time taken by the ice to slide down the frictional plane and t_2 down the frictionless plane.

Then s (length of the plane), $s = \frac{1}{2}\frac{g}{\sqrt{2}}(1-\mu)t_1^2$

and $s = \frac{1}{2}\frac{g}{\sqrt{2}}t_2^2$

$$\therefore \frac{1}{2}\frac{g}{\sqrt{2}}(1-\mu)t_1^2 = \frac{1}{2}\frac{g}{\sqrt{2}}t_2^2$$

But $t_1 = 2t_2$

$$\therefore (1-\mu)4t_2^2 = t_2^2 \text{ or } 1-\mu = \frac{1}{4}$$

$$\text{or } \mu = 1 - \frac{1}{4} = \frac{3}{4} = 0.75$$

21. (0.34): Here, $m = 250$ g $= 0.25$ kg, $k = 85$ N m^{-1}
$b = 70$ g s^{-1} $= 0.07$ kg s^{-1}

$$\therefore \frac{k}{m} = \frac{85\text{ N m}^{-1}}{0.25\text{ kg}} = 340\text{ s}^{-2}$$

$$\text{and } \frac{b^2}{4m^2} = \frac{(0.07\text{ kg s}^{-1})^2}{4(0.25\text{ kg})^2} = 0.02\text{ s}^{-2}$$

$$\text{As } \frac{k}{m} >> \frac{b^2}{4m^2}$$

$\therefore$ Angular frequency of damped oscillator

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} \simeq \sqrt{\frac{k}{m}}$$

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$\therefore$ Time period of motion

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 2\times\frac{22}{7}\sqrt{\frac{0.25\ \text{kg}}{85\ \text{N m}^{-1}}} = 0.34\ \text{s}$$

22. (4650.4) : By conservation of momentum

$$(M+m)v = M\sqrt{2gh} \quad \text{or} \quad v = \frac{M\sqrt{2gh}}{M+m}$$

By work-energy theorem, $-R\delta + (M+m)g\delta$

$$= 0 - \frac{1}{2}(M+m)v^2$$

or $$R = \frac{1}{2}(M+m)\frac{M^2 2gh}{(M+m)^2\delta} + (M+m)g$$

$$= \frac{M^2 gh}{(M+m)\delta} + (M+m)g$$

or $$R = \frac{10^2\times 9.8\times 5}{(10+3)\times 0.05} + (10+3)\,9.8$$

$$= 4523 + 127.4 = 4650.4\ \text{N}$$

23. (4) : The pressure in both the columns is same.

$$l_{50}\times\rho_{50}\times g = l_{100}\,\rho_{100}\times g$$

$$(50\ \text{cm})\times\rho_{50}\times g = (60\ \text{cm})\times\rho_{100}\times g \Rightarrow \frac{\rho_{100}}{\rho_{50}} = \frac{5}{6}$$

But $$\rho_{100} = \frac{\rho_{50}}{(1+\gamma\Delta T)}$$

$$\Rightarrow 1+\gamma\Delta T = \frac{6}{5} \quad \text{or} \quad \gamma\Delta T = \frac{1}{5}$$

$$\gamma = \frac{1}{5\Delta T} = \frac{1}{5(100-50)} = \frac{1}{250} = 0.004\ ^\circ\text{C}^{-1} \Rightarrow x = 4.$$

24. (1.6) : Let there by n atoms per unit volume.

$$u\ (\text{drift velocity}) = \frac{I}{ne}$$

$\therefore$ $l = ut$ and $s = \text{speed}\times t$

$$\langle v\rangle\ (\text{average speed}) = \sqrt{\frac{3kT}{m_e}}$$

$$s = \langle v\rangle t, \quad \therefore\ s = l\,\frac{\langle v\rangle}{u}$$

M kg of copper contains N atoms $= N$ electrons

$\therefore$ n = number of atoms in ρ kg $= \dfrac{N\rho}{M}$

$$\therefore\ u = \frac{IM}{N\rho e} = \frac{10^3\times 63.546}{6.02\times10^{26}\times 8900\times 1.6\times10^{-19}}$$

$$= 7.4\times10^{-5}\ \text{m s}^{-1}$$

$$\langle v\rangle = \sqrt{\frac{3\times1.38\times10^{-23}\times300}{9.1\times10^{-31}}} = 1.17\times10^5\ \text{m s}^{-1}$$

$$\therefore\ s = 10^{-3}\times\frac{1.17\times10^5}{7.4\times10^{-5}} = 1.6\times10^6\ \text{m} = 1.6\times10^3\ \text{km}$$

25. (90) : Here $\phi_B = \displaystyle\int_{x=0}^{x=a} a\,dx\,B(x,t)$

$$\therefore\ \frac{d\phi_B}{dt} = \frac{d}{dt}\int_0^a a\,dx\,B(x,t) = a\int_0^a dx\,\frac{dB}{dt}$$

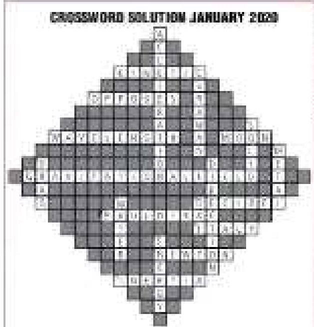
Now by the rule of partial differentiation

$$\frac{dB}{dt} = \frac{\partial B}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial B}{\partial t}\cdot\frac{dt}{dt} = v\frac{\partial B}{\partial x} + \frac{\partial B}{\partial t}$$

$$\therefore\ \varepsilon\frac{d\phi_B}{dt} = a\int_0^a\left(v\frac{\partial B}{\partial x} + \frac{\partial B}{\partial t}\right)dx = a^2\left(v\frac{\partial B}{\partial x} + \frac{\partial B}{\partial t}\right)$$

$$= 0.1^2\,(0.08\times0.1 + 10^{-3}) = 9\times10^{-5}\ \text{V} = 90\ \mu\text{V}$$

CROSSWORD SOLUTION JANUARY 2020



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1. Ashutosh Mohapatra, Ambala Cantt (Haryana)
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3. Shitali Bhagat
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5. Shivanshi Sharma, Kangra (H.P.)
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7. Pooja Kaushal (Mumbai)
8. Akshita Singh, Patna (Bihar)
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10. Prem Bali, Rajouri (J & K)
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16. Abdul H Shaikh, Mangalore (Karnataka)
17. Vikash Meena, Sirohi (Rajasthan)
18. Mukul Prajapat (Rajasthan)
19. Arun Bandyopadhyay, Gwalior (M.P.)

$\frac{m}{k} = \frac{1}{400\pi^2}$

$v_{max} = A\omega = \frac{g}{400\pi^2} \times 20\pi = \frac{1}{2\pi}$ m s^{-1}

8. (d) : At $x_1 = \frac{\pi}{3k}$ and $x_2 = \frac{3\pi}{2k}$, $\sin kx_1$ or $\sin kx_2$ is not zero.
Therefore, neither x_1 nor x_2 is a node.

$\Delta x = x_2 - x_1 = \left(\frac{3}{2} - \frac{1}{3}\right)\frac{\pi}{k} = \frac{7\pi}{6k}$

since $\frac{2\pi}{k} > \Delta x > \frac{\pi}{k}$

or $\lambda > \Delta x > \frac{\lambda}{2}\left(k = \frac{2\pi}{\lambda}\right)$

Therefore, $\phi_1 = \pi$

and $\phi_2 = k.\Delta x = \frac{7\pi}{6}$

$\therefore \frac{\phi_1}{\phi_2} = \frac{6}{7}$

9. (a) : Let x_1 and x_2 be the distances of the two positions from centre.
Then with usual notations

$u^2 = \omega^2 (A^2 - x_1^2)$ ...(i)

$v^2 = \omega^2 (A^2 - x_2^2)$ ...(ii)

$a = -\omega^2 x_1$ ...(iii)

$b = -\omega^2 x_2$ ...(iv)

Subtracting eqn. (ii) from eqn. (i),

$u^2 - v^2 = \omega^2 (x_2^2 - x_1^2)$ ...(v)

Adding eqs. (iii) and (iv),

$a + b = -\omega^2 (x_1 + x_2)$ ...(vi)

Dividing eqn. (v) by eqn. (vi) $\frac{u^2 - v^2}{a + b} = x_1 - x_2$

10. (c) : $n = \frac{1}{2L}\sqrt{\frac{T}{m}} = \frac{1}{2Lr}\sqrt{\frac{T}{\pi\rho}}$ ...(i)

as $m = \pi r^2 \rho$

Here L remains constant when the bridges are separated by a fixed distance. When tension is increased from T to T', the radius decreases from r to r', as the wire is not exactly elastic. Hence, the frequency n' is

$n' = \frac{1}{2Lr'}\sqrt{\frac{T'}{\pi\rho}}$ ...(ii)

$\therefore \frac{n'}{n} = \left(\sqrt{\frac{T'}{T}}\right)\left(\frac{r}{r'}\right)$

Here $\frac{T'}{T} = 4$ and $\frac{r'}{r}$ is slightly smaller than one

i.e., $\left(\frac{r}{r'}\right) > 1 \Rightarrow \frac{n'}{n} = 2\left(\frac{r}{r'}\right) \quad \therefore \frac{n'}{n} > 2$

11. (b) : $v = \omega\sqrt{A^2 - x^2} \Rightarrow$ or $x^2 = A^2 - \frac{v^2}{\omega^2}$ ...(i)

As $a = -\omega^2 x \Rightarrow x^2 = \frac{a^2}{\omega^4}$ ...(ii)

Equating eqns. (i) and (ii) we get.

$$A^2 - \frac{v^2}{\omega^2} = \frac{a^2}{\omega^4} \Rightarrow v^2 = \left(-\frac{1}{\omega^2}\right)a^2 + A^2\omega^2$$

12. (c)

13. (d) : $T = 2\pi\sqrt{\frac{I_{O'}}{mgd}}$

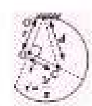
$I_{O'} = I_C + mr'^2$

$\Rightarrow mr^2 = I_C + mr'^2$

$\Rightarrow I_C = m(r^2 - r'^2)$

$I_{O'} = I_C + m(r^2 - r'^2) + m(r^2 + r'^2) = 2mr^2$

$d = \sqrt{r^2 + \left(\frac{2r}{\pi}\right)^2} = r\sqrt{1 + \frac{4}{\pi^2}}$



$\Rightarrow T = 2\pi\sqrt{\frac{2r}{g\left(1 + \frac{4}{\pi^2}\right)^{\frac{1}{2}}}}$

14. (a) : $y_i = a\sin\frac{2\pi}{\lambda}(vt - x)$

and $y_r = -a\sin\frac{2\pi}{\lambda}(vt + x)$

$\therefore y = y_i + y_r = -2a\sin\frac{2\pi x}{\lambda}\cos\frac{2\pi vt}{\lambda}$

Now amplitude A of stationary wave is given

$A = -2a\sin\frac{2\pi x}{\lambda}$

Here, $\lambda = 120$ cm, when $x = 10$ cm.

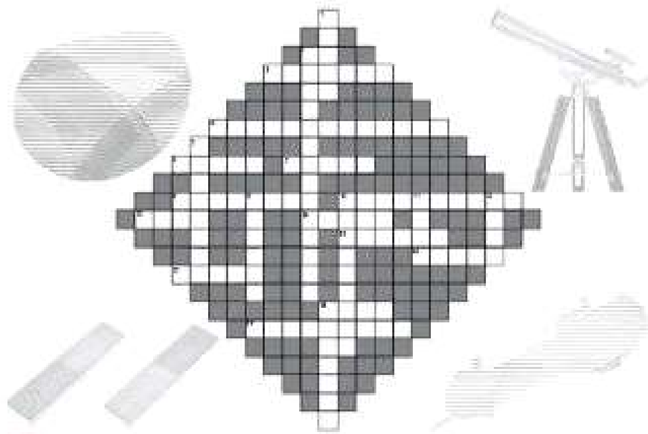
Then $A' = -2a\sin 2\pi\left(\frac{10}{120}\right) = -2a\sin\frac{2\pi}{12} = -a$

When $x = 30$ cm, then

$A'' = -2a\sin 2\pi\left(\frac{30}{120}\right) = -2a \quad \therefore \frac{A'}{A''} = \frac{-a}{-2a} = \frac{1}{2}$

CROSS WORD

Readers can send solutions at editor@mtg.in or post us with their complete address by 30th of every month to get their names published in next issue.



DOWN

1. SI unit of frequency (5)
2. The first scientist who uses optical telescope systematically to study the night sky (7, 7)
5. Product of mass and velocity (8)
9. A piece of material that can attract and pick up other metal objects (6)
10. The detector of radioactivity (1, 1,7)
11. Prefix used in the metric system to denote 10^{-6} (4)
12. Difference of potential between two points on a current carrying conductor (4)
18. Instrument that measures electric unit (7)

ACROSS

3. Branch of mechanics, deals with the study of objects at rest or in equilibrium (7)
4. Measures components of solar radiation (13)
6. It is pentavalent impurity (5)
7. A moving component of an electromagnetic system in electric motor (5)
8. The region in which partial light reaches (8)
10. A radio wave that travels in approximately a straight line between points on the Earth's surface (6, 4)
13. It is the unit of loudness (4)
14. Unit of luminous flux (5)
15. The luminous plasma atmosphere of the Sun (6)
16. The path through space of one celestial body about another (5)
17. A representation of a physical object formed by a lens or mirror (5)
18. Smallest part of an element that can exist (4)
19. A device used for measuring pressure difference in two liquid columns (9)

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Vector component of B along x-axis

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Vector component of B along y-axis $= 0$

Hence the resultant of B is in the xz plane and

$$\tan\theta_x = \frac{\frac{\mu_0 I}{8R}\left(1+\frac{1}{\pi}\right)}{\frac{\mu_0 I}{4\pi R}} = \frac{1}{2}(1+\pi) \text{ or } \theta_x = 64°, 244°$$

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When the plane is frictionless, $\mu = 0$

Thus, $a = g\sin\theta = g\sin 45° = \frac{g}{\sqrt{2}}$

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$$=\frac{M^2gh}{(M+m)\delta}+(M+m)g$$

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$$=4523+127.4=4650.4\ \text{N}$$

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Now by the rule of partial differentiation

$$\frac{dB}{dt}=\frac{\partial B}{\partial x}\cdot\frac{dx}{dt}+\frac{\partial B}{\partial t}\cdot\frac{dt}{dt}=v\frac{\partial B}{\partial x}+\frac{\partial B}{\partial t}$$

$$\therefore\ \varepsilon\frac{d\phi_B}{dt}=a\int_0^a\left(v\frac{\partial B}{\partial x}+\frac{\partial B}{\partial t}\right)dx=a^2\left(v\frac{\partial B}{\partial x}+\frac{\partial B}{\partial t}\right)$$

$$=0.1^2\,(0.08\times0.1+10^{-3})=9\times10^{-5}\ \text{V}=90\ \mu\text{V}$$

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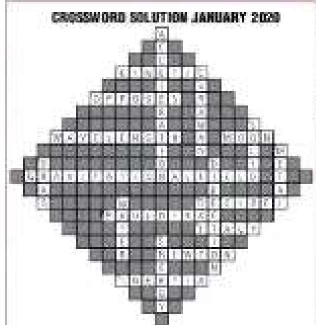

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9. Anhad Sharma
10. Pavan Bali, Rajouri (J & K)
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$\frac{m}{k} = \frac{1}{400\pi^2}$

$v_{max} = A\omega = \frac{g}{400\pi^2} \times 20\pi = \frac{1}{2\pi}\ \text{m s}^{-1}$

8. (d): At $x_1 = \frac{\pi}{3k}$ and $x_2 = \frac{3\pi}{2k}$, $\sin kx_1$ or $\sin kx_2$ is not zero.

Therefore, neither x_1 nor x_2 is a node.

$\Delta x = x_2 - x_1 = \left(\frac{3}{2} - \frac{1}{3}\right)\frac{\pi}{k} = \frac{7\pi}{6k}$

since $\frac{2\pi}{k} > \Delta x > \frac{\pi}{k}$

or $\lambda > \Delta x > \frac{\lambda}{2}\left(k = \frac{2\pi}{\lambda}\right)$

Therefore, $\phi_1 = \pi$

and $\phi_2 = k\,\Delta x = \frac{7\pi}{6}$

$\therefore \frac{\phi_1}{\phi_2} = \frac{6}{7}$

9. (a): Let x_1 and x_2 be the distances of the two positions from centre.

Then with usual notations

$u^2 = \omega^2 (A^2 - x_1^2)$ ...(i)

$v^2 = \omega^2 (A^2 - x_2^2)$ ...(ii)

$a = -\omega^2 x_1$ ...(iii)

$b = -\omega^2 x_2$ ...(iv)

Subtracting eqn. (ii) from eqn. (i),

$u^2 - v^2 = \omega^2 (x_2^2 - x_1^2)$ ...(v)

Adding eqs. (iii) and (iv),

$a + b = -\omega^2 (x_1 + x_2)$ ...(vi)

Dividing eqn. (v) by eqn. (vi) $\frac{u^2 - v^2}{a + b} = x_1 - x_2$

10. (c): $n = \frac{1}{2L}\sqrt{\frac{T}{m}} = \frac{1}{2Lr}\sqrt{\frac{T}{\pi\rho}}$ ...(i)

as $m = \pi r^2 \rho$

Here L remains constant when the bridges are separated by a fixed distance. When tension is increased from T to T', the radius decreases from r to r', as the wire is not exactly elastic. Hence, the frequency n' is

$n' = \frac{1}{2Lr'}\sqrt{\frac{T'}{\pi\rho}}$ ...(ii)

$\therefore \frac{n'}{n} = \left(\sqrt{\frac{T'}{T}}\right)\left(\frac{r}{r'}\right)$

Here $\frac{T'}{T} = 4$ and $\frac{r'}{r}$ is slightly smaller than one

i.e., $\left(\frac{r}{r'}\right) > 1 \Rightarrow \frac{n'}{n} = 2\left(\frac{r}{r'}\right) \quad \therefore \frac{n'}{n} > 2$

11. (b): $v = \omega\sqrt{A^2 - x^2} \Rightarrow$ or $x^2 = A^2 - \frac{v^2}{\omega^2}$ ...(i)

As $a = -\omega^2 x \Rightarrow x^2 = \frac{a^2}{\omega^4}$ ...(ii)

Equating eqns. (i) and (ii) we get.

$$A^2 - \frac{v^2}{\omega^2} = \frac{a^2}{\omega^4} \Rightarrow v^2 = \left(-\frac{1}{\omega^2}\right)a^2 + A^2\omega^2$$

12. (c)

13. (d): $T = 2\pi\sqrt{\frac{I_{O'}}{mgd}}$

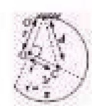
$I_{O'} = I_C + mr'^2$

$\Rightarrow mr^2 = I_C + mr'^2$

$\Rightarrow I_C = m(r^2 - r'^2)$

$I_{O'} = I_C + m(r^2 - r'^2) + m(r^2 + r'^2) = 2mr^2$

$d = \sqrt{r^2 + \left(\frac{2r}{\pi}\right)^2} = r\sqrt{1 + \frac{4}{\pi^2}}$



$\Rightarrow T = 2\pi\sqrt{\frac{2r}{g\left(1 + \frac{4}{\pi^2}\right)^{\frac{1}{2}}}}$

14. (a): $y_i = a\sin\frac{2\pi}{\lambda}(vt - x)$

and $y_r = -a\sin\frac{2\pi}{\lambda}(vt + x)$

$\therefore y = y_i + y_r = -2a\sin\frac{2\pi x}{\lambda}\cos\frac{2\pi vt}{\lambda}$

Now amplitude A of stationary wave is given

$A = -2a\sin\frac{2\pi x}{\lambda}$

Here, $\lambda = 120$ cm, when $x = 10$ cm.

Then $A' = -2a\sin 2\pi\left(\frac{10}{120}\right) = -2a\sin\frac{2\pi}{12} = -a$

When $x = 30$ cm, then

$A'' = -2a\sin 2\pi\left(\frac{30}{120}\right) = -2a \quad \therefore \frac{A'}{A''} = \frac{-a}{-2a} = \frac{1}{2}$

15. (d): $k_1 = 2k\sin^2\theta$
and $k_2 = 2(2k)\sin^2\beta$

Then, $k_{eq} = k_1 + k_2 = 2k\,[\sin^2\theta + 2\sin^2\beta]$

$= 2k[\sin^2 45^\circ + 2\sin^2 30^\circ] = 2k\left(\frac{1}{2} + \frac{1}{2}\right) = 2k$

Then $T = 2\pi\sqrt{\frac{m}{k_{eq}}} = 2\pi\sqrt{\frac{m}{2k}} = \pi\sqrt{\frac{2m}{k}}$

16. (c)

17. (401): $v = \sqrt{(\gamma RT/M)}$
Molecular weight of the mixture

$= \frac{\mu_1 M_1 + \mu_2 M_2}{\mu_1 + \mu_2} = \frac{1\times 4 + 2\times 32}{1+2} = \frac{68}{3}\times 10^{-3}\ \text{kg mole}^{-1}$

$(C_v)_{mix} = \frac{\mu_1 C_{v_1} + \mu_2 C_{v_2}}{\mu_1 + \mu_2} = \frac{1\times\frac{3R}{2} + 2\left(\frac{5R}{2}\right)}{1+2} = \frac{13R}{6}$

$(C_p)_{mix} = (C_v)_{mix} + R = (19R/6)$

$\therefore\ \gamma_{mix} = \frac{(C_p)_{mix}}{(C_v)_{mix}} = (19/13)$

$\therefore\ v = \sqrt{(\gamma RT/M)} = \sqrt{\frac{19}{13}\times\frac{8.31\times 300}{\left(\frac{68}{3}\times 10^{-3}\right)}} = 401\ \text{m s}^{-1}$.

18. (1.2): $f \propto \sqrt{T}$

$\frac{f_{air}}{f_{water}} = \sqrt{\frac{w_{air}}{w_{water}}} = \sqrt{\frac{V\rho g}{V\rho g - V\rho_w g}}$

or $\frac{f}{f/2} = \sqrt{\frac{\rho}{\rho - \rho_w}} \Rightarrow 2 = \sqrt{\frac{\rho}{\rho - \rho_w}}$

$\Rightarrow 4\rho - 4\rho_w = \rho \Rightarrow \rho = \frac{4}{3}\rho_w$ ...(i)

Similarly in second case

$\frac{f}{f/3} = \sqrt{\frac{\rho}{\rho - \rho_L}} \Rightarrow 3 = \sqrt{\frac{\frac{4}{3}\rho_w}{\frac{4}{3}\rho_w - \rho_L}} = \sqrt{\frac{4}{4 - 3\frac{\rho_L}{\rho_w}}}$

Here, $\frac{\rho_L}{\rho_w}$ = specific gravity (say s)

$\therefore\ 9 = \frac{4}{4-3s} \Rightarrow s = \frac{32}{27} = 1.2$

19. (1.4): $U = \int(-2x - x^3)\,dx = x^2 + \frac{x^4}{4}$

Since there is no loss of energy, so

$\frac{x^4}{4} + x^2 = E_i = \frac{1}{2}mv^2 = 3$

$x^4 + 4x^2 - 12 = 0$

$\Rightarrow\ x^2 = \frac{-4 \pm\sqrt{16+48}}{2} = \frac{-4\pm 8}{2} = -6, +2$

$\Rightarrow\ x = \sqrt{2} = 1.41$ m

20. (826): $V_s\cos\theta$ = component of $\vec{v}_s$ along $\vec{r}$

$= \frac{\vec{v}_s\cdot\vec{r}}{r}$

After 2 s: $\vec{v}_s = 60\hat{i} + 40\hat{j}$

$\vec{r} = (60\times 2)\hat{i} + \left(60\times 2 - \frac{1}{2}\times 10\times 4\right)\hat{j} = 120\hat{i} + 100\hat{j}$

$\vec{v}_s\cdot\vec{r} = (120\times 60) + (40\times 100) = 11200\ \text{m}^2\,\text{s}^{-1}$

$r = \sqrt{(120)^2 + (100)^2} = 156.2$ m

$\therefore\ v_s\cos\theta = 71.7\ \text{m s}^{-1}$

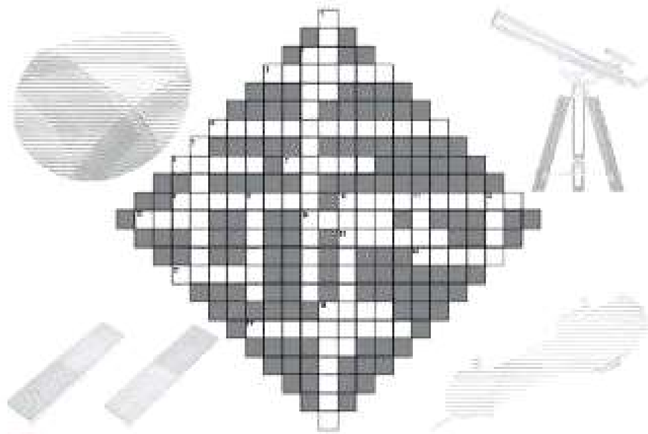
Since, $v_s\cos\theta$ is +ve *i.e.*, distance between S and O is increasing. Hence,

$f' = 1000\left(\frac{340}{340 + 71.7}\right) = 826$ Hz

COMIC CAPSULE

CROSS WORD

Readers can send solutions at editor@mtg.in or post us with their complete address by 10th of every month to get their names published in next issue.



DOWN

1. SI unit of frequency (5)
2. The first scientist who used optical telescope systematically to study the night sky (7, 7)
5. Product of mass and velocity (8)
9. A piece of material that can attract and pick up other metal objects (6)
10. The detector of radioactivity (1, 1,7)
11. Prefix used in the metric system to denote 10^{-9} (4)
12. Difference of potential between two points on a current carrying conductor (4)
18. Instrument that measures electric unit (7)

ACROSS

3. Branch of mechanics, deals with the study of objects at rest or in equilibrium (7)
4. Measures components of solar radiation (13)
6. It is pentavalent impurity (5)
7. A moving component of an electromagnetic system in electric motor (5)
8. The region in which partial light reaches (8)
10. A radio wave that travels in approximately a straight line between points on the Earth's surface (6, 4)
13. It is the unit of loudness (4)
14. Unit of luminous flux (5)
15. The luminous plasma atmosphere of the Sun (6)
16. The path through space of one celestial body about another (5)
17. A representation of a physical object formed by a lens or mirror (5)
18. Smallest part of an element that can exist (4)
19. A device used for measuring pressure difference in two liquid columns (9)